

3D Vision: Coordinate Spaces



A lot of slides from Noah Snively +
Shree Nayar's YT series: First principals of Computer Vision

CS180: Intro to Computer Vision and Comp. Photo
Angjoo Kanazawa & Alexei Efros, UC Berkeley, Fall 2023

House keeping

Project 4 Part 1 due tomorrow

Final Project

Easy path: Pre-canned

- Group of 1 : 2 projects
- Group of 2 : 3 projects

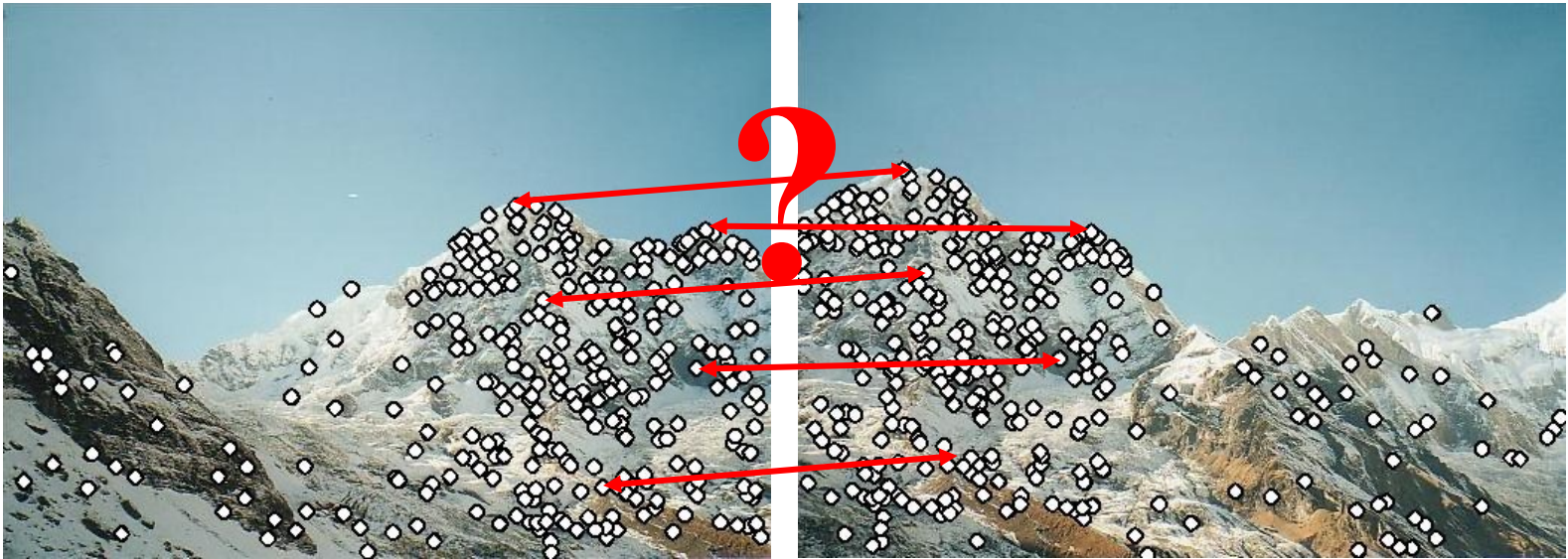
Grad students: Your own project

- 1 page Proposal with pictures **due 11/9**

Recap: Feature descriptors

We know how to detect points

Next question: **How to match them?**



Point descriptor should be:

1. Invariant

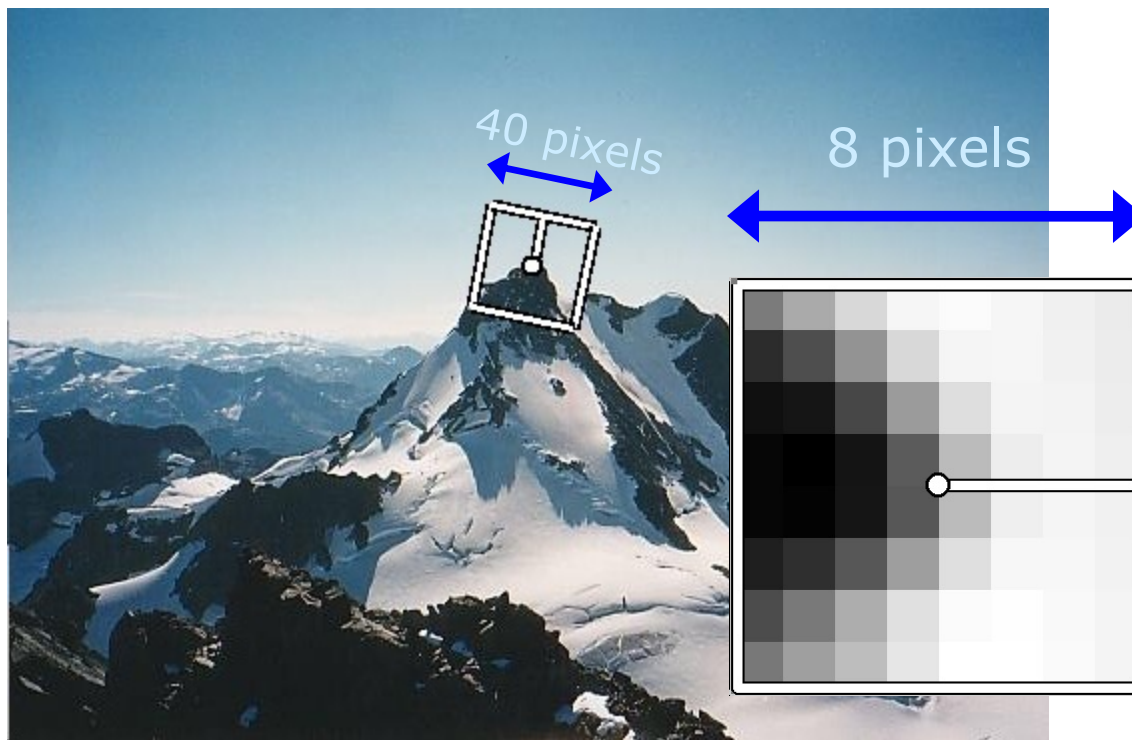
2. Distinctive

MOPS descriptor vector

8x8 oriented patch

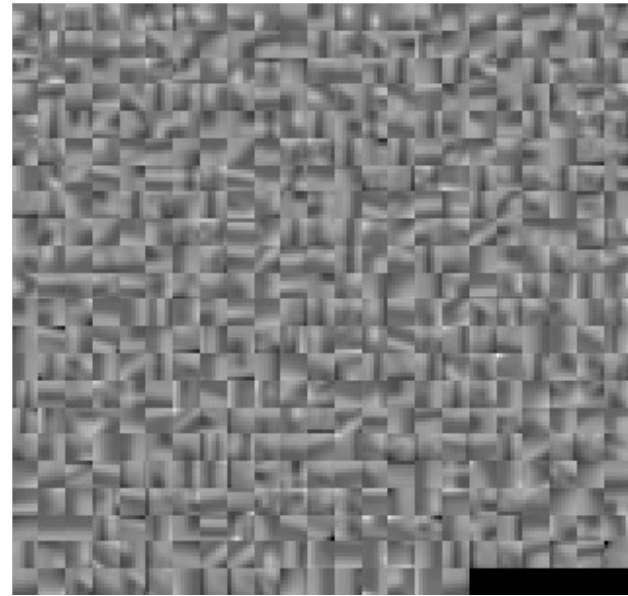
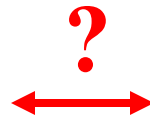
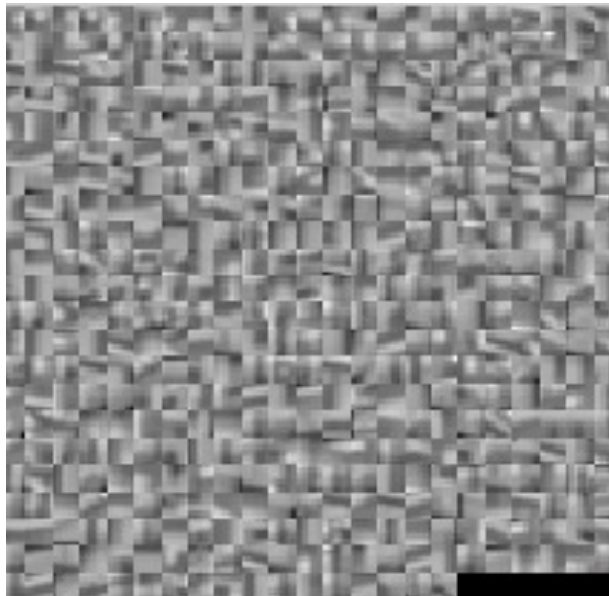
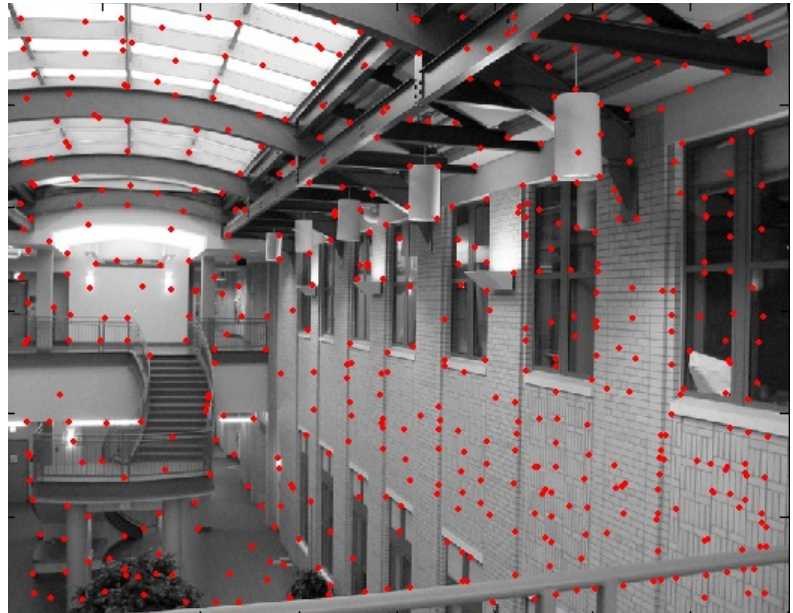
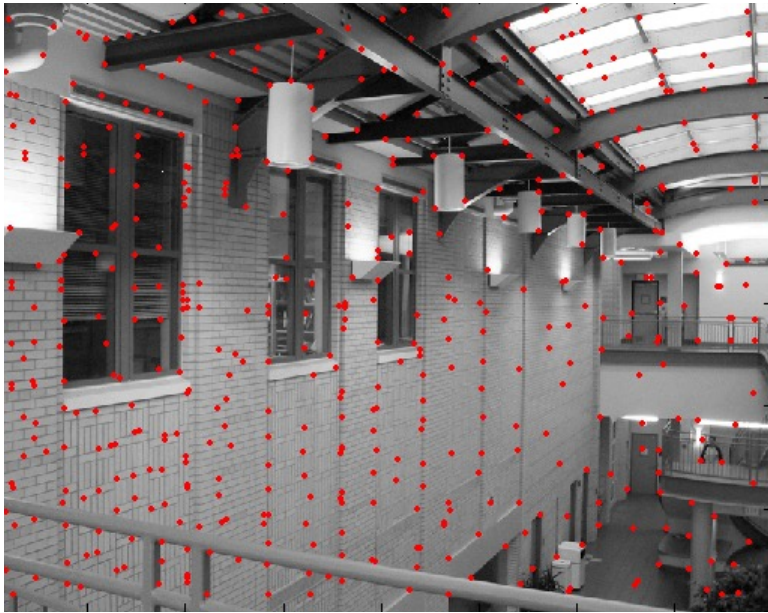
- Sampled at 5 x scale

Bias/gain normalisation: $I' = (I - \mu)/\sigma$



Automatic Feature Matching

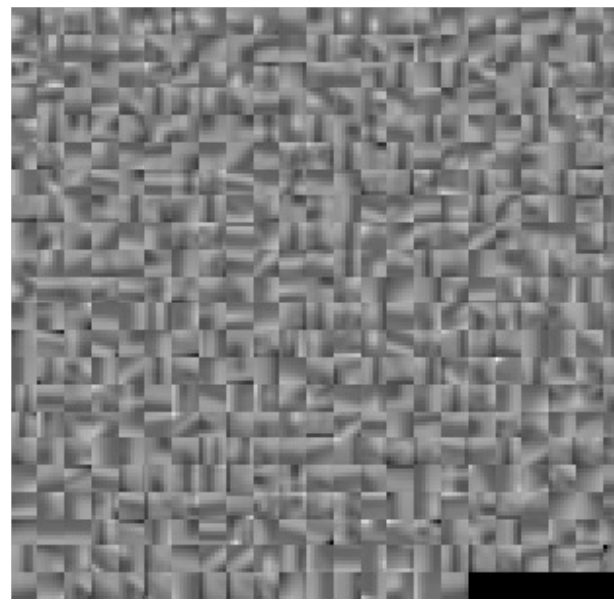
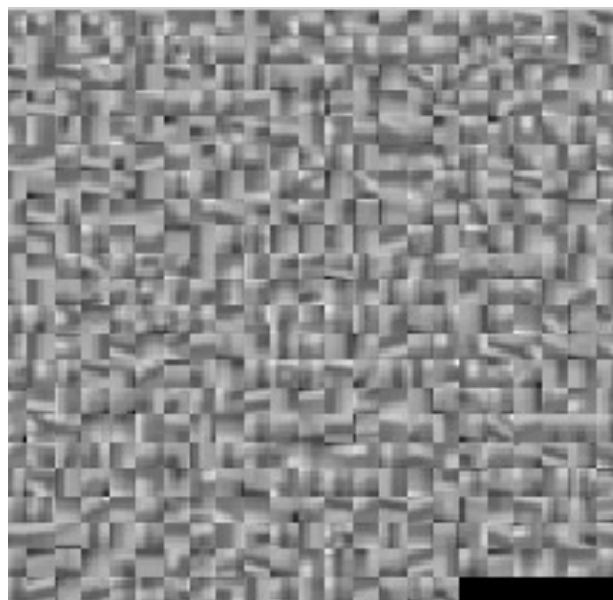
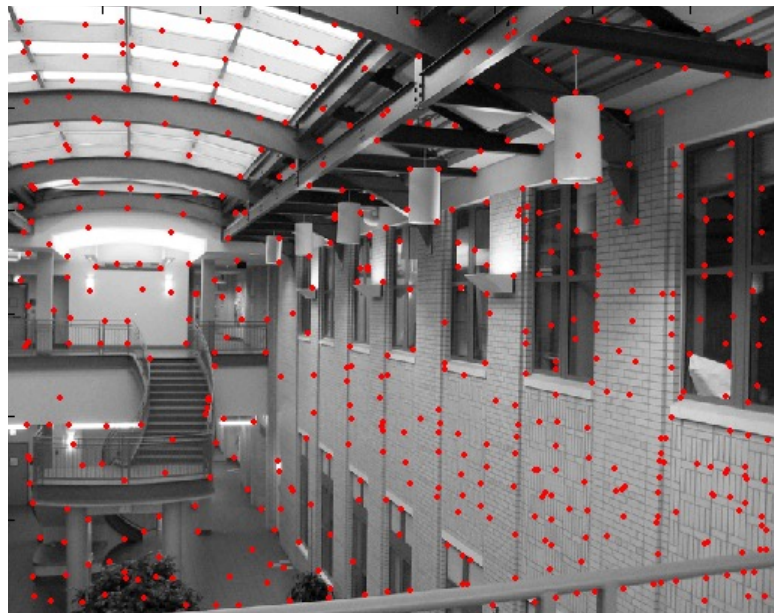
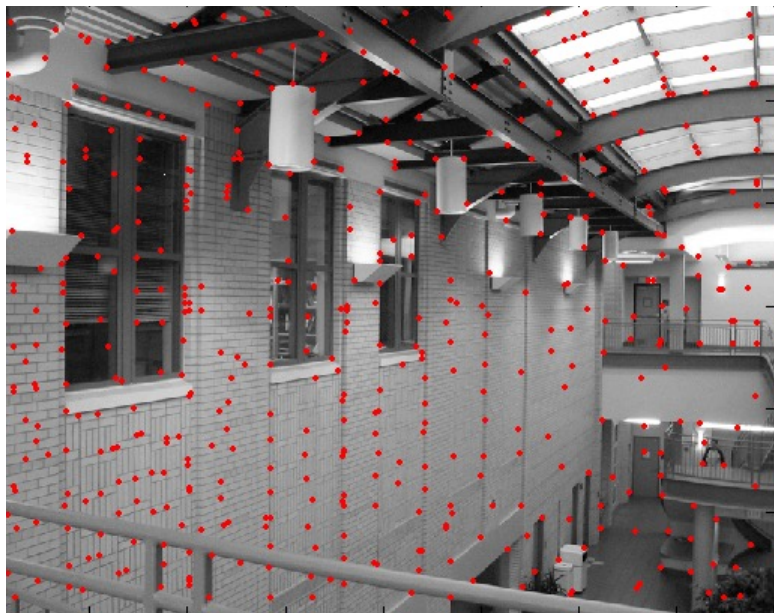
Feature matching



Feature matching

- Pick best match!
 - For every patch in image 1, find the most similar patch (e.g. by SSD).
 - Called “nearest neighbor” in machine learning
- Can do various speed ups:
 - Hashing
 - compute a short descriptor from each feature vector, or hash longer descriptors (randomly)
 - Fast Nearest neighbor techniques
 - *kd*-trees and their variants
 - Clustering / Vector quantization
 - So called “visual words”

What about outliers?

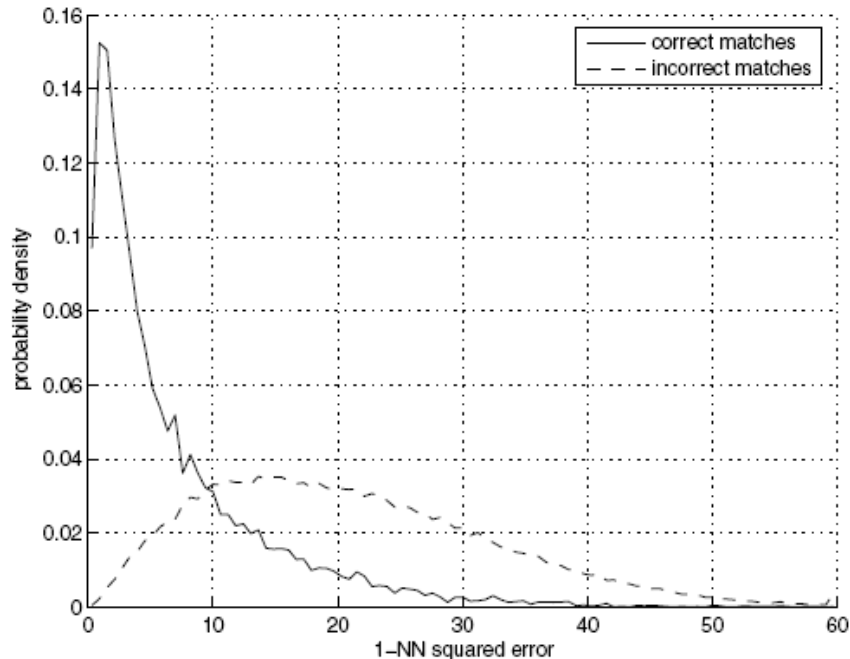


Feature-space outlier rejection

Let's not match all features, but only these that have "similar enough" matches?

How can we do it?

- $SSD(patch1, patch2) < threshold$
- How to set threshold?



Feature-space outlier rejection: symmetry

Let's not match all features, but only these that have "similar enough" matches?

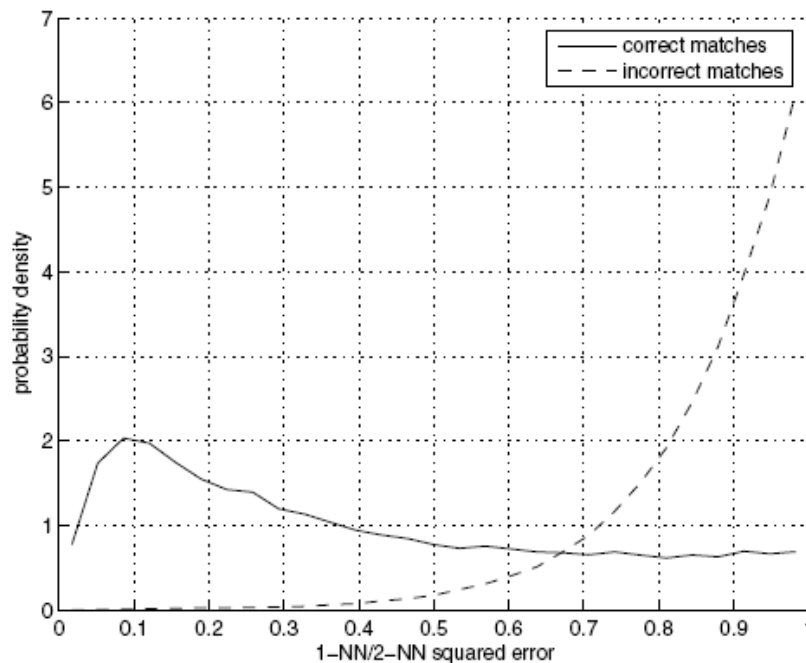
How can we do it?

- Symmetry: x 's NN is y , and y 's NN is x

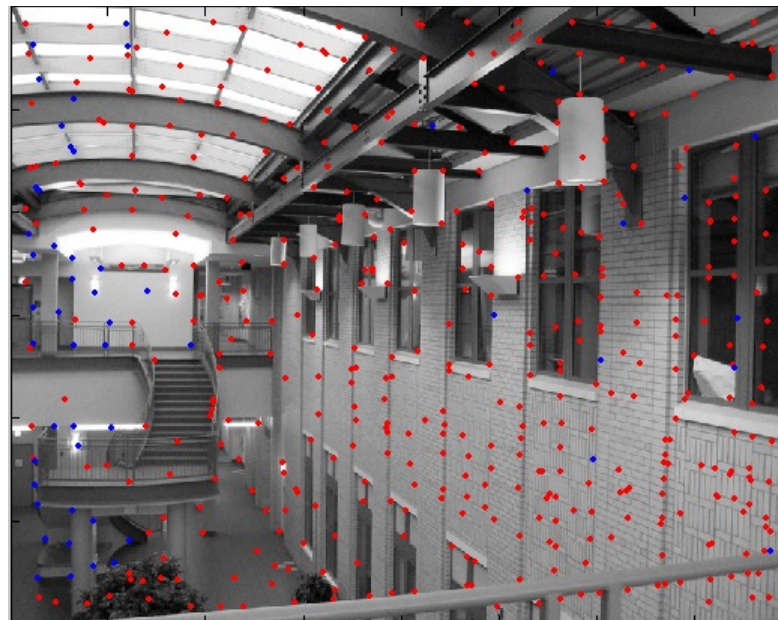
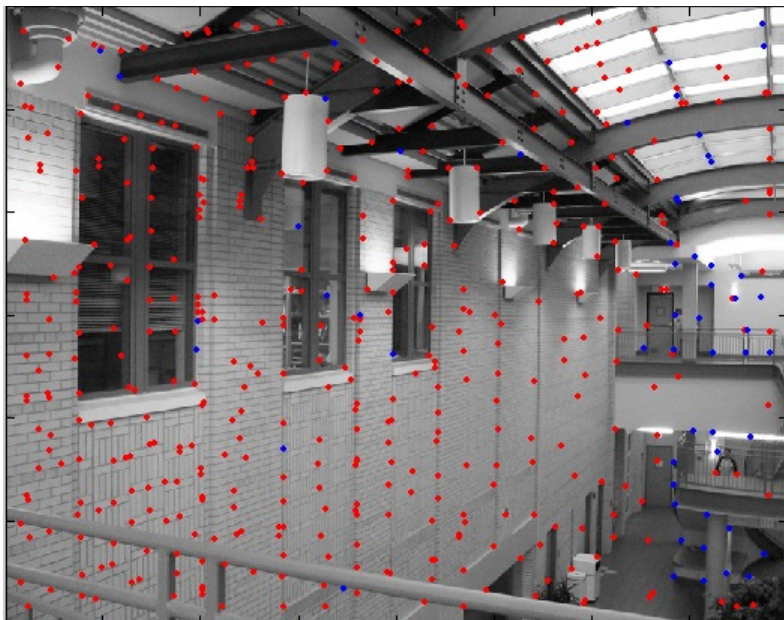
Feature-space outlier rejection: Lowe's trick

A better way [Lowe, 1999]:

- 1-NN: SSD of the closest match
- 2-NN: SSD of the second-closest match
- Look at how much better 1-NN is than 2-NN, e.g. 1-NN/2-NN
- That is, is our best match so much better than the rest?



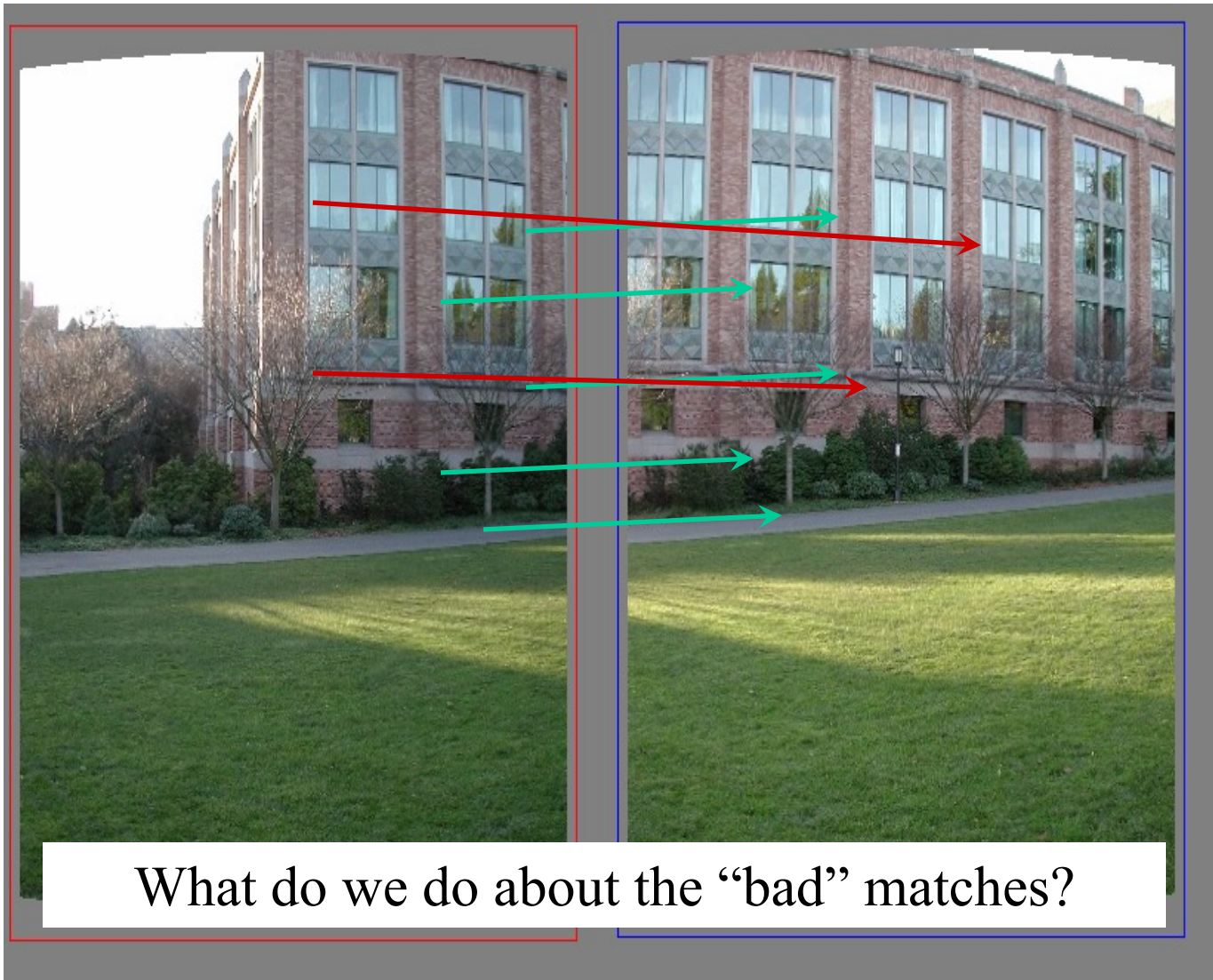
Feature-space outlier rejection



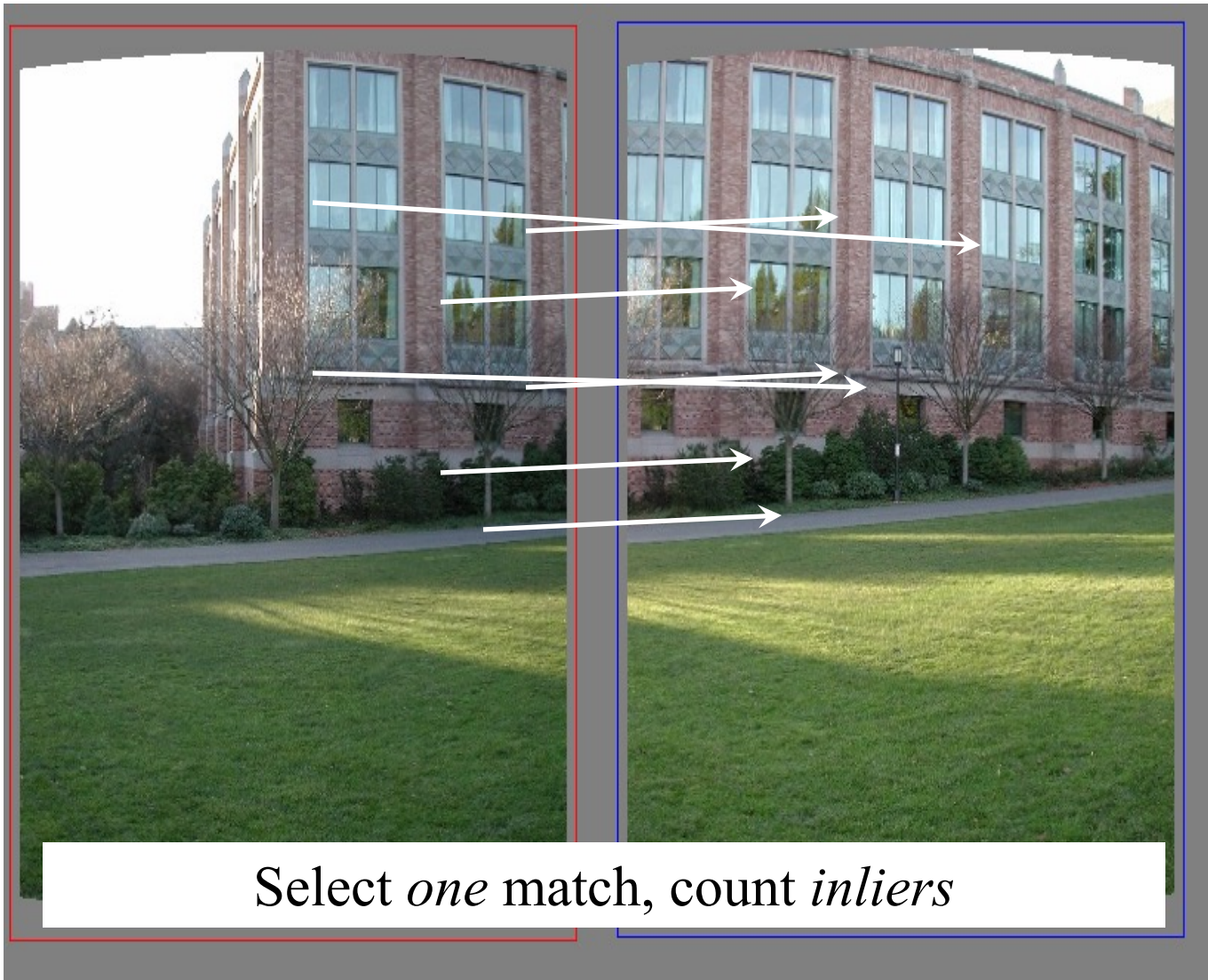
Can we now compute H from the blue points?

- No! Still too many outliers...
- What can we do?

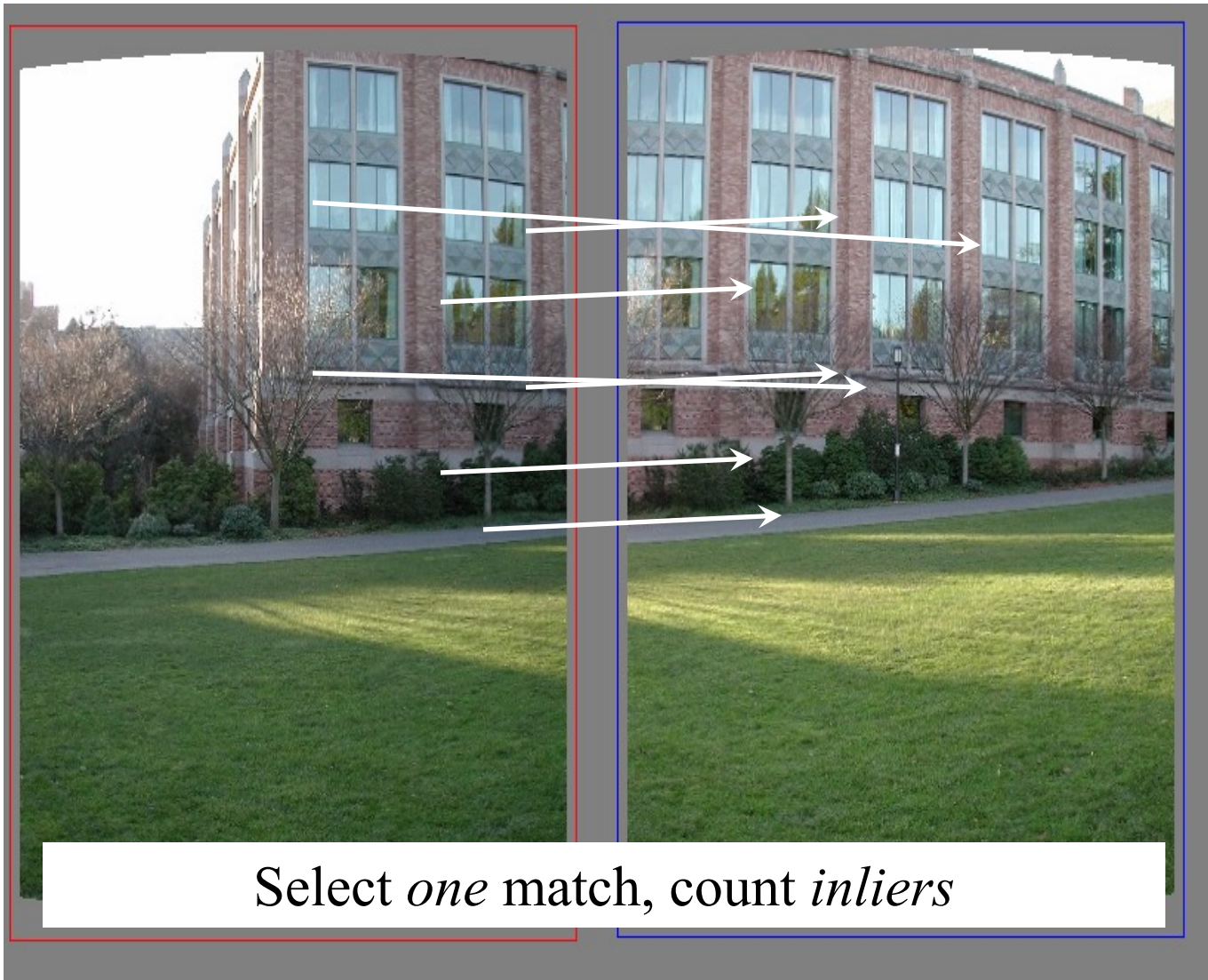
Matching features



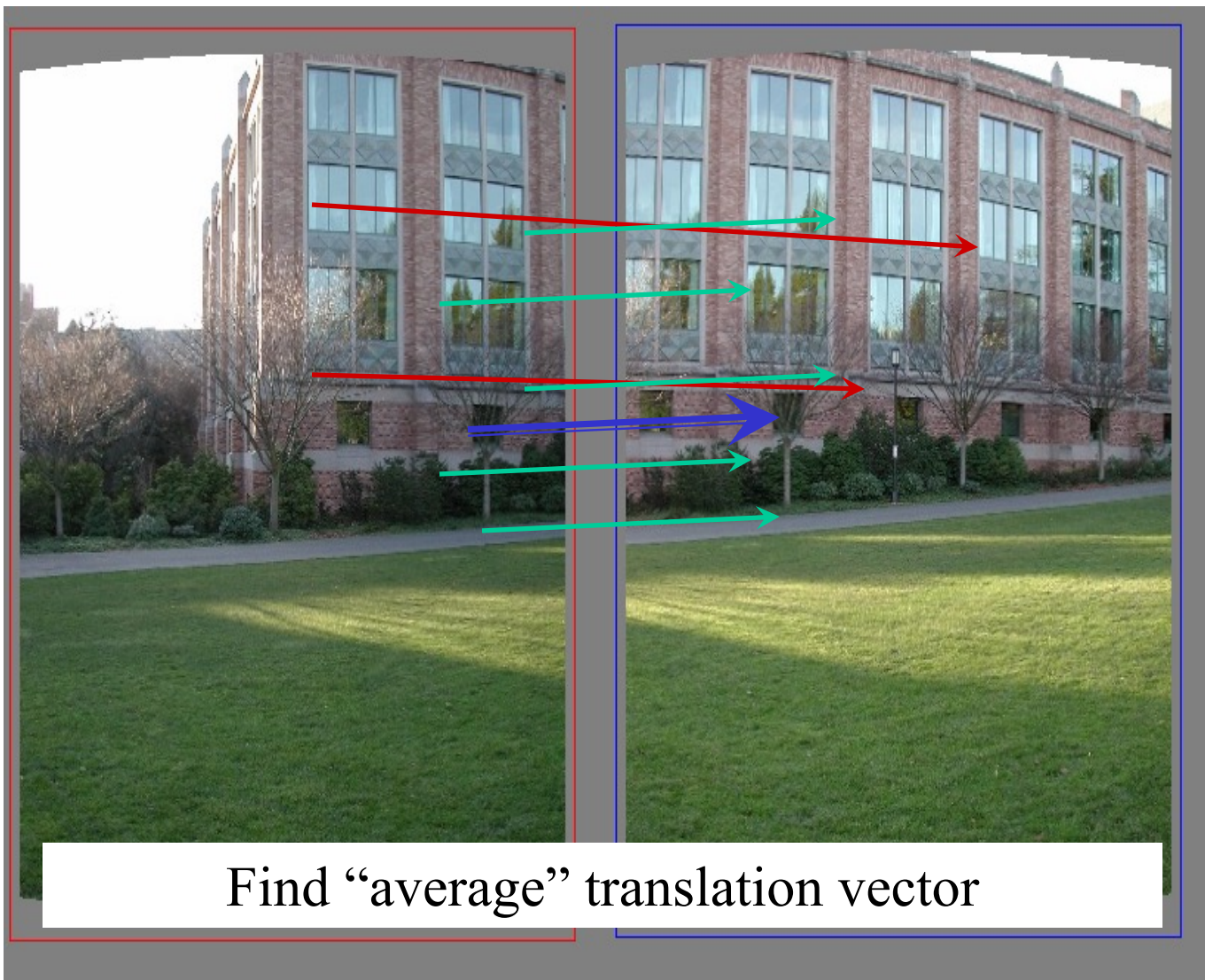
Random Sample Consensus



Random Sample Consensus




Least squares fit

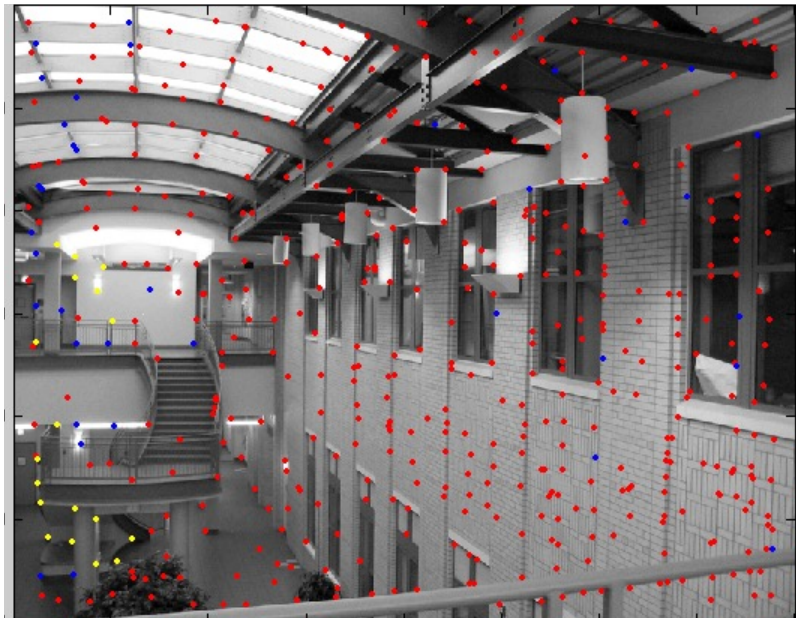
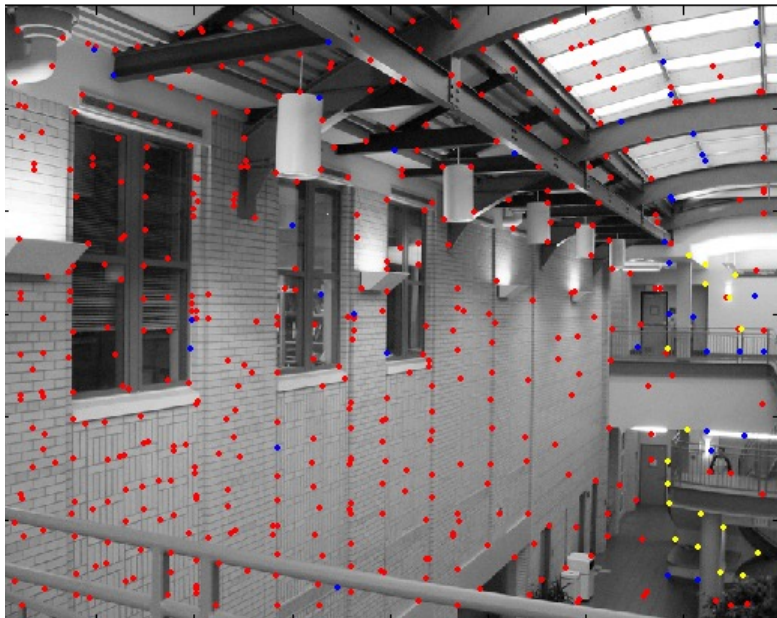


RANSAC for estimating homography

RANSAC loop:

1. Select four feature pairs (at random)
 2. Compute homography H (exact)
 3. Compute *inliers* where $dist(p_i', \mathbf{H} p_i) < \varepsilon$
 4. Keep largest set of inliers
 5. Re-compute least-squares H estimate on all of the inliers
- 

RANSAC



Limitations of Alignment

We need to know the global transform
(e.g. affine, homography, etc)

Breaking out of 2D

...now we are ready to break out of 2D



And enter the real world!



on to 3D...

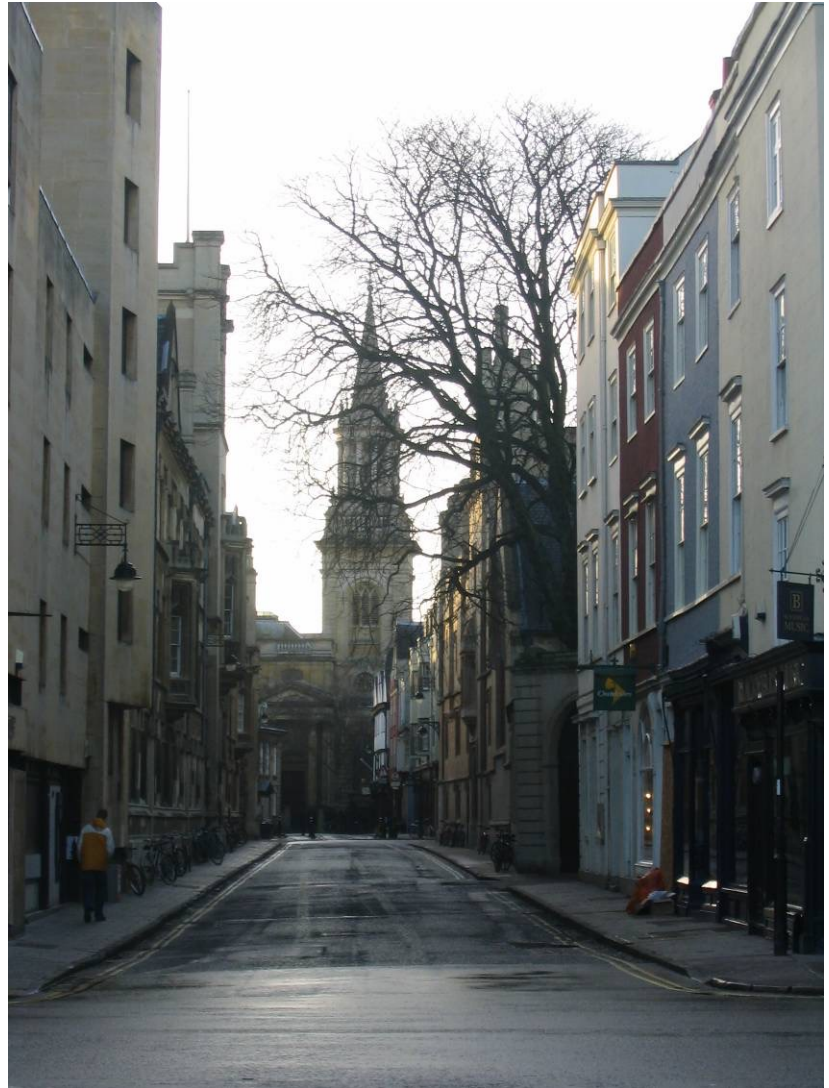
Enough of images!

We want more of the
plenoptic function

We want real 3D scene
walk-throughs:

- Camera rotation

- Camera translation



3D is super cool!



<https://rd.nytimes.com/projects/reconstructing-journalistic-scenes-in-3d>

3D is super cool!



@capturingreality



[@organiccomputer](#)

NeRF in the wild (will get to in few more lectures)



Not just about 3D reconstruction



[The Chemical Brothers - Wide Open ft. Beck, MV]

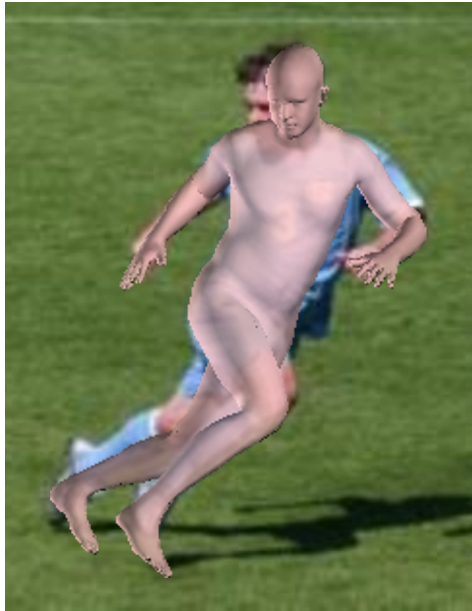
3D for video editing



@blottermedia

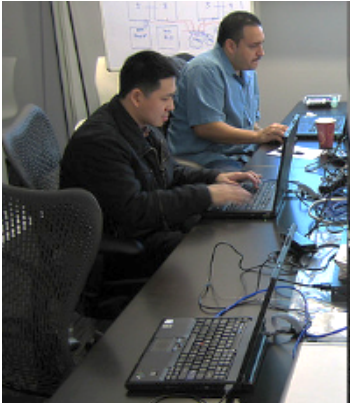
My Research

Single-View 3D Human Mesh Recovery



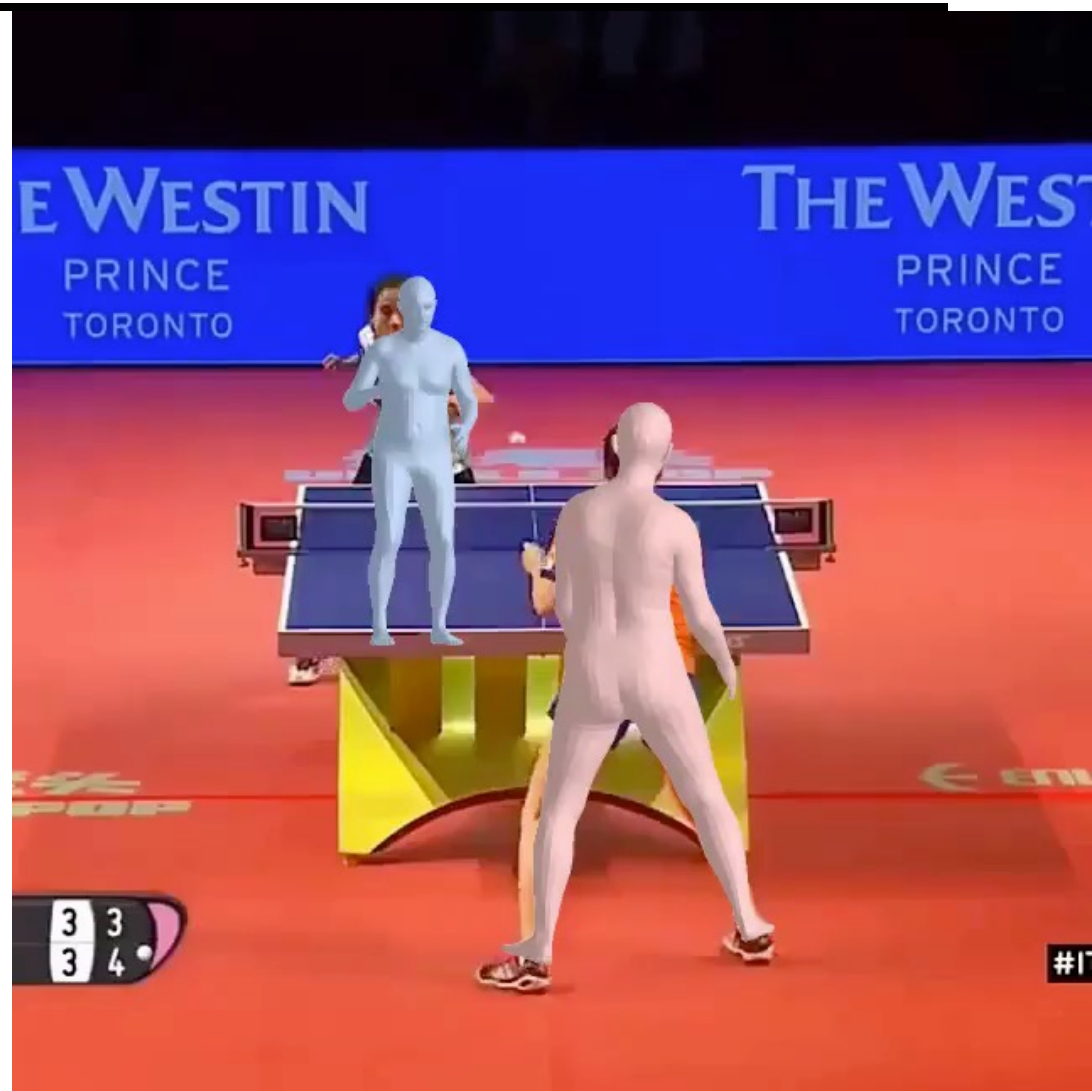
[Bogo*, **Kanazawa***, Lassner, Gehler, Romero, Black ECCV '16]

In everyday photos



Kanazawa, Black, Jacobs, Malik. CVPR 2018

Or from Video



Kanazawa, Zhang, and Felsen et al. CVPR 2019

In more detail

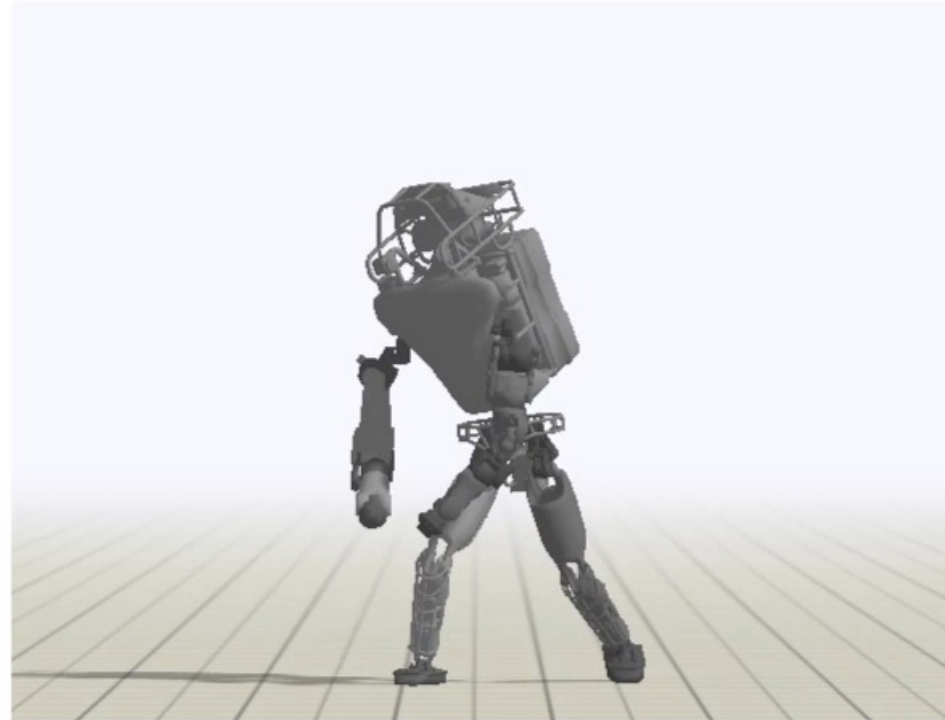


Pixel-Aligned Implicit Function for High-Resolution Clothed Human Digitization,
Saito, Huang, Natsume, Morishima, **Kanazawa**, Li, ICCV 2019

Teaching robots how to dance from watching YouTube



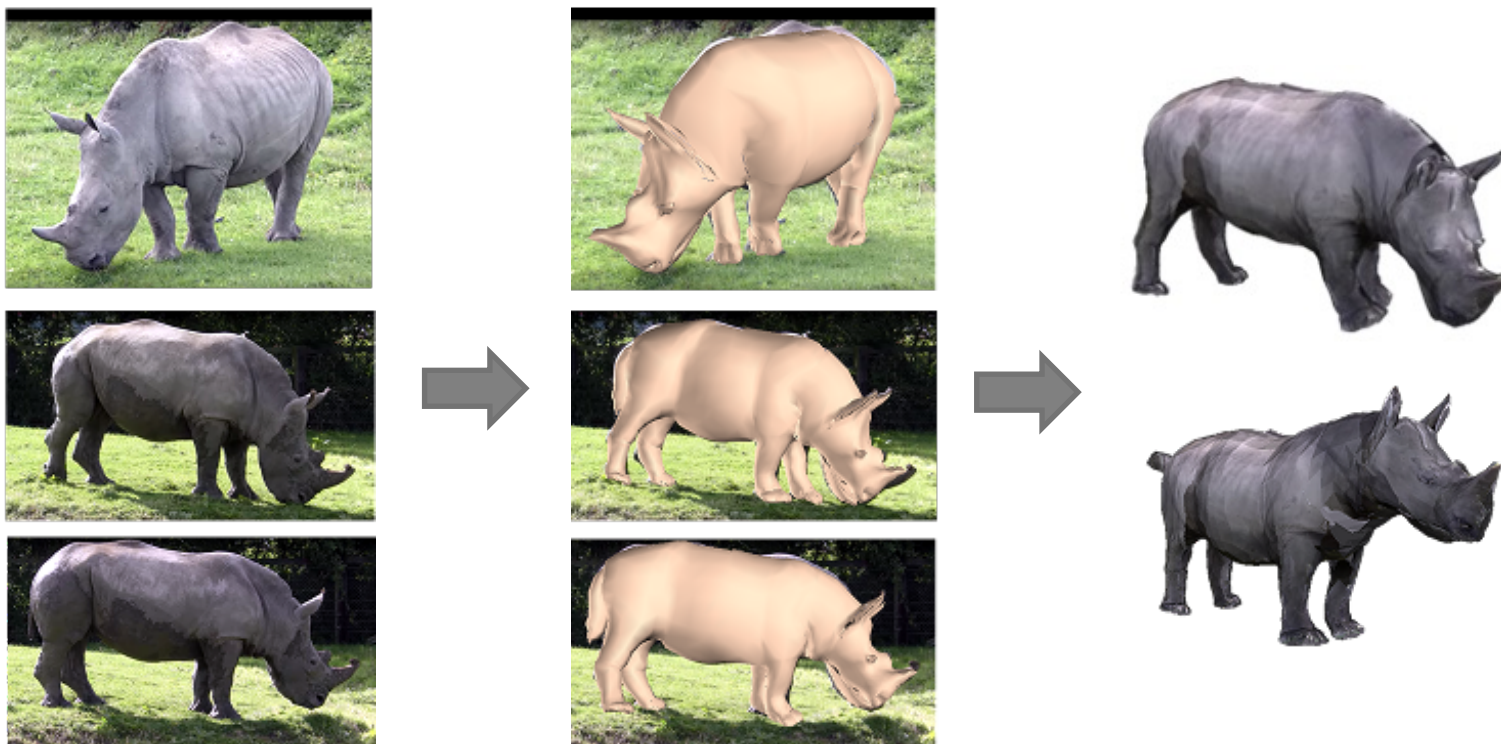
Video



Policy

Peng, Kanazawa, Malik, Abbeel, Levine
“SFV: Reinforcement Learning of Physical Skills from Videos”, SIGGRAPH Asia 2018

Reconstructing Animals with Human Input



Zuffi, Kanazawa, Black, *"Lions and Tigers and Bears: Capturing Non-Rigid, 3D, Articulated Shape from Images"*, CVPR 2018

Print it!!



[Kanazawa*, Tulsiani*, Efros, Malik, ECCV 2018]



Zuffi, Kanazawa, Black, "Lions and Tigers and Bears: Capturing Non-Rigid, 3D, Articulated Shape from Images", CVPR 2018

Flying into an image



Infinite Nature: Perpetual View Generation of Natural Scenes from a Single Image, ICCV 2021



Matthew Tancik*, Ethan Weber*, Evonne Ng*, Ruilong Li, Brent Yi, Terrance Wang, Alexander Kristoffersen, Jake Austin, Kamyar Salahi, Abhik Ahuja, David McAllister, Angjoo Kanazawa



100+ additional Github contributors



Matt

Ethan

Evonne

3D Capture

GETTING STARTED [GITHUB](#) [DOCUMENTATION](#)

 nerfstudio



VIEWPORT RENDER VIEW

▶ RESUME TRAINING

Show Scene

Show Images

Refresh Page

Resolution: 640x1024px

Time Allocation: 100% spent on viewer

Server Connected | Render Connected

CONTROLS RENDER SCENE

LOAD PATH

EXPORT PATH



Height
1080

Width
1920

FOV
50

Seconds
4

FPS
24

ADD CAMERA



Smoothness 

0.00

0 1 2 3

⏪

⏴

⏵

⏩

⏹

CAMERA 0





so on to 3D...

Enough of images!

We want more of the
plenoptic function

We want real 3D scene
walk-throughs:

Camera rotation

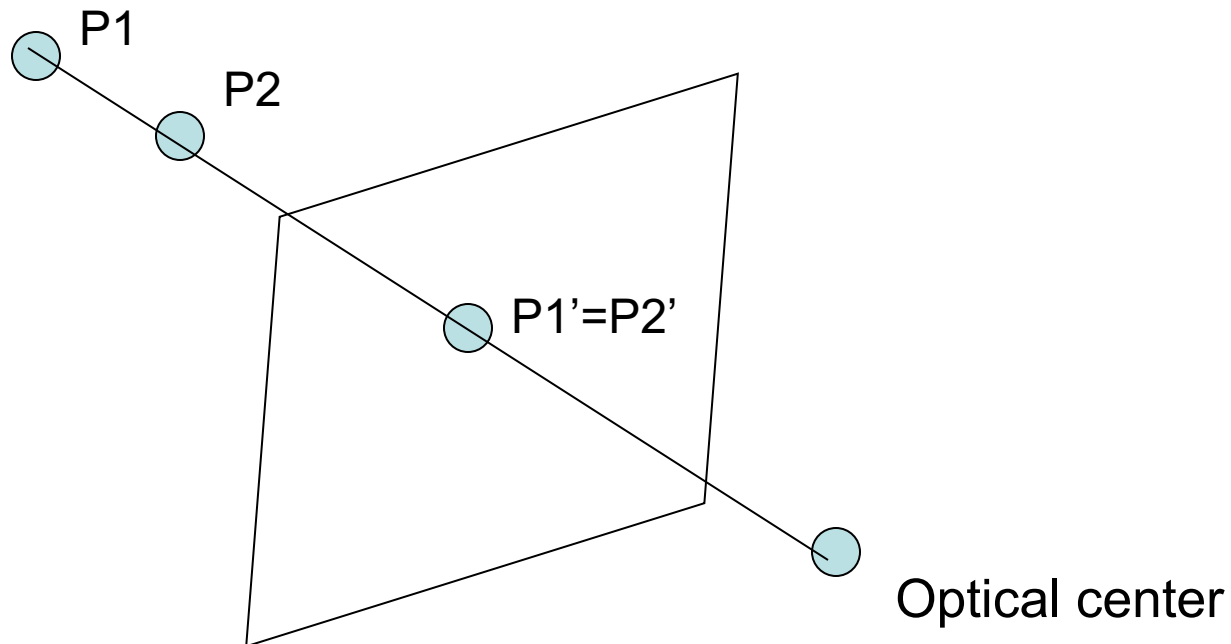
Camera translation

Can we do it from a single
photograph?



Why multiple views?

- Structure and depth are inherently ambiguous from single views.



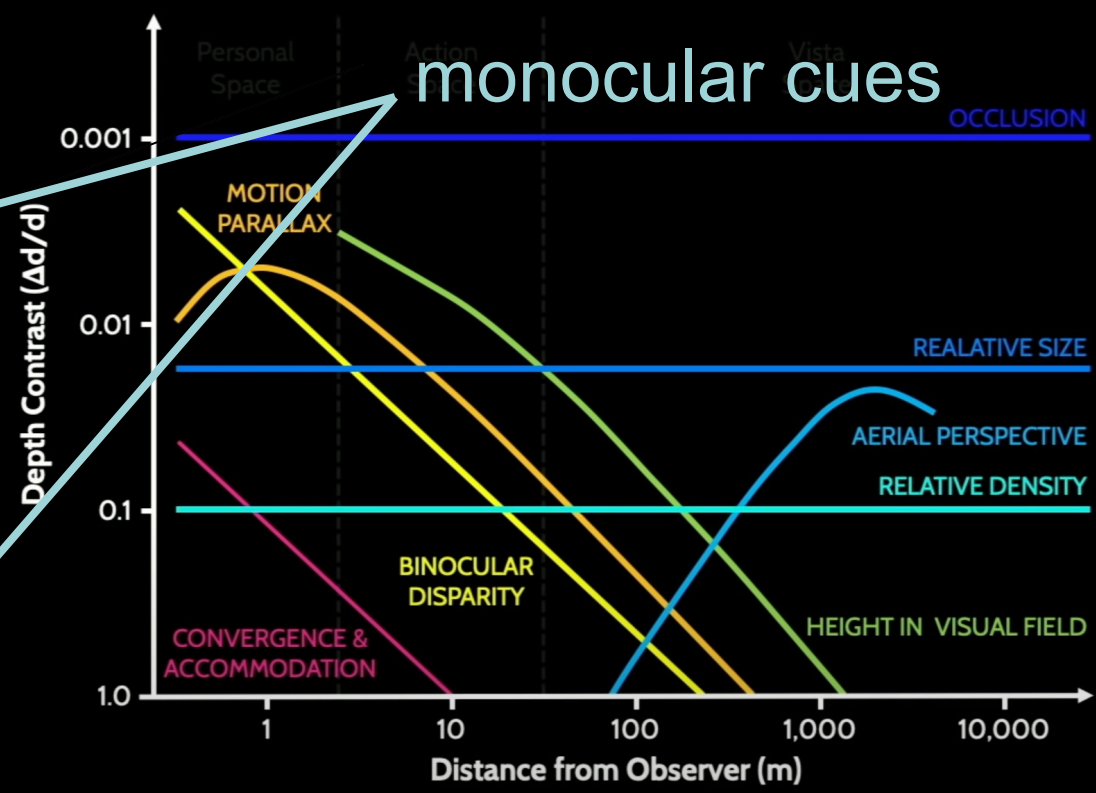
Why multiple views?

- Structure and depth are inherently ambiguous from single views.



Human Depth Cues

- OCCLUSION**
- RELATIVE SIZE**
- AERIAL PERSPECTIVE**
- RELATIVE DENSITY**
- HEIGHT IN VISUAL FIELD**
- BINOCULAR DISPARITY**
- MOTION PARALLAX**
- CONVERGENCE & ACCOMMODATION**



Cutting and Vishton, Perceiving layout and knowing distances, 1995.

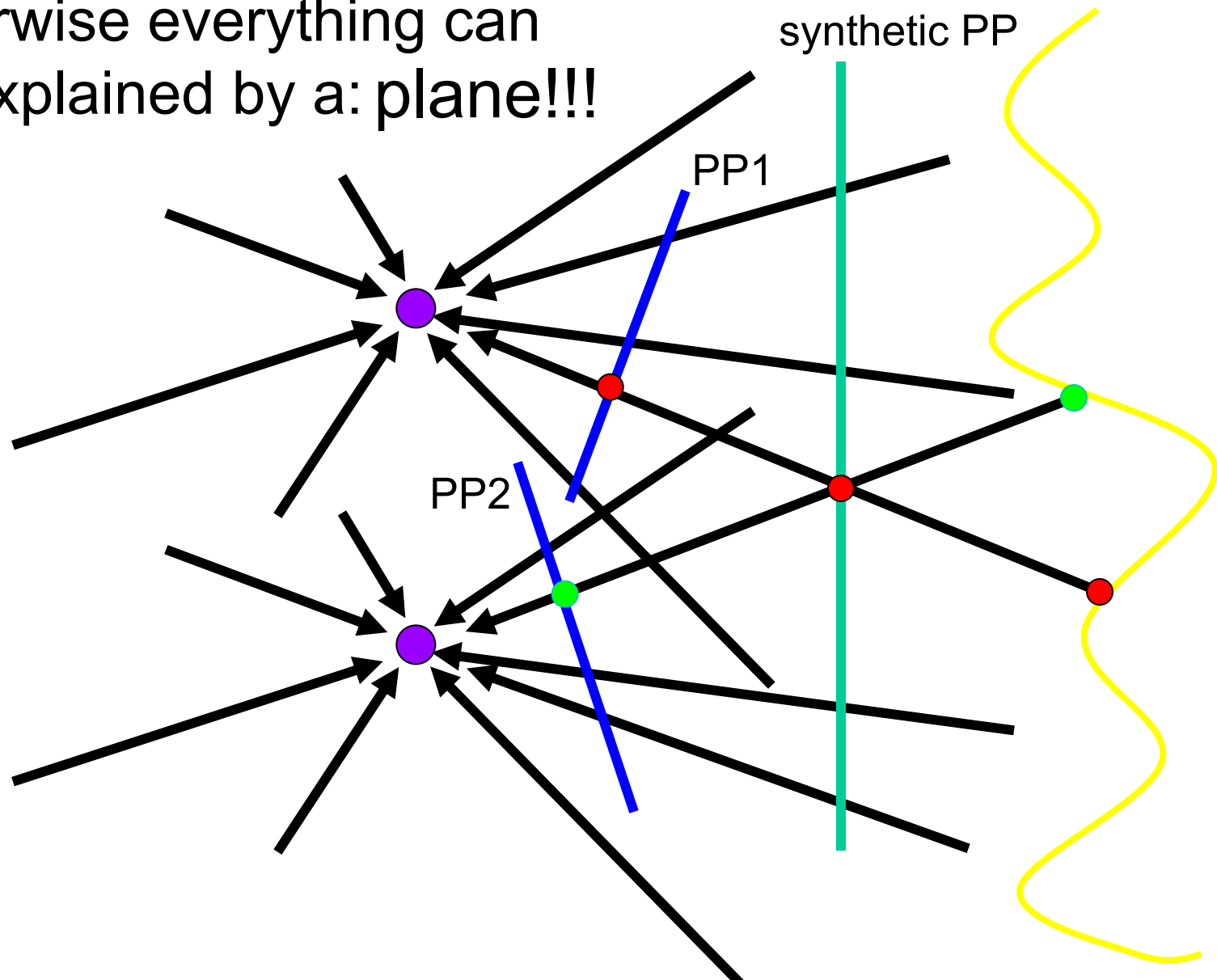
<https://youtu.be/LQwMAI9bGNY?si=6rf4F9NmB7o8vfgY>

Geometric Depth Understanding

- Ambiguous from a single image
- Why?

Need two different camera center

otherwise everything can
be explained by a: plane!!!



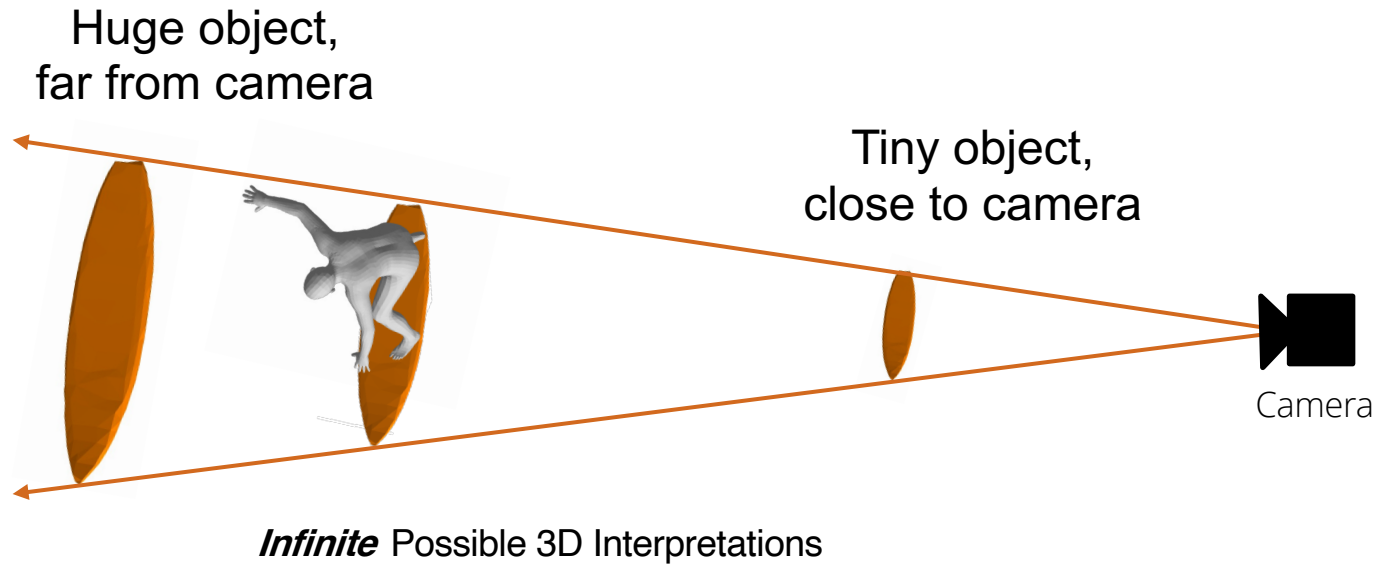
Fundamental Depth Ambiguity in 2D \rightarrow 3D



Original Image

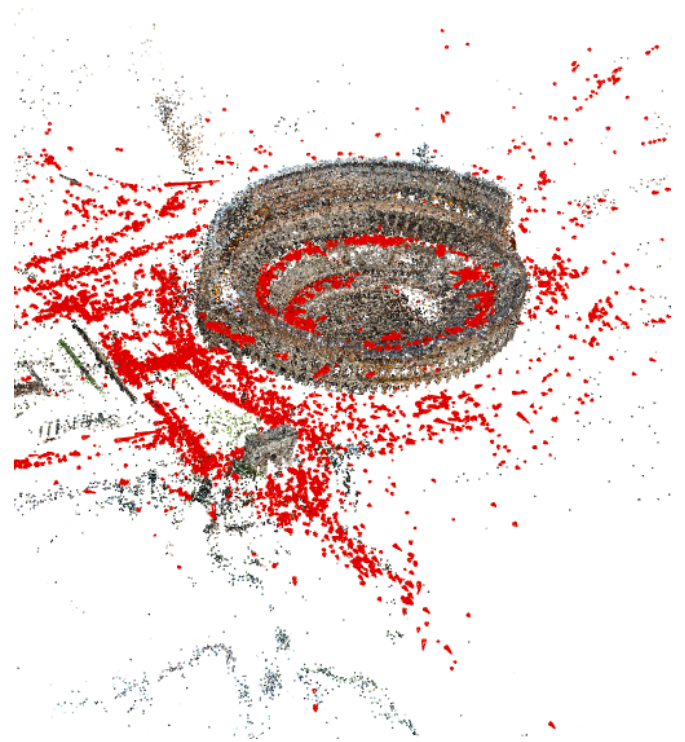


Same 2D Projection



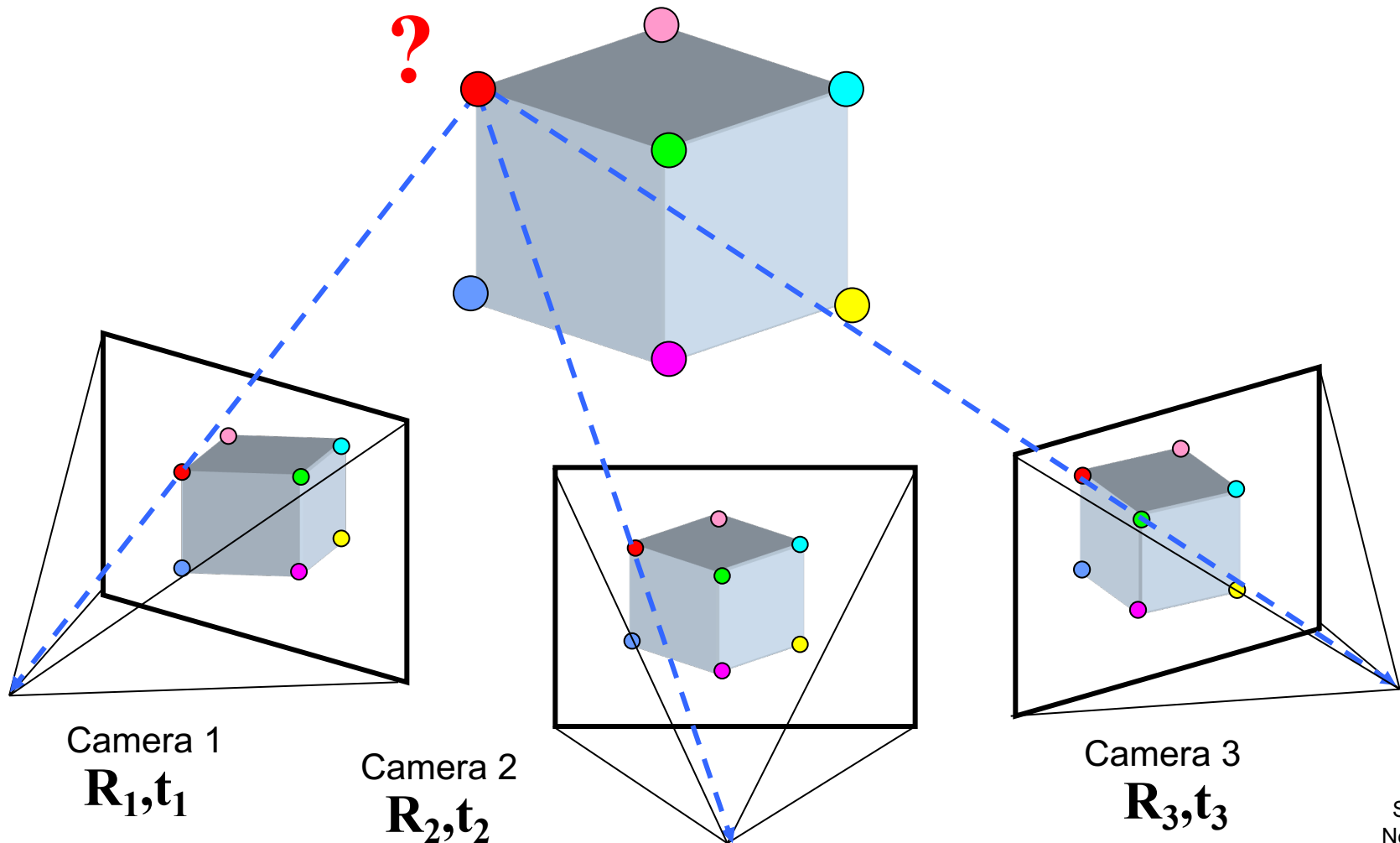
2.5D vs 3D

- is 3D = depth from a single image?



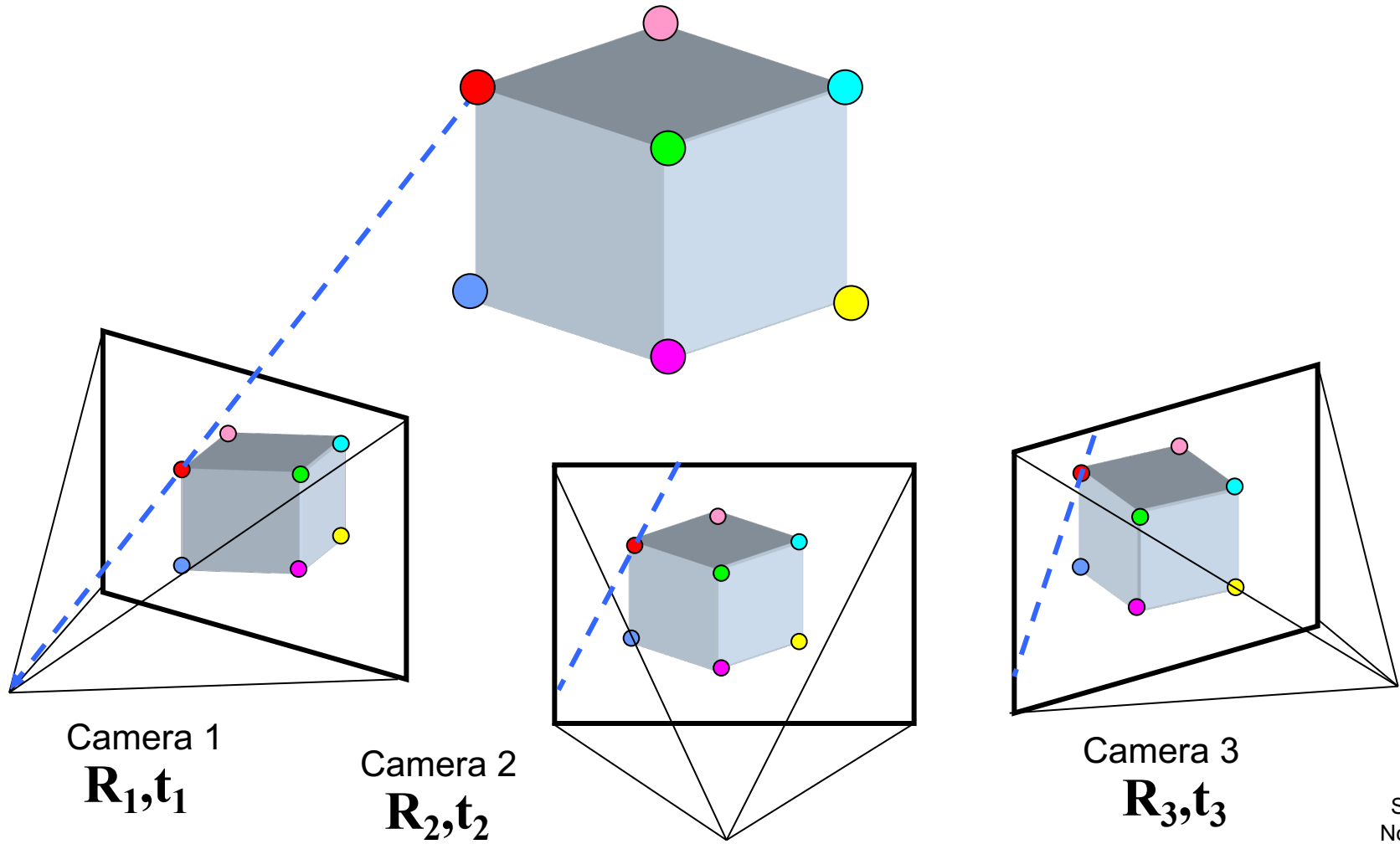
Multi-view geometry problems

- **Structure:** What is the 3D coordinate of a point that can be seen in multiple images?



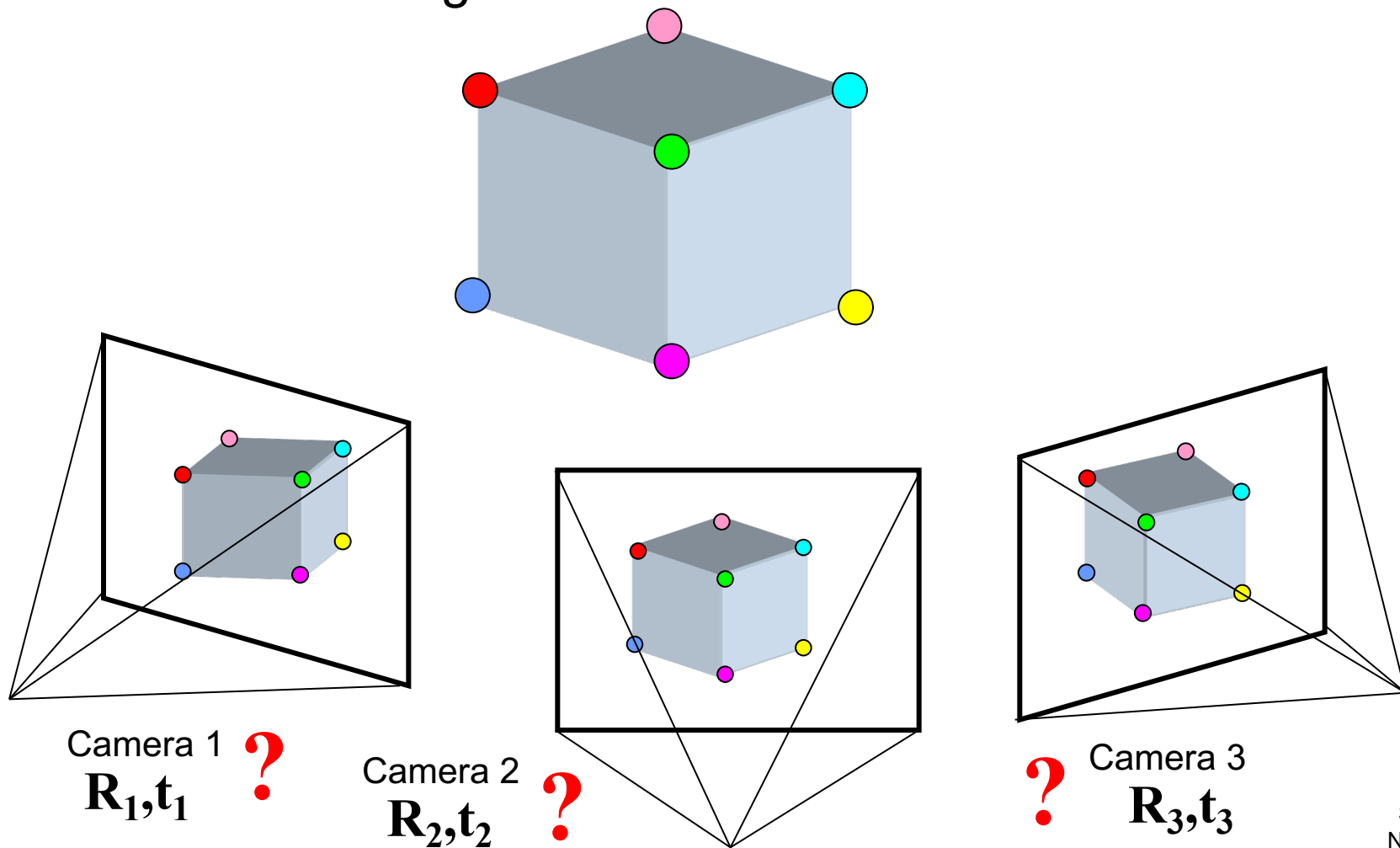
Multi-view geometry problems

- **Correspondence:** Given a point in one of the images, where are the corresponding points in the other images?

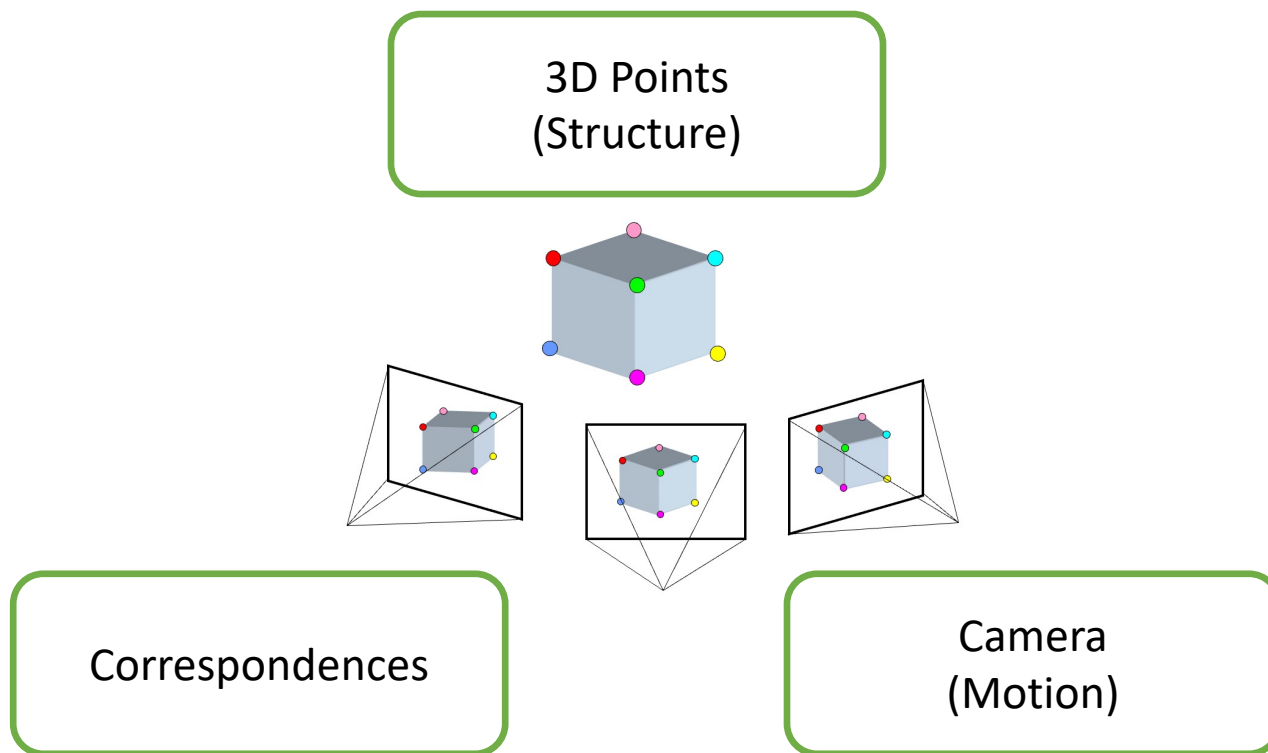


Multi-view geometry problems

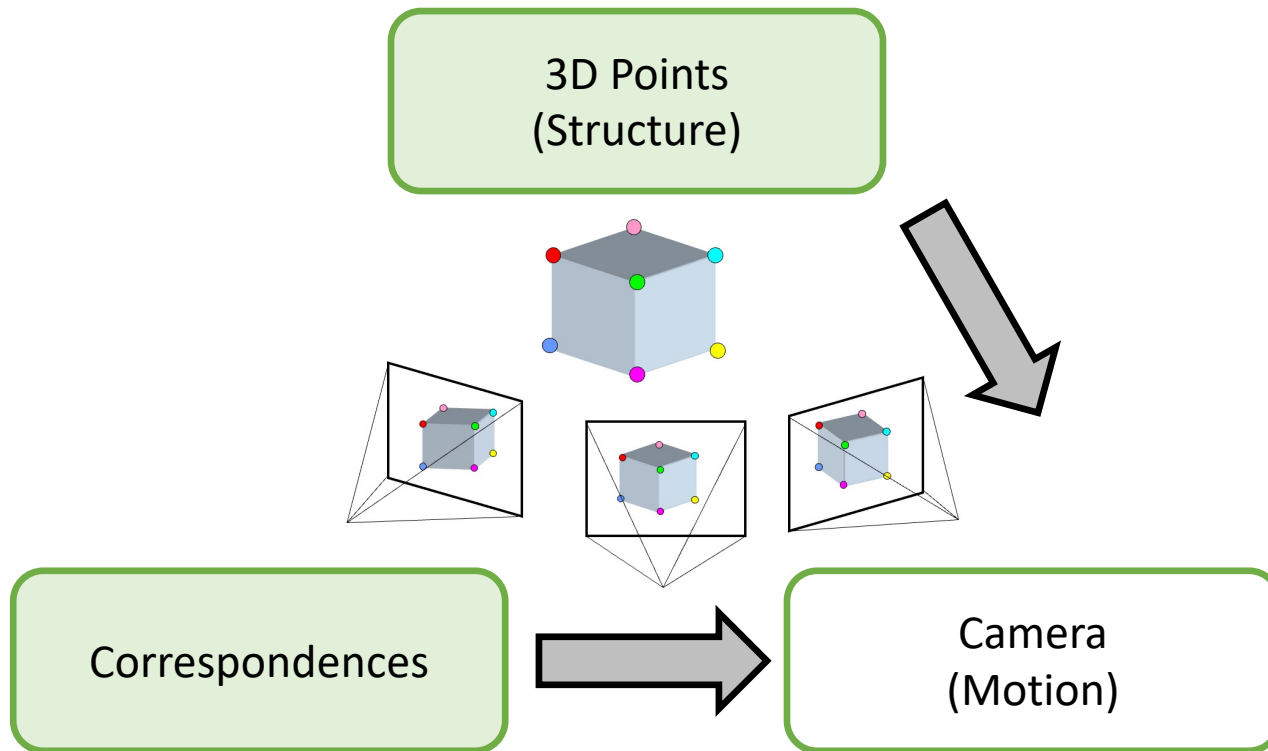
- **Motion:** Given a set of corresponding points in two or more images, what is the relative camera parameters between the images?



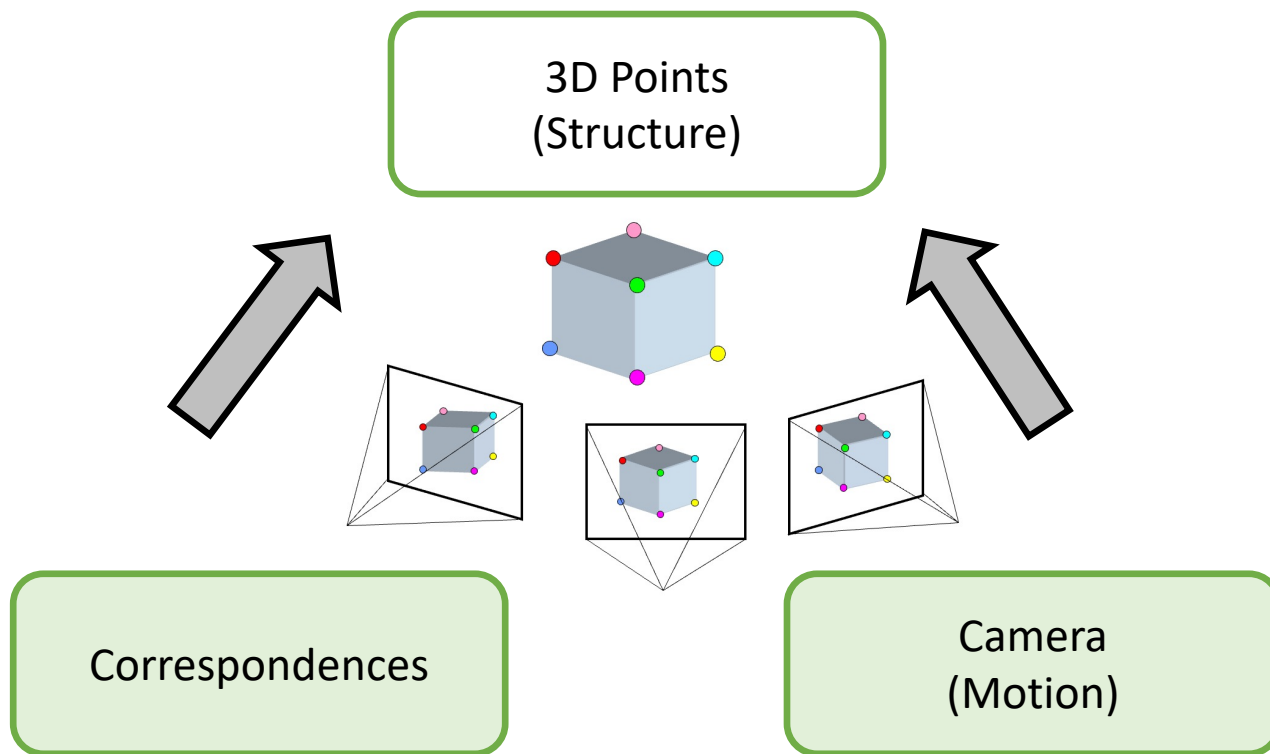
Big picture: 3 key components in 3D



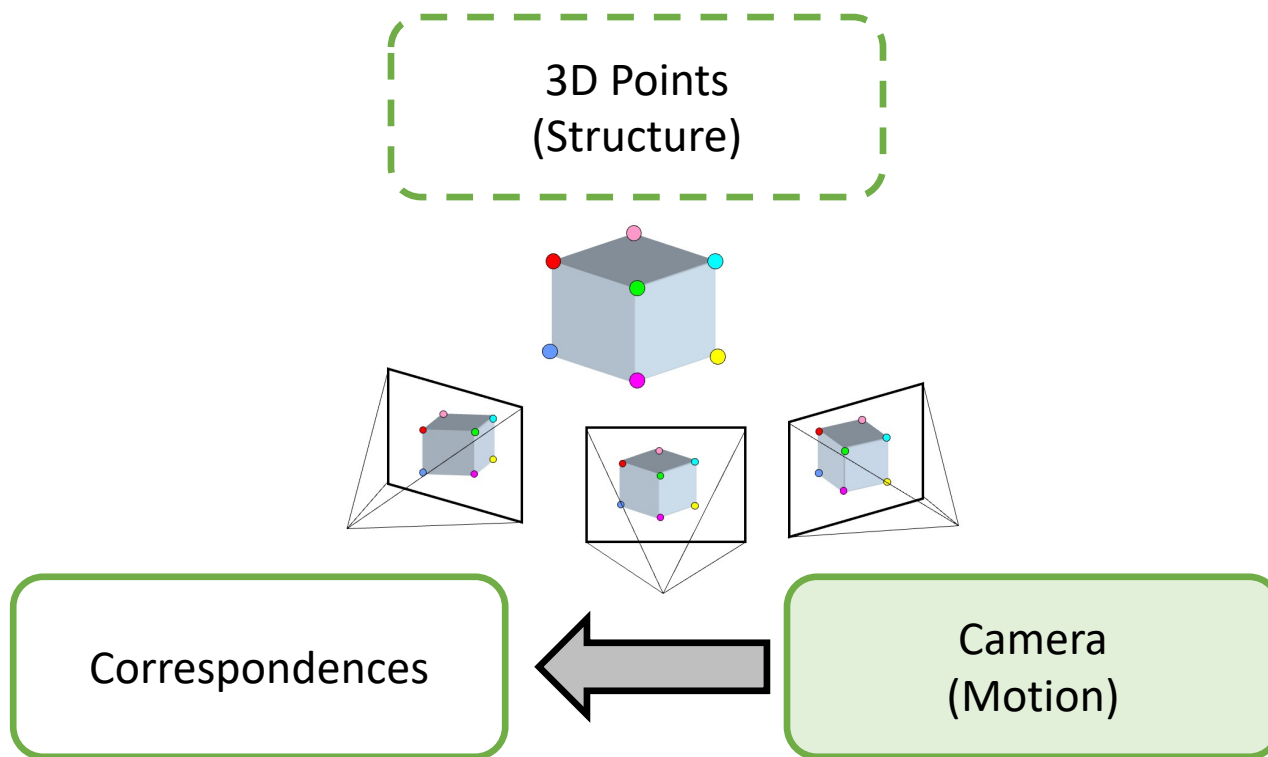
Big picture: 3 key components in 3D



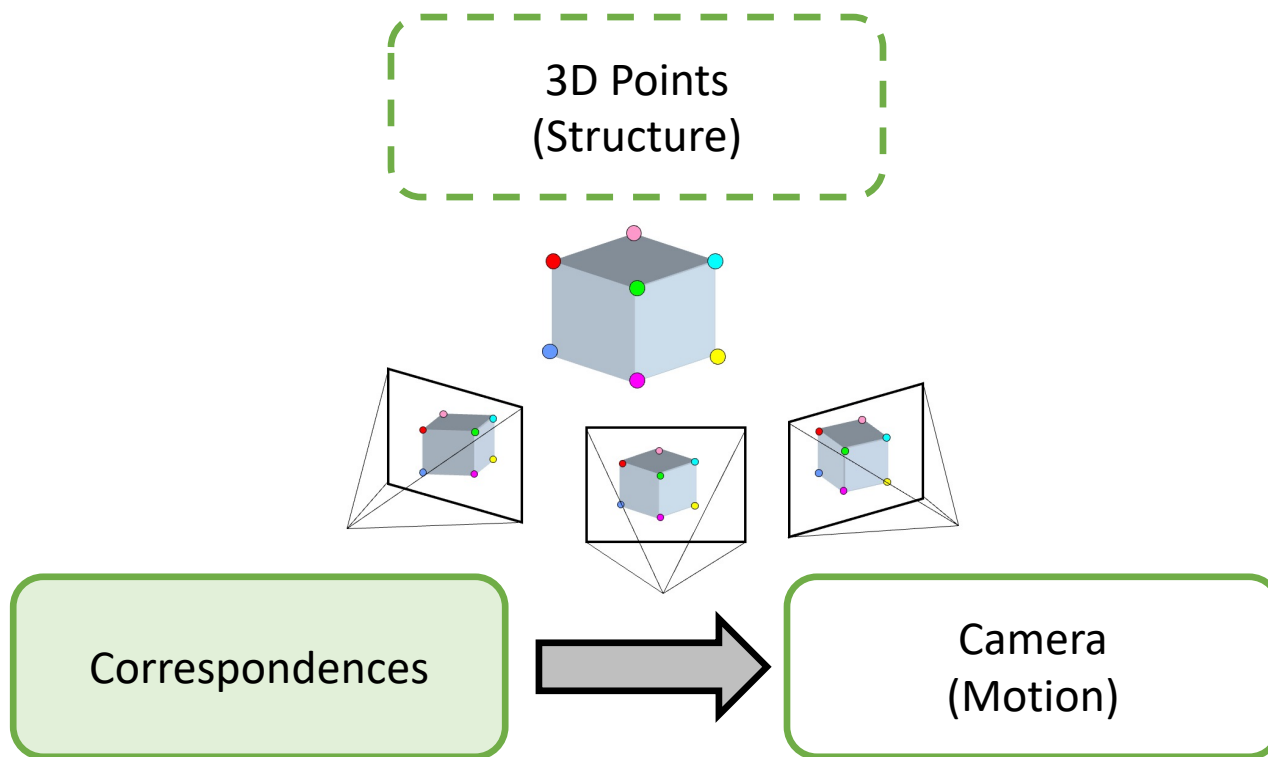
Big picture: 3 key components in 3D



Big picture: 3 key components in 3D



Big picture: 3 key components in 3D



From pixels to the 3D world

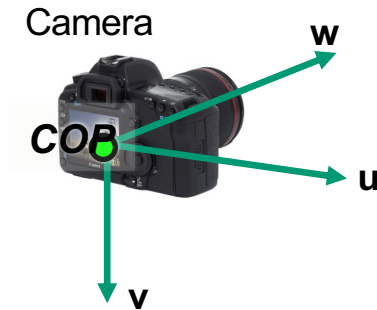
To go from pixels to 3D location in the **world coordinates**, we need to know two things about the camera:

1. Position of the camera with respect to the world (extrinsics)
2. How the camera maps a point in the world to image (intrinsics)

Problem setup

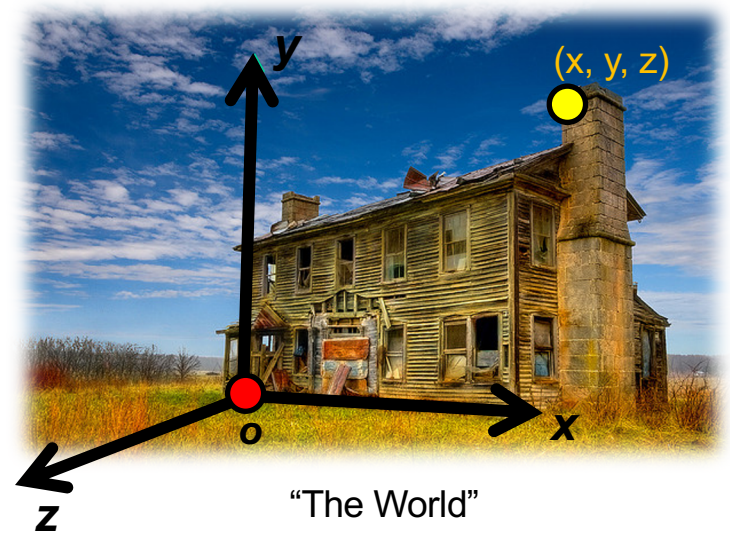
There is a world coordinate frame and camera looking at the world

How can we model the geometry of a camera?



Three important coordinate systems:

1. *World* coordinates
2. *Camera* coordinates
3. *Image* coordinates



Coordinate frames + Transforms

Orientation + Location of
the camera in the World

How the camera maps a
point in 3D to image

Extrinsics (R, T)

Intrinsics (K)

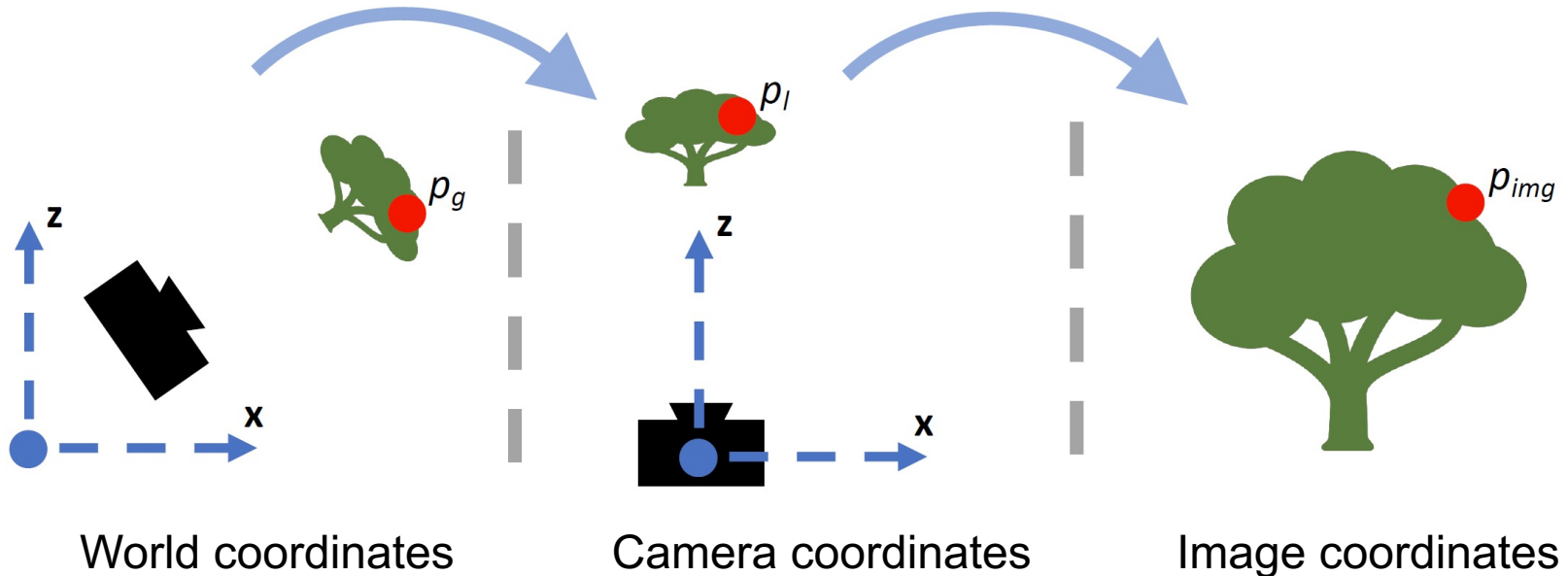


Figure credit: Peter Hedman

Camera: Specifics

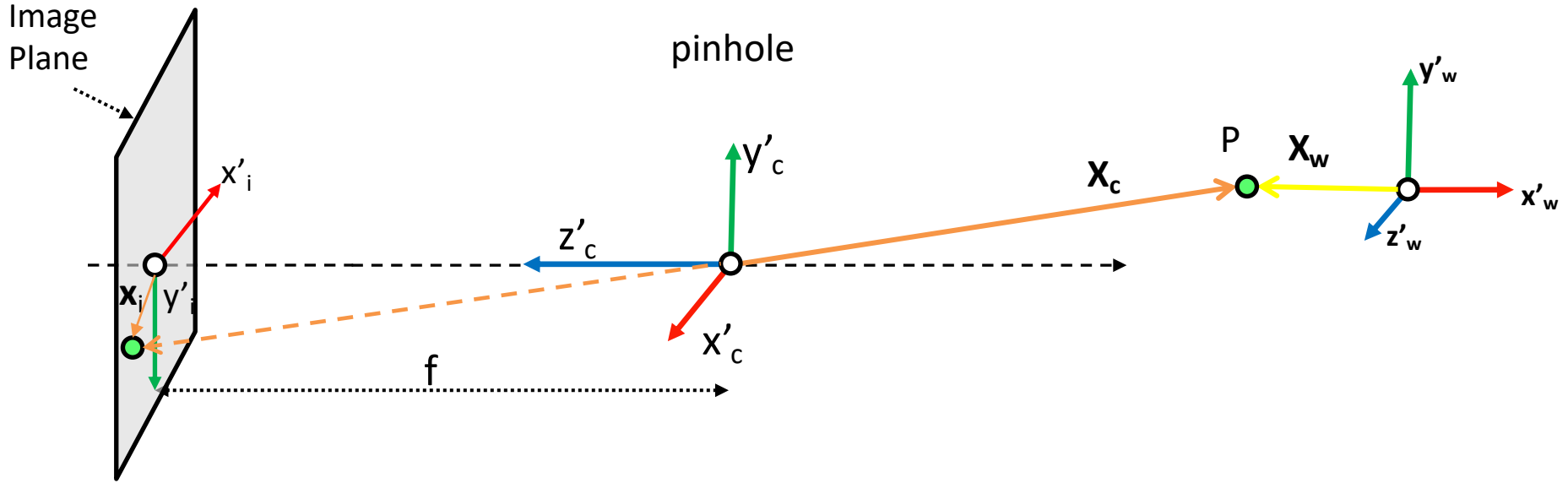
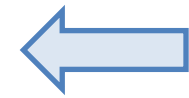


Image Coordinates

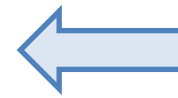
$$\mathbf{X}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$



Perspective
Projection
(3D to 2D)

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

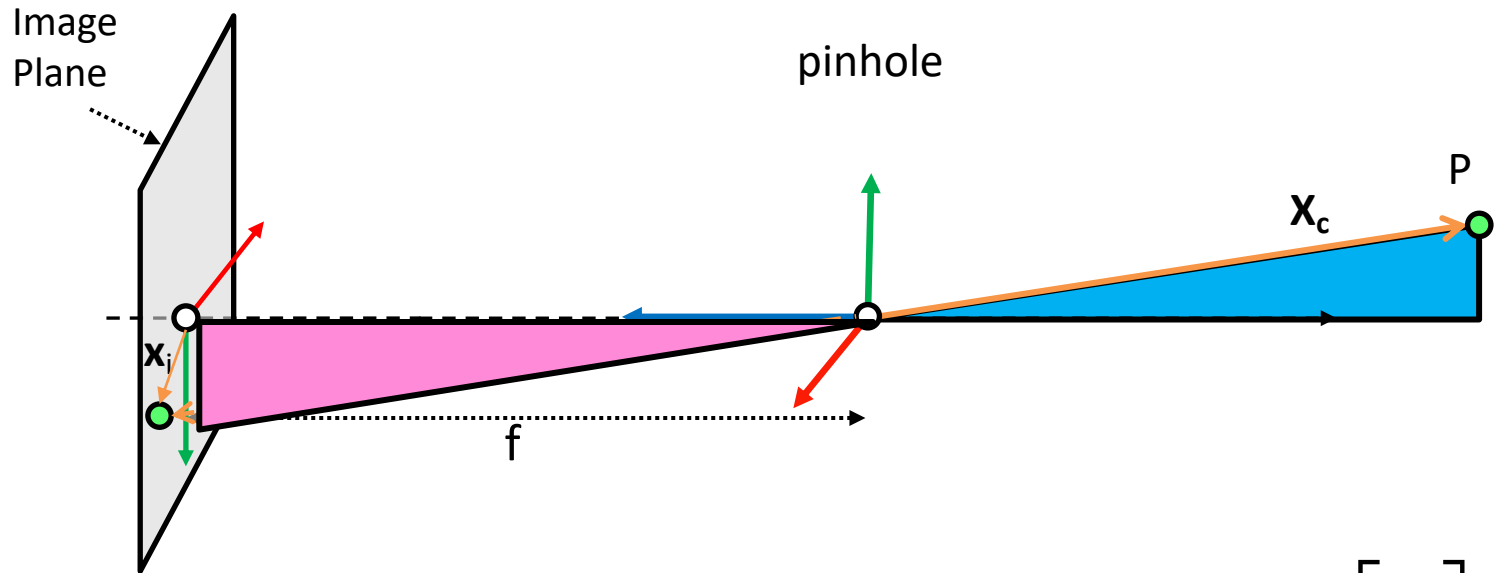


Coordinate
Transformation
(3D to 3D)

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Perspective Projection



$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Image Coordinates

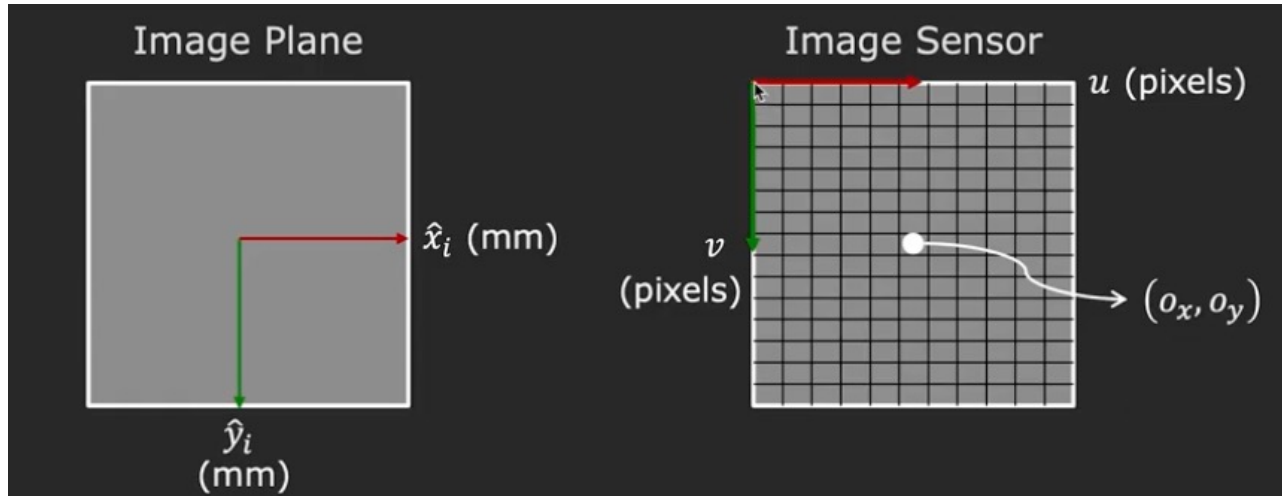
$$\frac{x_i}{f} = \frac{x_c}{z_c}$$

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Camera Coordinates

$$x_i = f \frac{x_c}{z_c}$$

Image Plane to Image Sensor Mapping



1. Account for pixel density (pixel/mm) & aspect ratio by scalars: $[m_x, m_y]$

$$m_x x_i, m_y y_i$$

2. Usually the top left corner is the origin. But in the image plane, the origin is where the optical axis pierces the plane! Need to shift by:

$$(o_x, o_y)$$

$$u_i = \alpha_x x_i + o_x = \alpha_x f \frac{x_c}{z_c} + o_x$$

$$\text{where } [f_x, f_y] = [m_x f, m_y f]$$

Pixel Coordinates:

$$u_i = \boxed{f_x} \frac{x_c}{z_c} + \boxed{o_x} \quad v_i = \boxed{f_y} \frac{y_c}{z_c} + \boxed{o_y}$$

With homogeneous coordinates

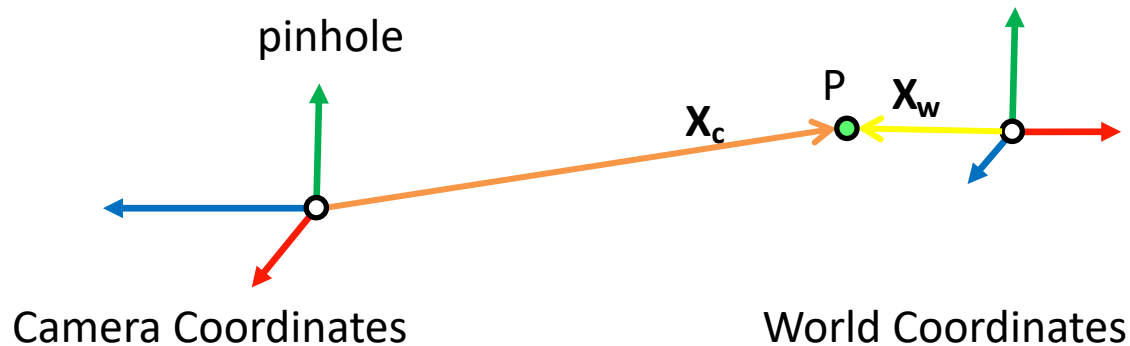
Perspective projection + Transformation to Pixel Coordinates:

$$u_i = f_x \frac{x_c}{z_c} + o_x \quad v_i = f_y \frac{y_c}{z_c} + o_y$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Intrinsic Matrix

Camera Transformation (3D-to-3D)



$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

←
Coordinate
Transformation

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

**Extrinsic
Matrix**

Putting it all together

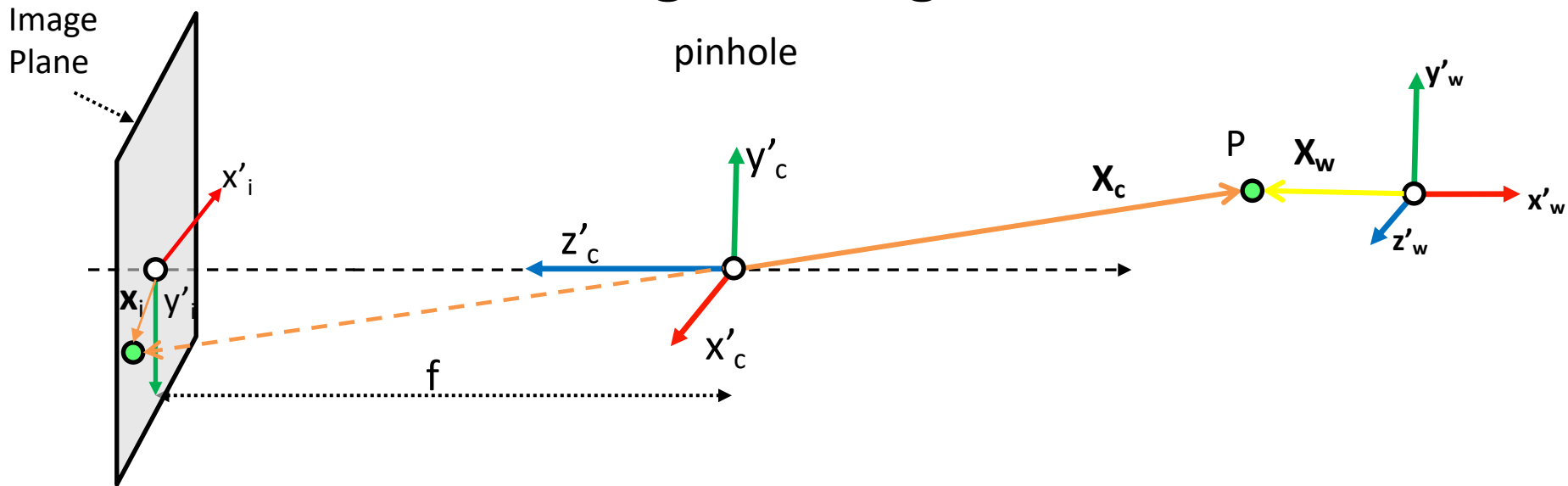
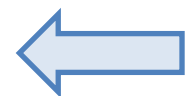


Image Coordinates

Camera Coordinates

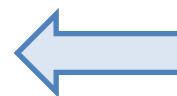
World Coordinates

$$\mathbf{X}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$



Perspective
Projection

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$



Coordinate
Transformation

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

$$\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$

Projection Matrix

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}}_{\text{3 x 4 Projection matrix}} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

3 x 4 Projection matrix

What's the Degrees of Freedom?

For completeness, we need to add **skew** (this is 0 unless pixels are shaped like rhombi/parallelograms)

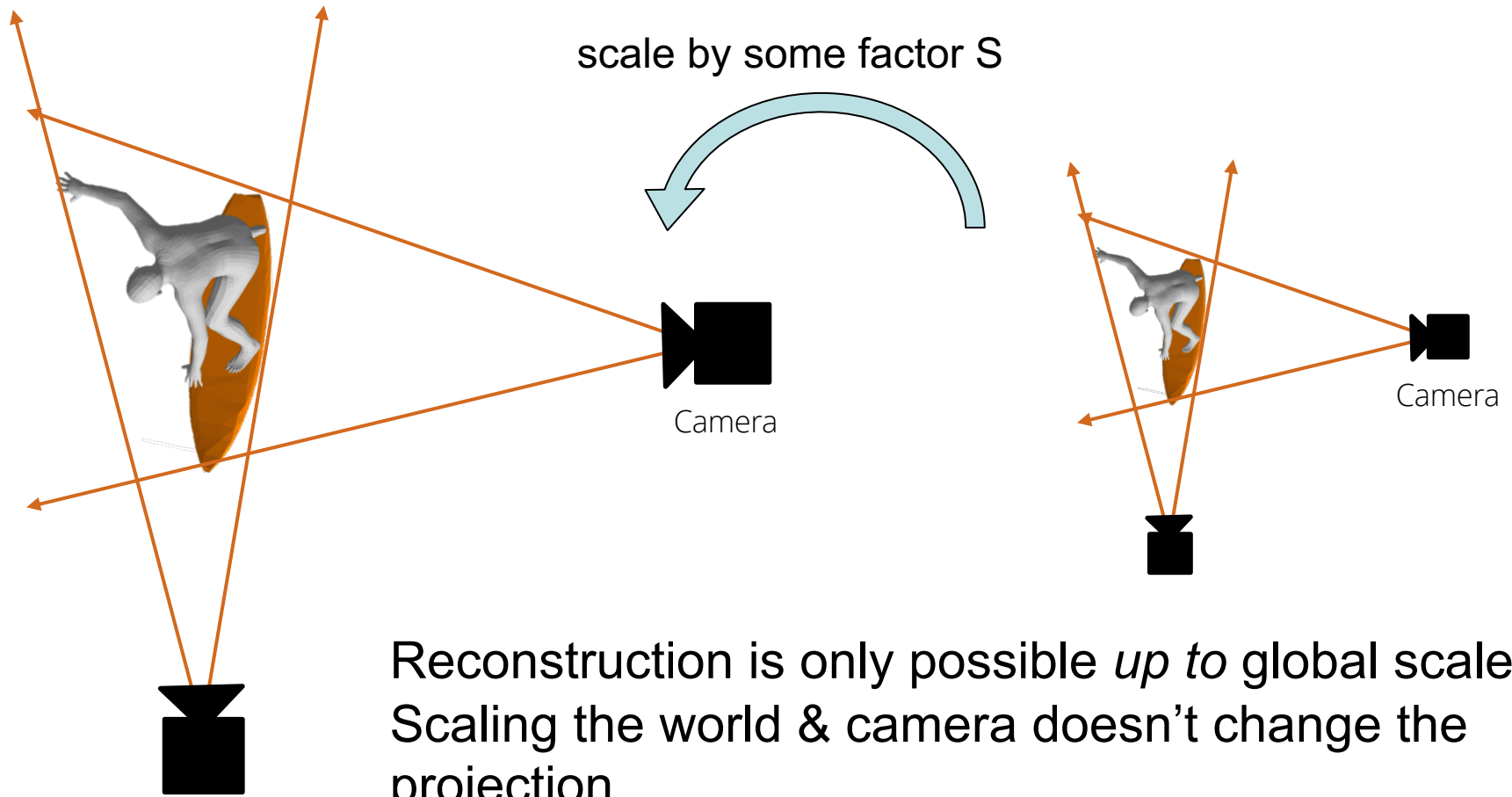
$$K = \begin{bmatrix} f_x & s & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Intrinsics: 4 + 1 (skew)

Extrinsic: 3 + 3 = 6

11 unknowns (up to scale)

Fundamental Scale Ambiguity



Reconstruction is only possible *up to* global scale
Scaling the world & camera doesn't change the projection
Unless you know something metric about the scene
e.g. surfboard is 2.1m

Going from World to Camera

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Extrinsic Matrix:

$$T_{w2c} = \begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$

$$\mathbf{X}_c = T_{w2c} \mathbf{X}_w$$

Going from Camera to World

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Extrinsic Matrix:

$$T_{w2c} = \begin{bmatrix} R_{3 \times 3} & \mathbf{t} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$

$$T_{w2c}^{-1} \mathbf{X}_c = \mathbf{X}_w$$

Camera to Image

Image Coordinates

$$\mathbf{X}_i = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix}$$

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Intrinsics Matrix:

$$K = \begin{bmatrix} f_x & s & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{x}_i = K \mathbf{X}_c$$

Image to Camera?

Image Coordinates

$$\mathbf{X}_i = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix}$$

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

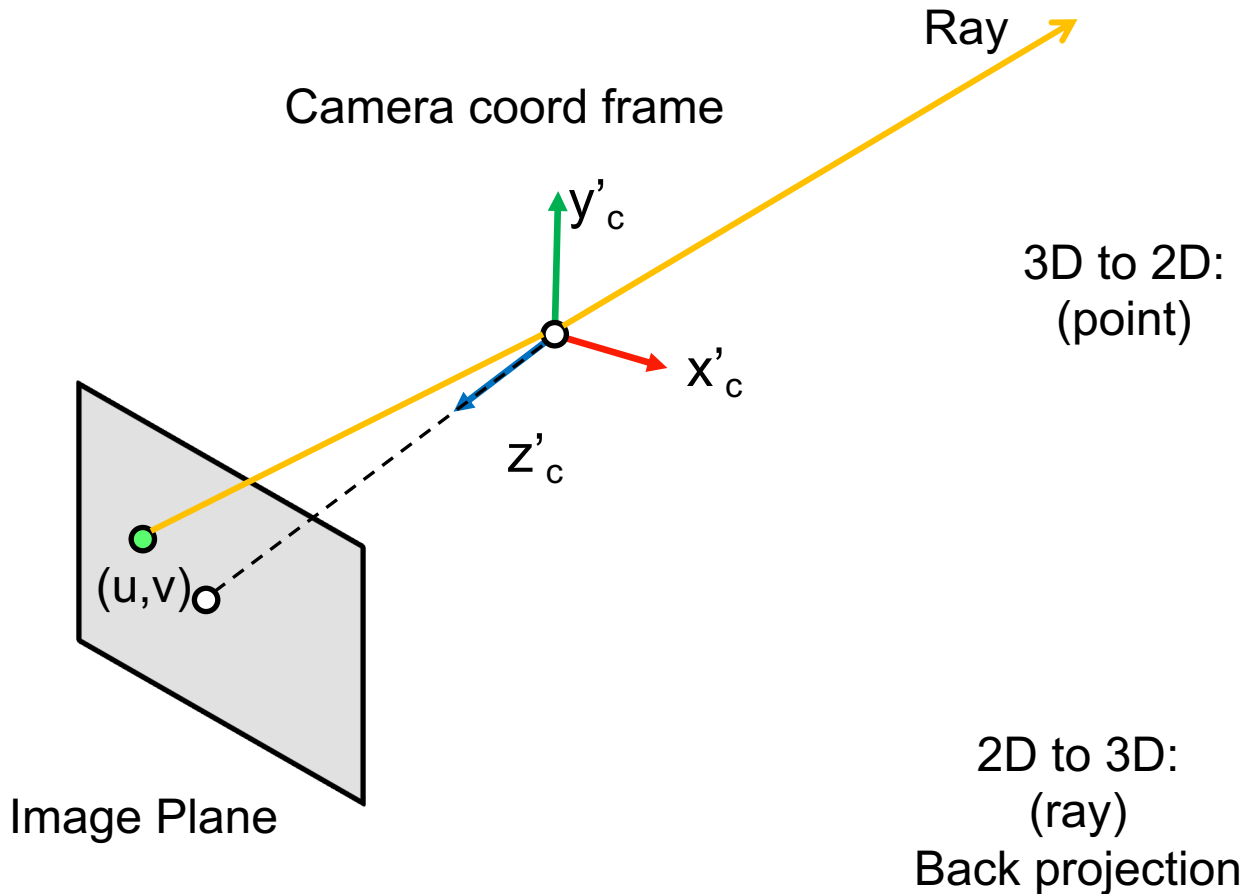
Intrinsics Matrix:

$$K = \begin{bmatrix} f_x & s & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$u = f_x \frac{x_c}{z_c} + o_x \quad \longrightarrow \quad x = \frac{z}{f_x} (u - o_x)$$

What's the problem?

We don't know the depth!
but at the least it will be:
on the ray!



$$u = f_x \frac{x_c}{z_c} + o_x$$
$$v = f_y \frac{y_c}{z_c} + o_y$$

$$x = \frac{z}{f_x} (u - o_x)$$
$$y = \frac{z}{f_y} (v - o_y)$$
$$z > 0$$

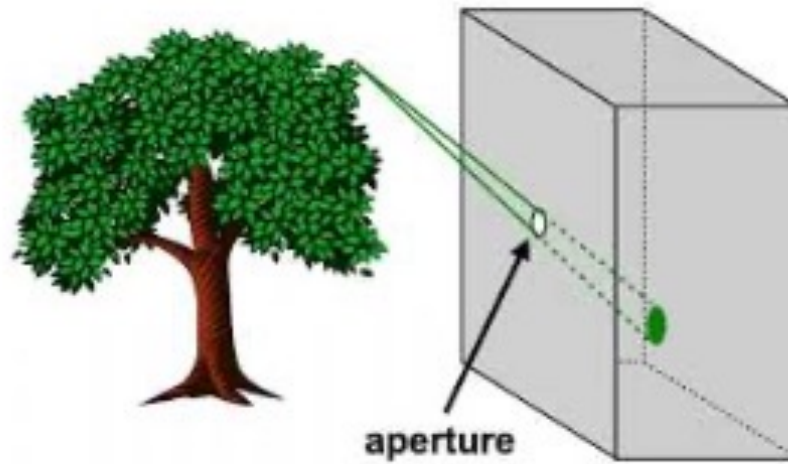
What is your coordinate space?

- In Project 5 (and in life) always make sure you're in the right coordinate space.
- eg. Which space is the ray defined in?

2D to 3D:
(ray)
Back projection

$$\begin{aligned}x &= \frac{z}{f_x} (u - o_x) \\y &= \frac{z}{f_y} (v - o_y) \\z &> 0\end{aligned}$$

Watch these 5 min videos



<https://www.youtube.com/watch?v=F5WA26W4JaM>
<https://www.youtube.com/watch?v=g7Pb8mrwcJ0>

Where are my cameras?

How to calibrate the camera?

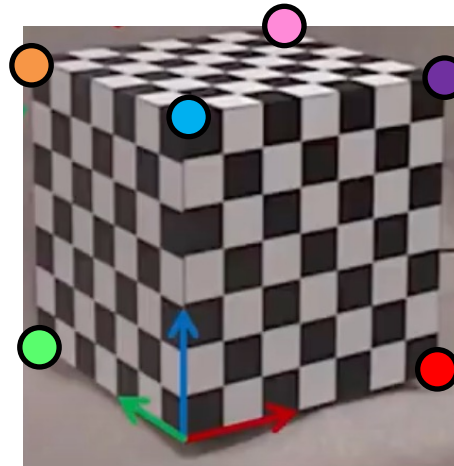
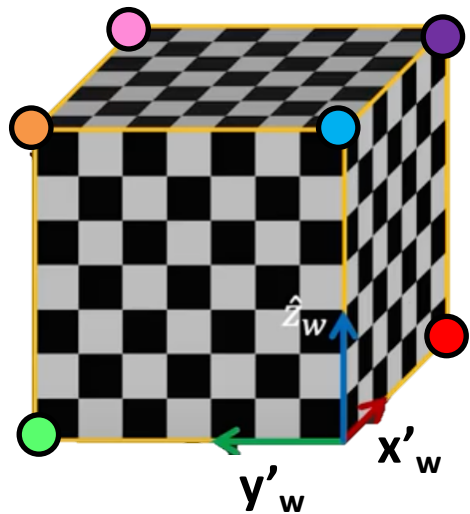
$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \mathbf{X}$$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

If we know the points in 3D we can estimate the camera!!

Step 1: With a known 3D object

1. Take a picture of an object with known 3D geometry



2. Identify correspondences

How do we calibrate a camera?

880 214
43 203
270 197
886 347
745 302
943 128
476 590
419 214
317 335
783 521
235 427
665 429
655 362
427 333
412 415
746 351
434 415
525 234
716 308
602 187

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

312.747 309.140 30.086
305.796 311.649 30.356
307.694 312.358 30.418
310.149 307.186 29.298
311.937 310.105 29.216
311.202 307.572 30.682
307.106 306.876 28.660
309.317 312.490 30.230
307.435 310.151 29.318
308.253 306.300 28.881
306.650 309.301 28.905
308.069 306.831 29.189
309.671 308.834 29.029
308.255 309.955 29.267
307.546 308.613 28.963
311.036 309.206 28.913
307.518 308.175 29.069
309.950 311.262 29.990
312.160 310.772 29.080
311.988 312.709 30.514

Method: Set up a linear system

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- Solve for m's entries using linear least squares

Ax=0 form

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1 X_1 & -u_1 Y_1 & -u_1 Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1 X_1 & -v_1 Y_1 & -v_1 Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_n X_n & -u_n Y_n & -u_n Z_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_n X_n & -v_n Y_n & -v_n Z_n & -v_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Similar to how you solved for homography!

Can we factorize M back to K [R | T]?

- Yes.
- Why? because K and R have a very special form:

$$\begin{bmatrix} f_x & s & O_x \\ 0 & f_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

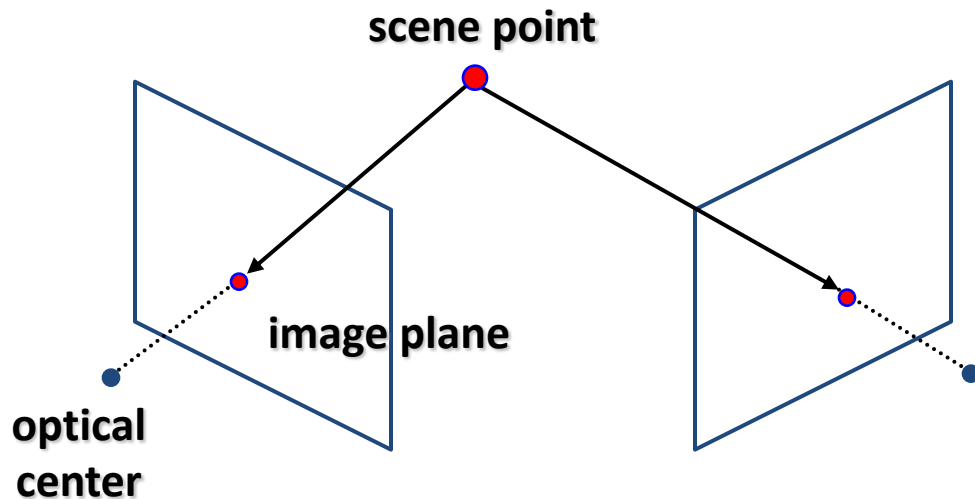
- QR decomposition
- Practically, use camera calibration packages (there is a good one in OpenCV)

Now that our cameras are calibrated, can we find the 3D scene point of a pixel?



Estimating depth with stereo

- **Stereo:** shape from “motion” between **two** views
- We’ll need to consider:
 - 1. Camera pose (“calibration”)
 - 2. Image point correspondences



Stereo vision



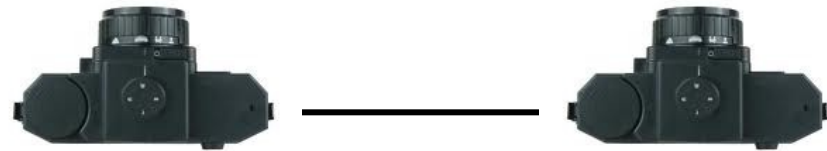
Two cameras, simultaneous views



Single moving camera and static scene

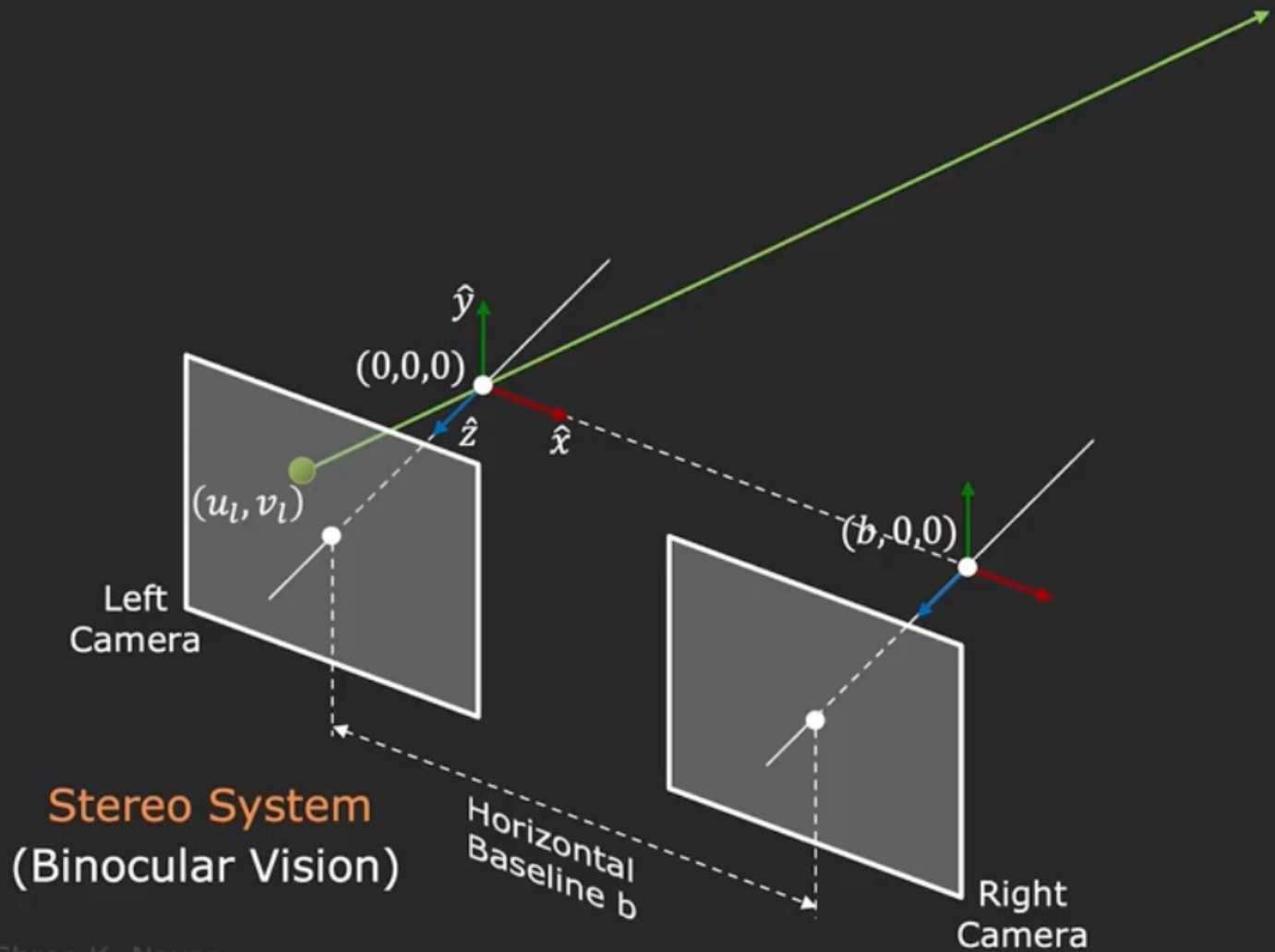
Simple Stereo Setup

- Assume **parallel** optical axes
- Two cameras are calibrated
- Find relative depth

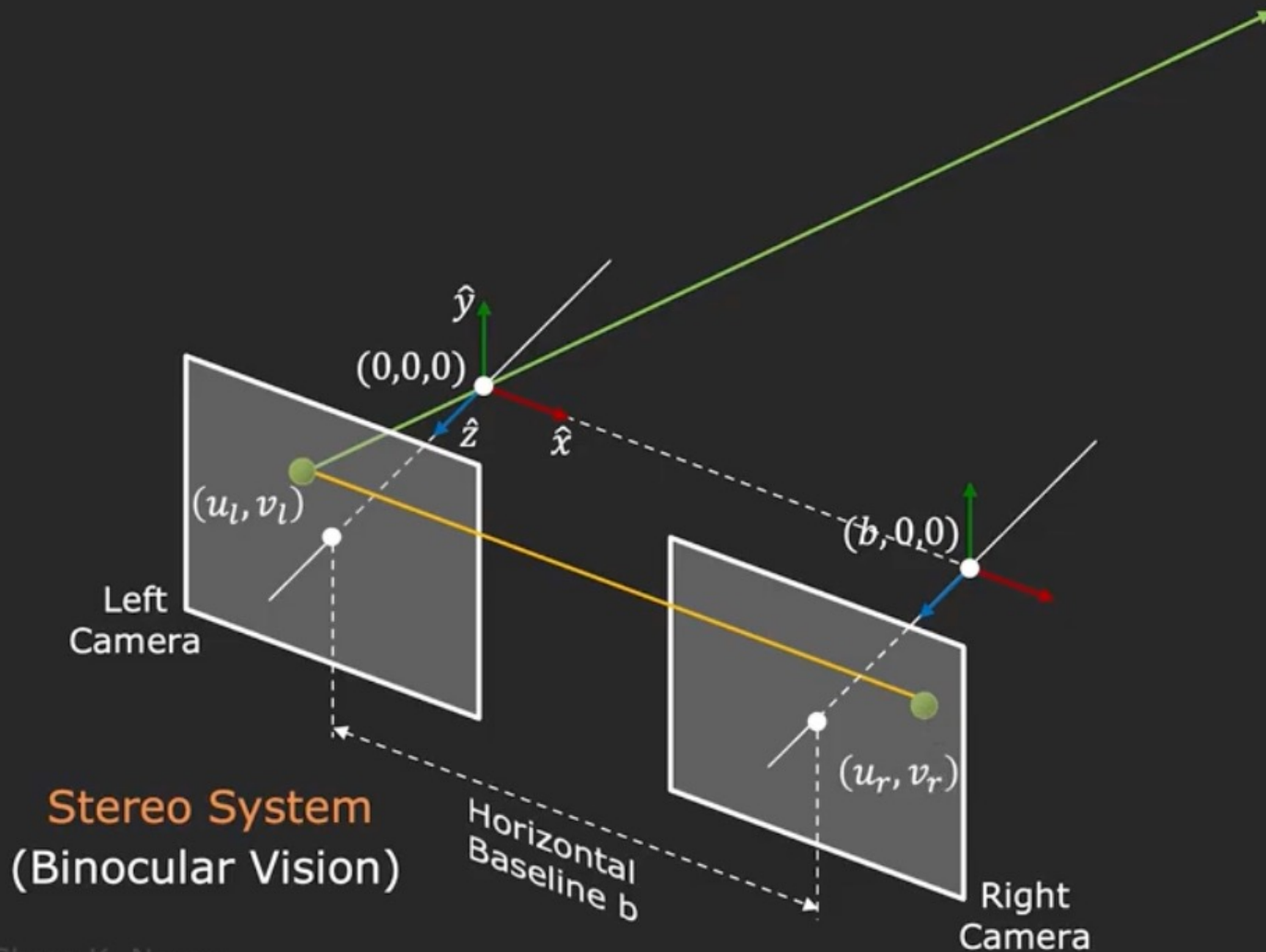


Key Idea: difference in corresponding points to understand shape

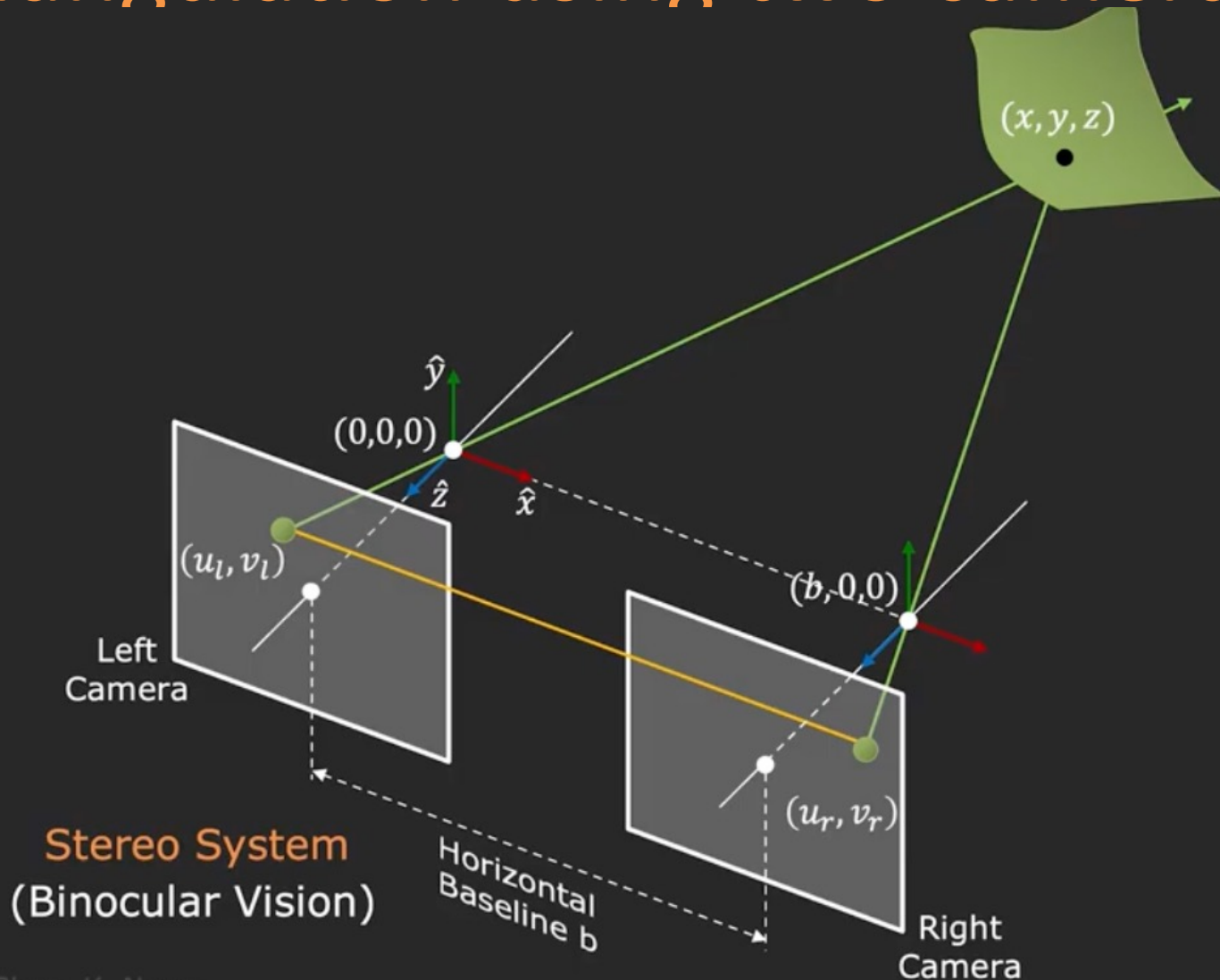
Triangulation using two cameras



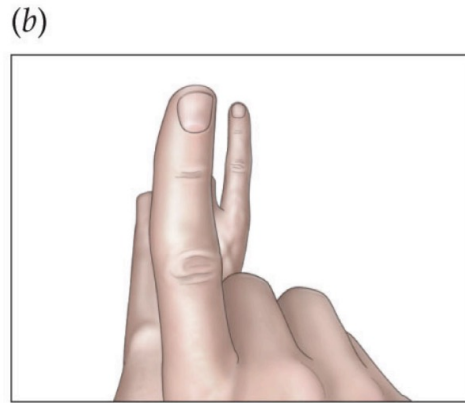
Triangulation using two cameras



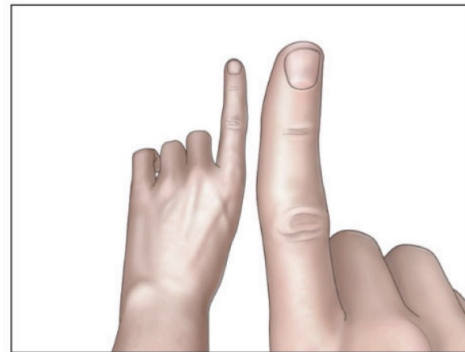
Triangulation using two cameras



We are equipped with binocular vision. Let's try!

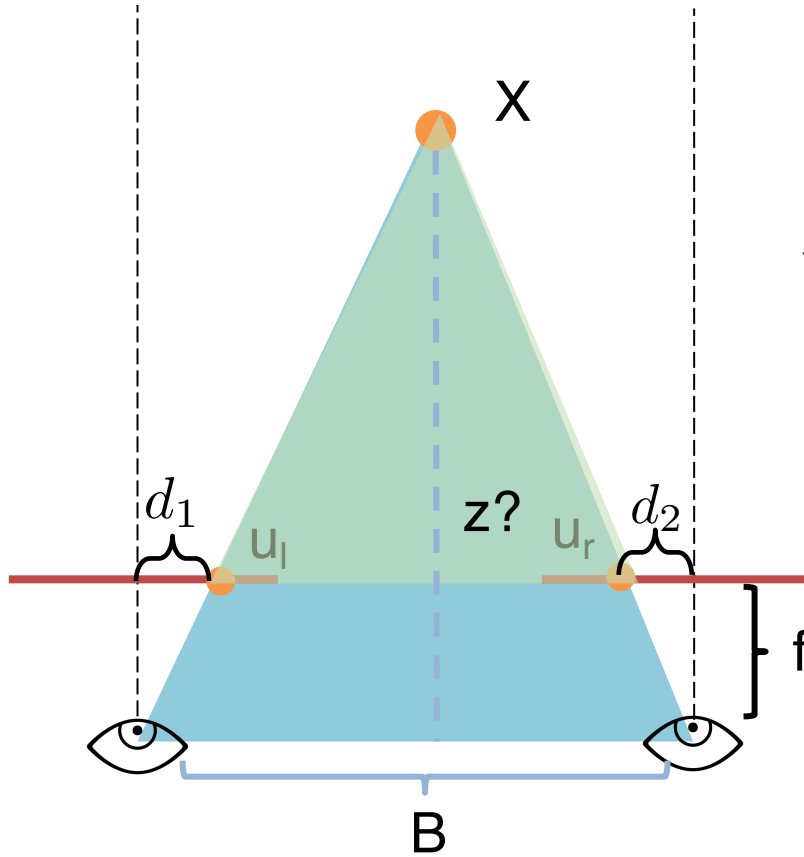


Right retinal image



Left retinal image

Solving for Depth in Simple Stereo



Do we have enough to know what is Z?

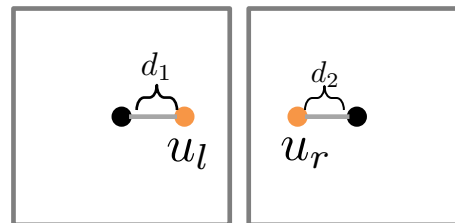
Yes, similar triangles!

$$\triangleleft \frac{B - (u_l - u_r)}{z - f} = \frac{B}{z} \triangleright$$

$$z = \frac{f B}{u_l - u_r}$$

disparity
(how much
corrsp. pixels
move)

Base of \triangleleft : $B - (d_1 + d_2)$
in image coordinates: $= B - (u_l - u_r)$

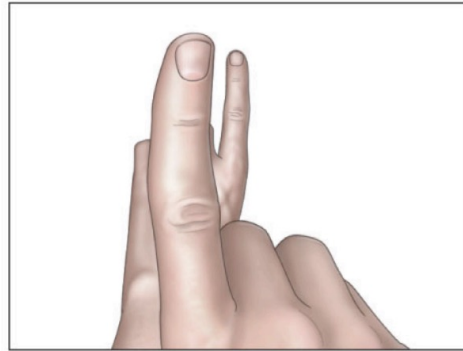


Try with your hands!

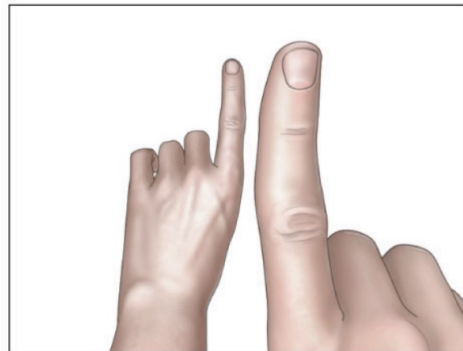
(a)



(b)

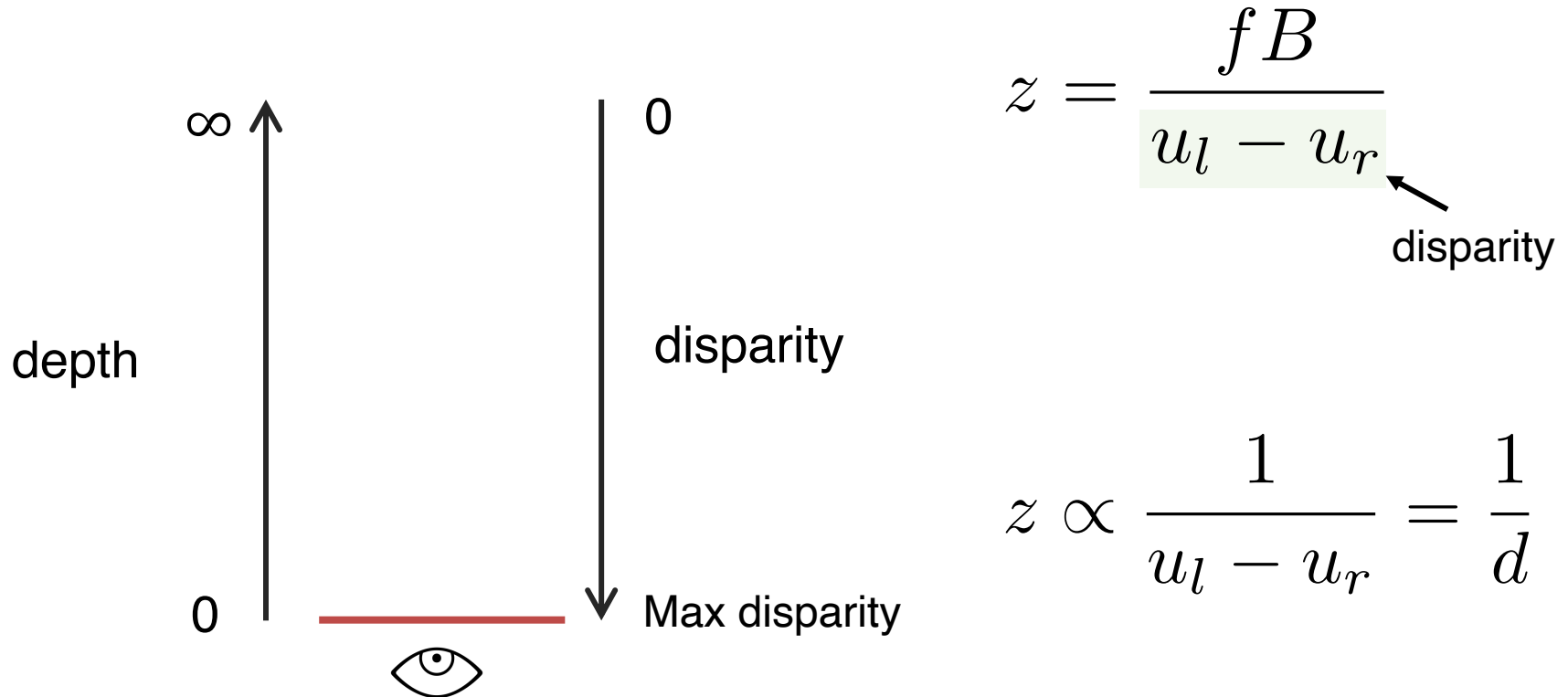


Right retinal image



Left retinal image

Depth is inversely proportional to disparity



what is the disparity of the closer point?

what is the disparity of the far away point?

Disparity gives you the depth information!

Try again

1. Setup so your fingers are on the same line of sight from one eye
2. Now look in the other eye

They move!

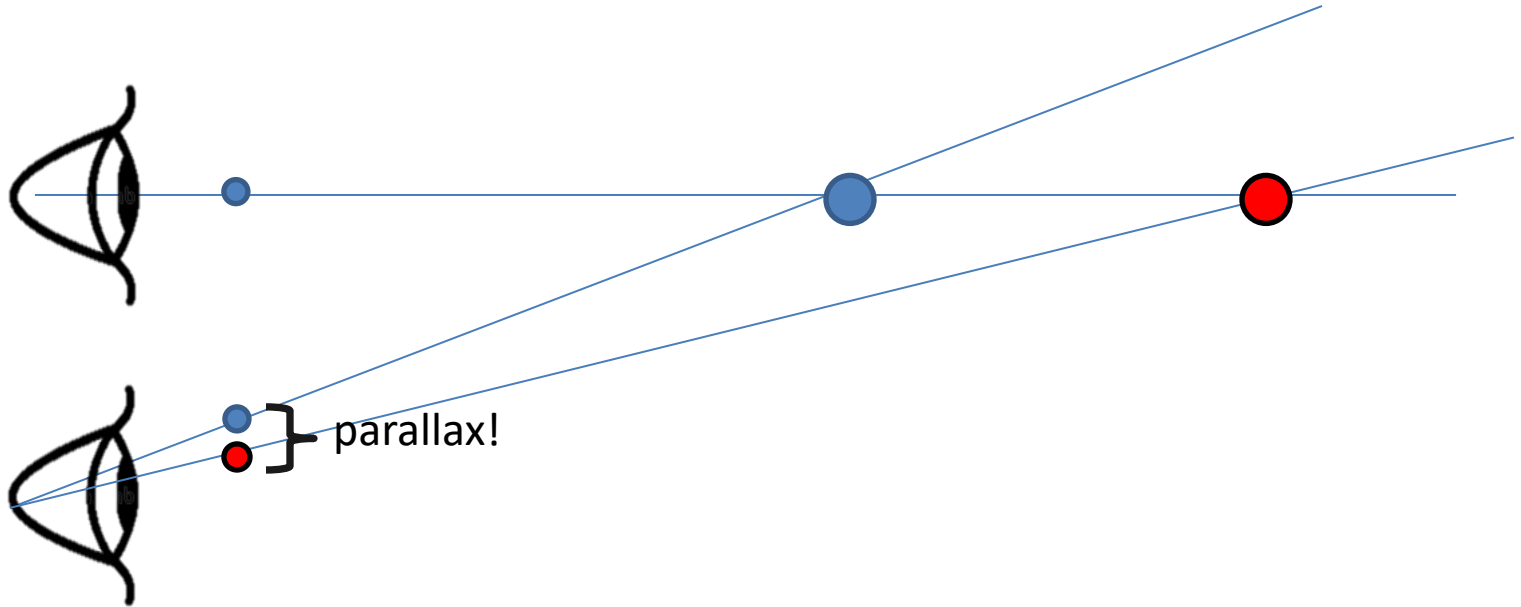
Relative displacement is higher as
the relative distance grows

== Parallax





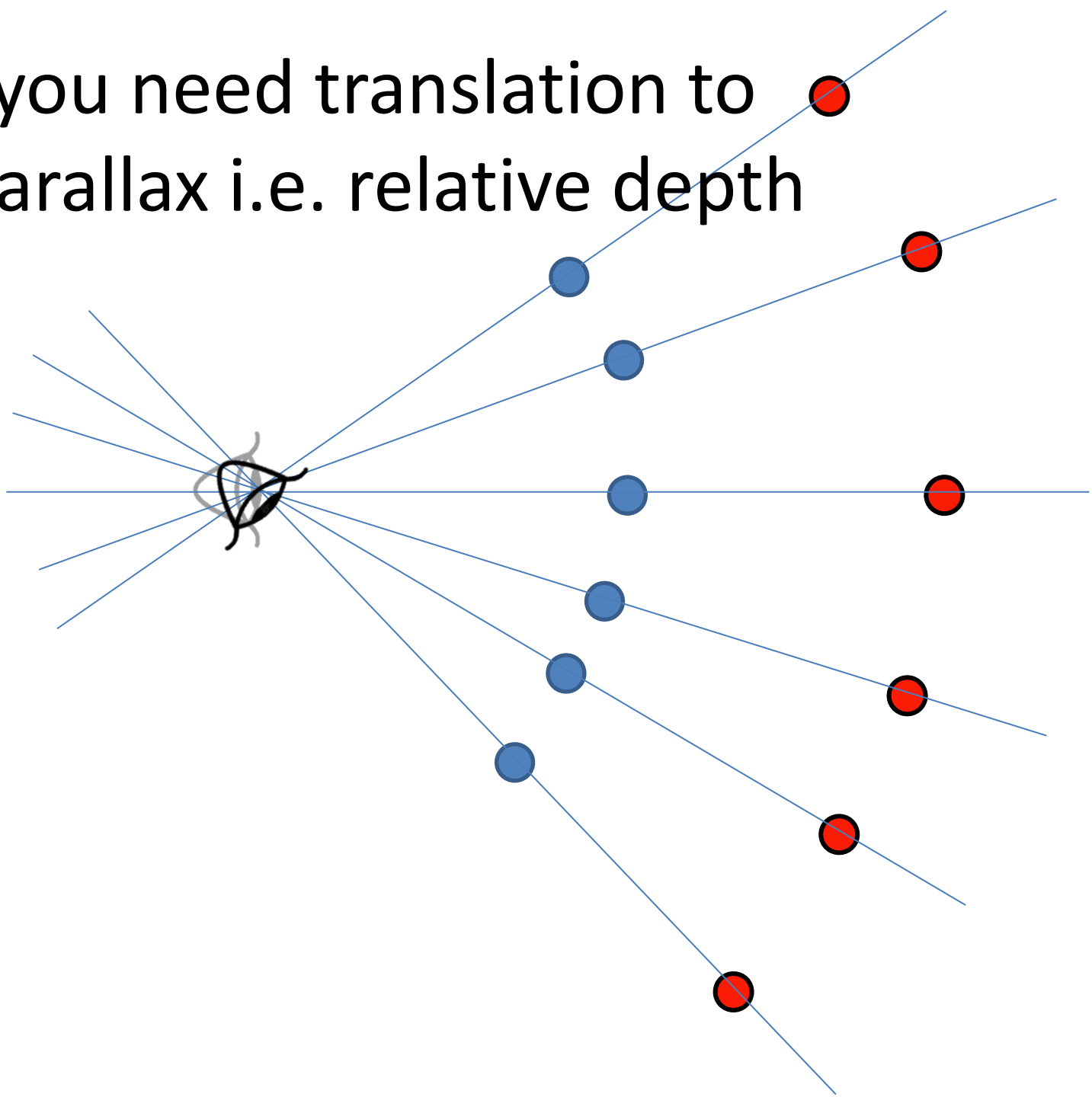
Parallax



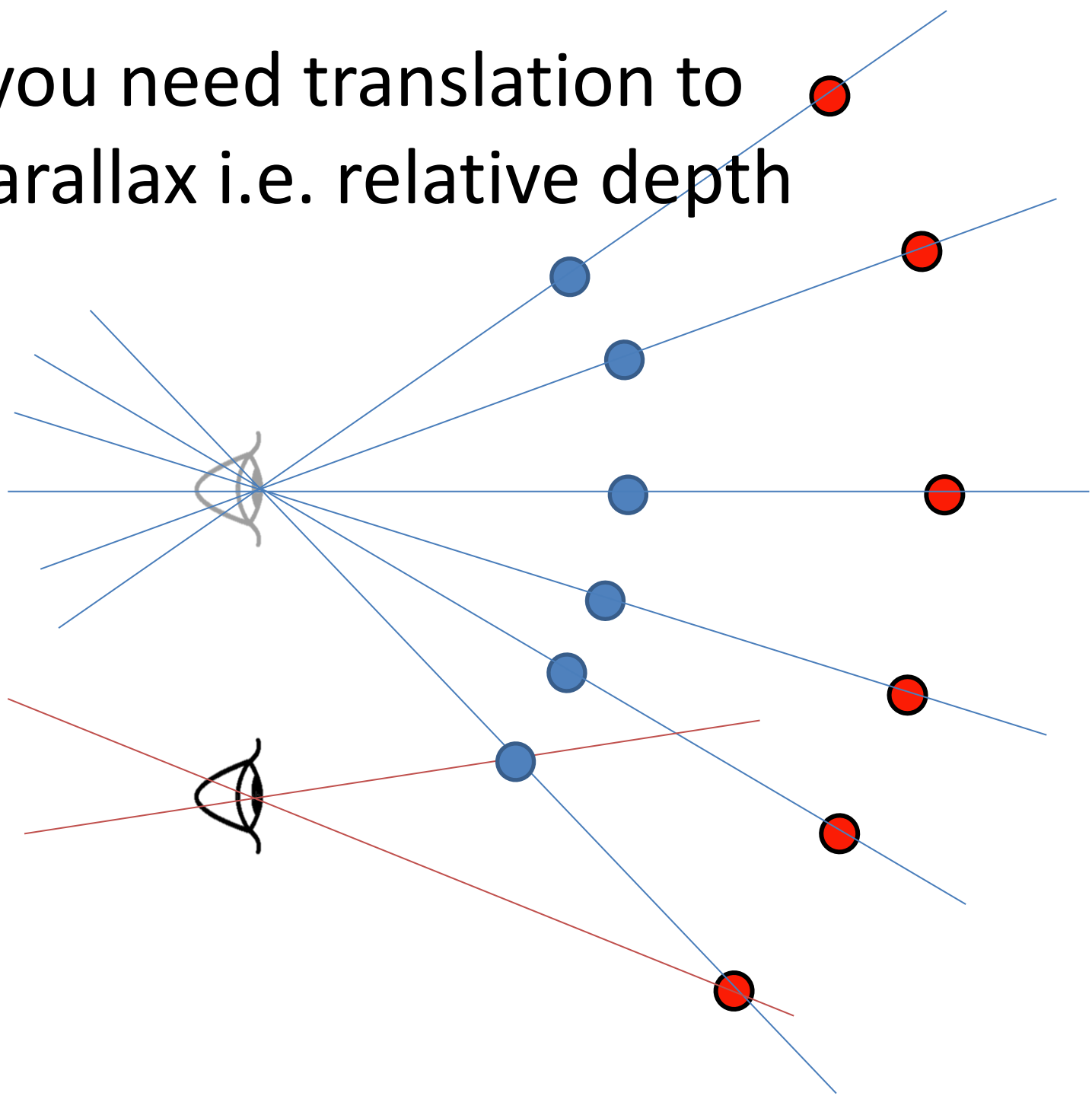
Parallax = *from ancient Greek parállaxis*
= *Para* (side by side) + *allássō*, (to alter)
= *Change in position from different view point*

Two eyes give you parallax, you can also move to see more
parallax = "Motion Parallax"

Why you need translation to see parallax i.e. relative depth



Why you need translation to see parallax i.e. relative depth



Stereo Matching: Finding Disparities

Goal: Find the disparity between left and right stereo pairs.



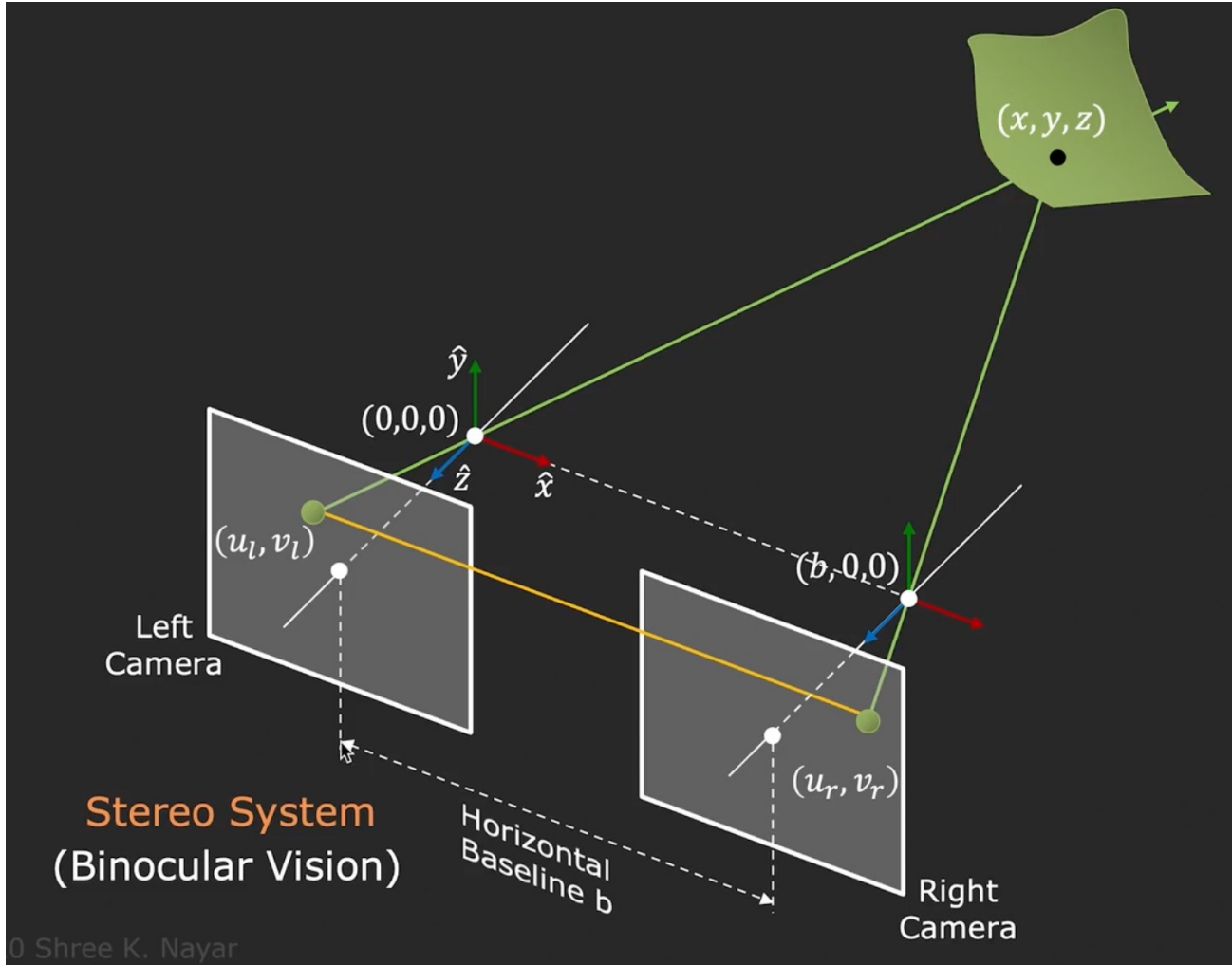
Left/Right Camera Images



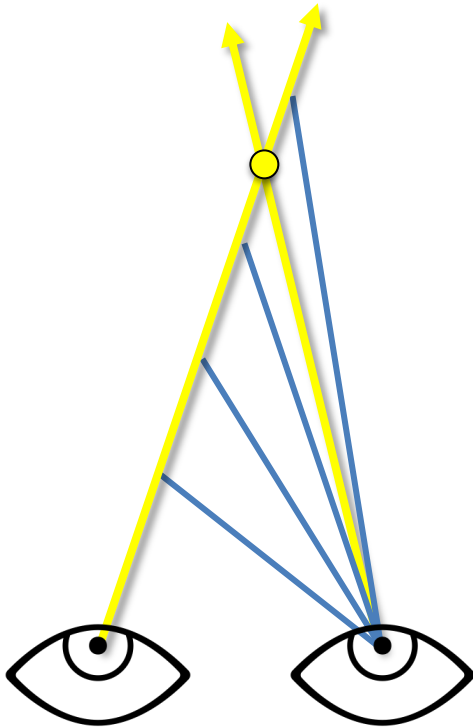
Disparity Map (Ground Truth)

Where is the corresponding point going to be?

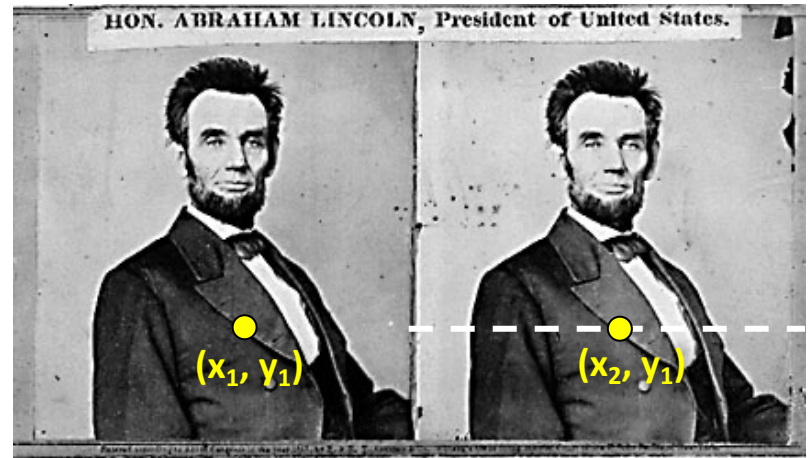
Hint



Epipolar Line



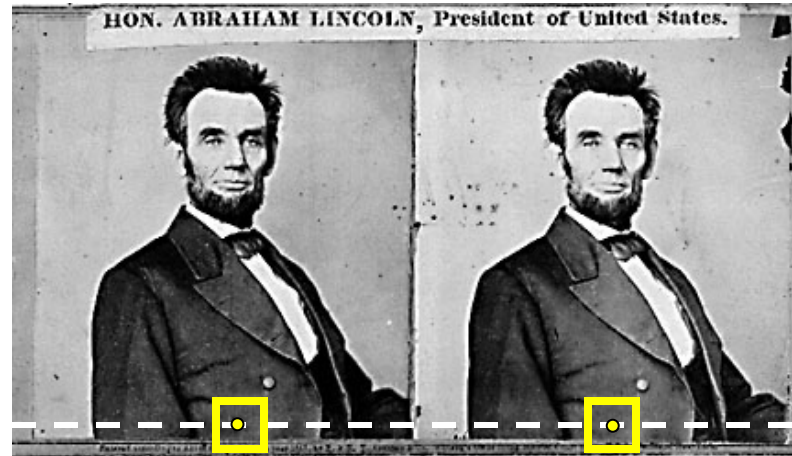
*epipolar
lines*



Two images captured by a purely horizontal translating camera
(*rectified* stereo pair)

$x_1 - x_2 =$ the *disparity* of pixel (x_1, y_1)

Your basic stereo algorithm



For every epipolar line:

For each pixel in the left image

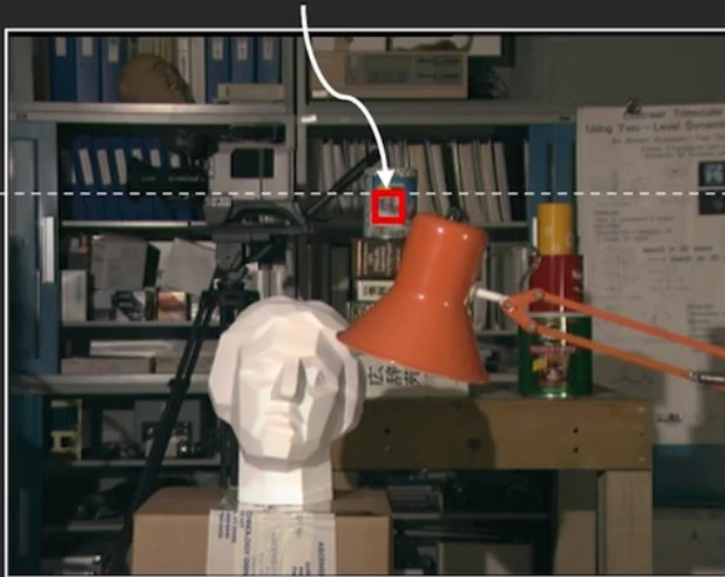
- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match *windows*, + clearly lots of matching strategies

Your basic stereo algorithm

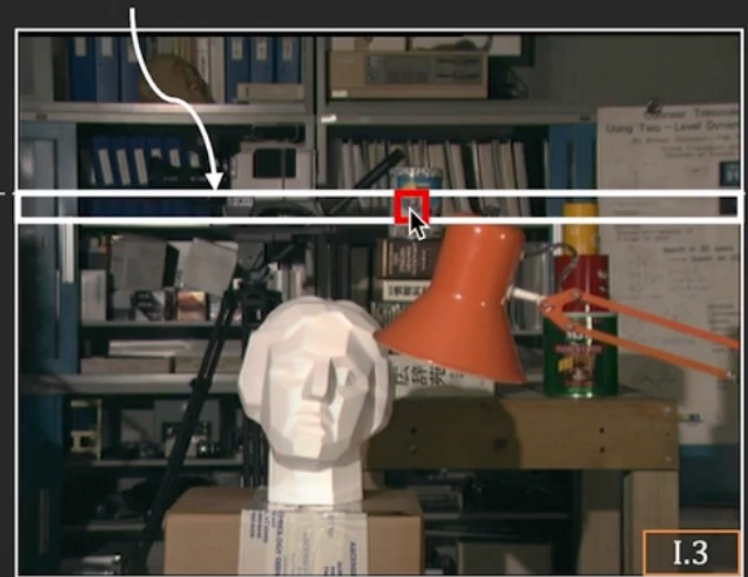
Determine Disparity using **Template Matching**

Template Window T



Left Camera Image E_l

Search Scan Line L



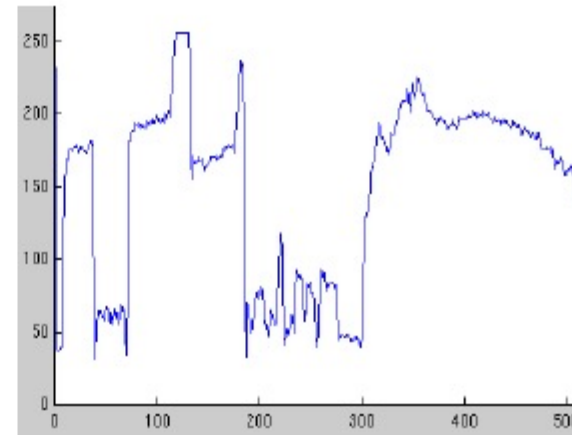
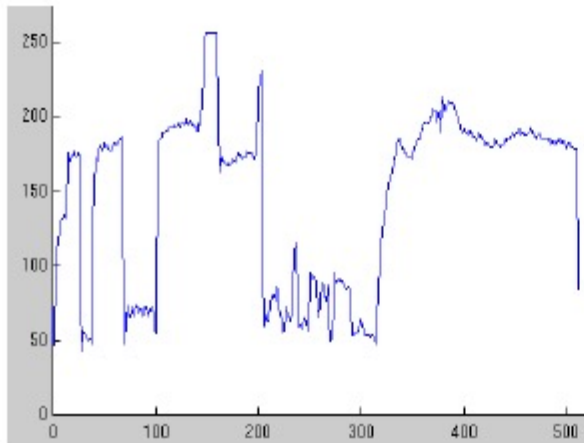
Right Camera Image E_r

Correspondence problem

Parallel camera example – epipolar lines are corresponding rasters

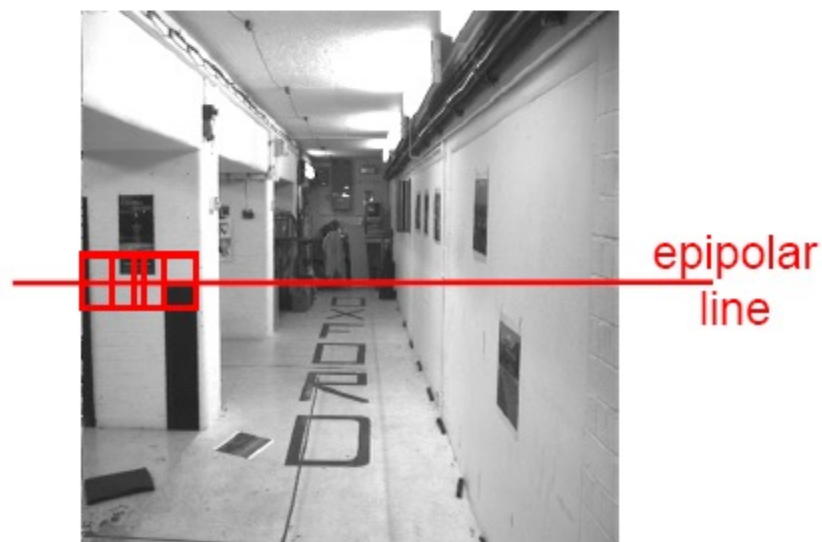


Intensity profiles



- Clear correspondence between intensities, but also noise and ambiguity

Correspondence problem



Neighborhood of corresponding points are similar in intensity patterns.

Normalized cross correlation

subtract mean: $A \leftarrow A - \langle A \rangle, B \leftarrow B - \langle B \rangle$

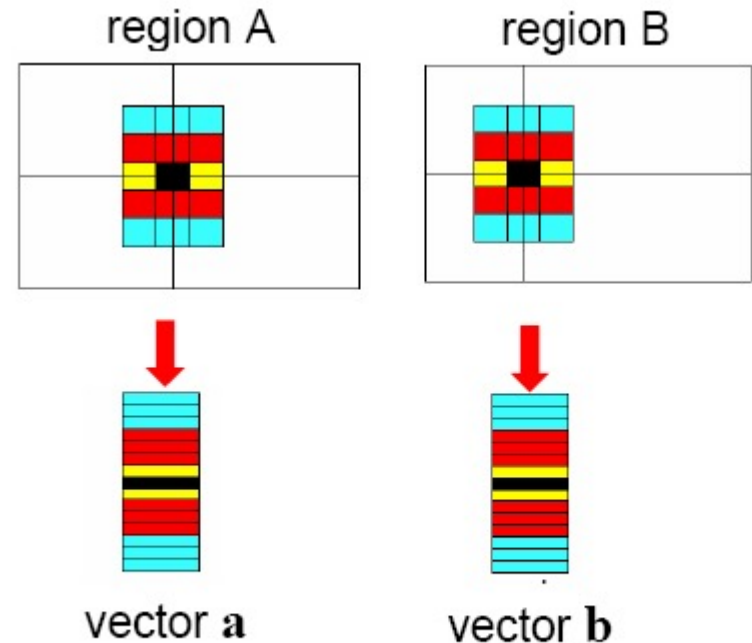
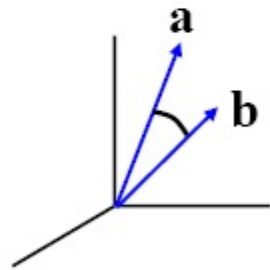
$$NCC = \frac{\sum_i \sum_j A(i, j) B(i, j)}{\sqrt{\sum_i \sum_j A(i, j)^2} \sqrt{\sum_i \sum_j B(i, j)^2}}$$

Write regions as vectors

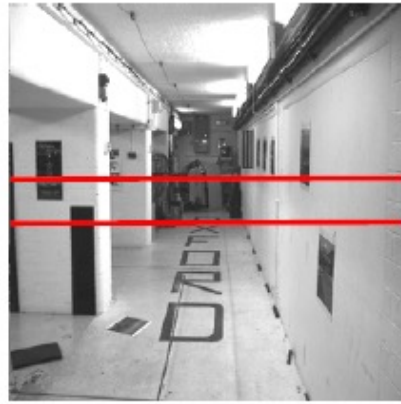
$A \rightarrow \mathbf{a}, B \rightarrow \mathbf{b}$

$$NCC = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$

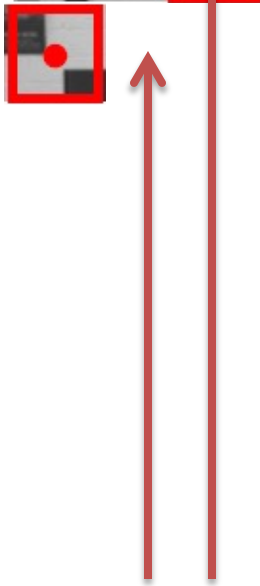
$$-1 \leq NCC \leq 1$$



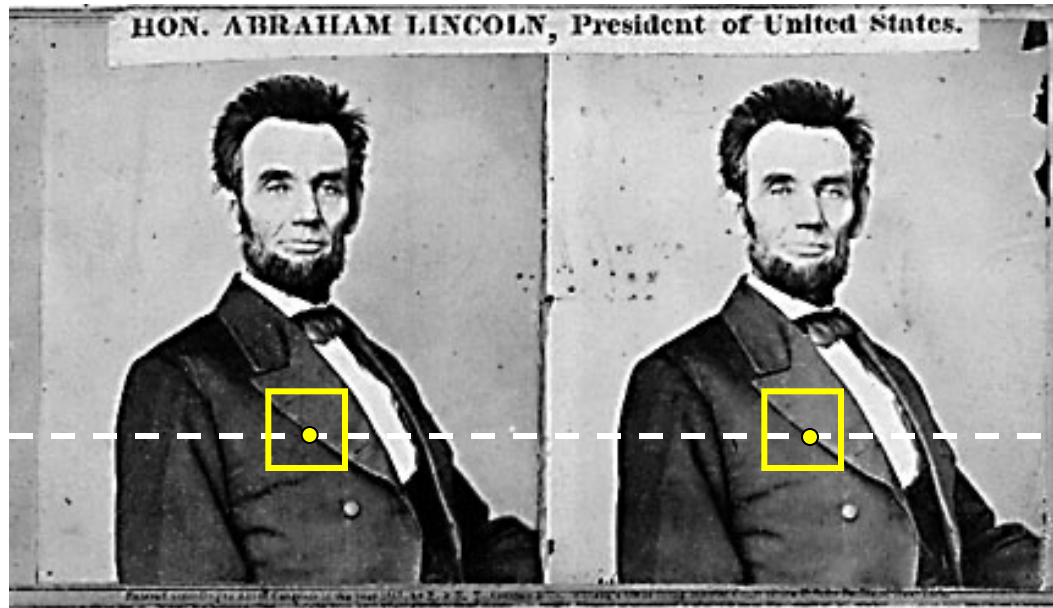
Correlation-based window matching



left image band (x)



Dense correspondence search

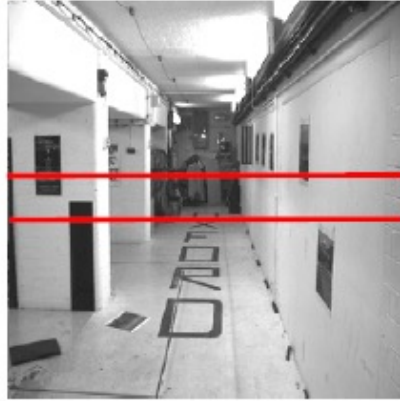


For each epipolar line

For each pixel / window in the left image

- compare with every pixel / window on same epipolar line in right image
- pick position with minimum match cost (e.g., SSD, correlation)

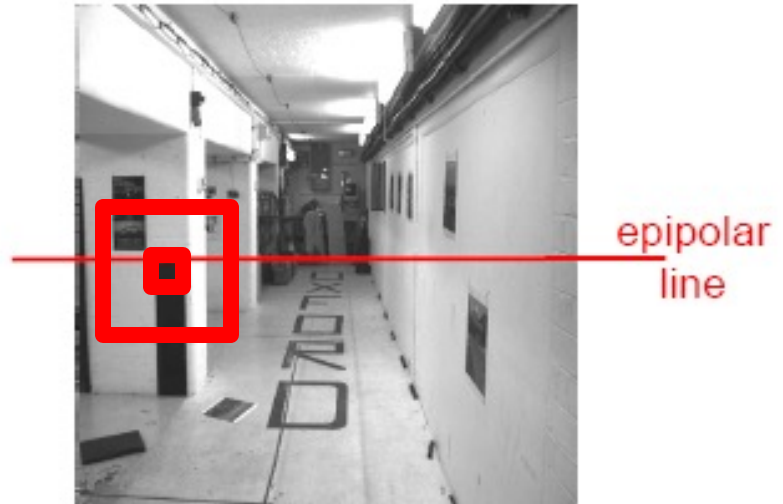
Textureless regions



target region

left image band (x)

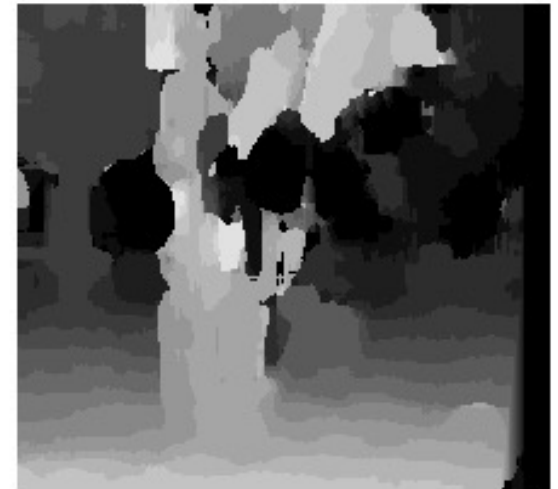
Effect of window size



Effect of window size



$W = 3$



$W = 20$

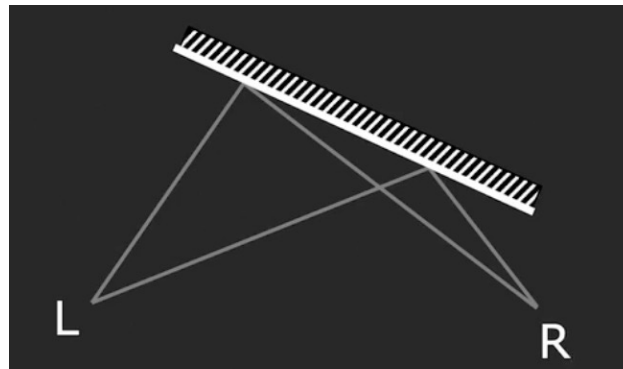
Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Issues with Stereo

- Surface must have non-repetitive texture

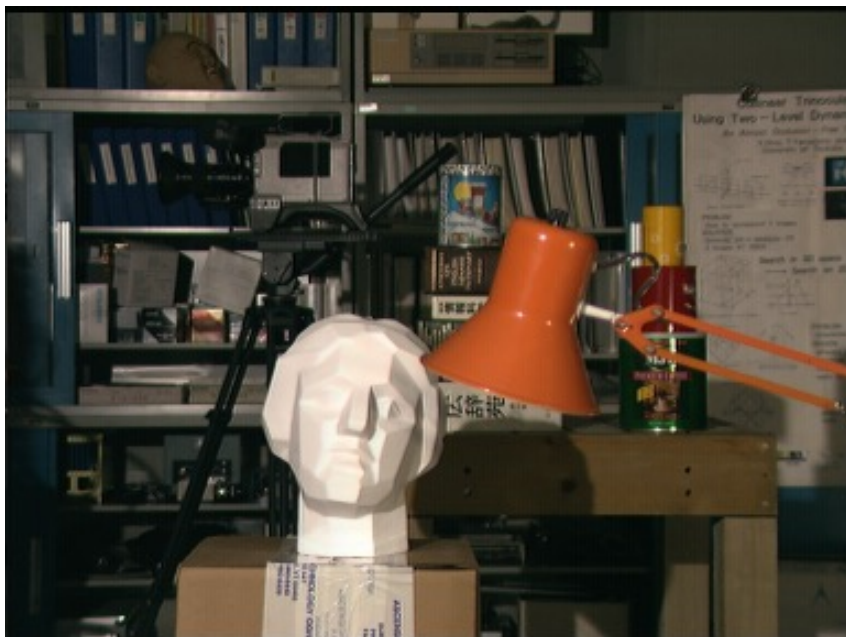


- Foreshortening effect makes matching a challenge



Stereo Results

- Data from University of Tsukuba

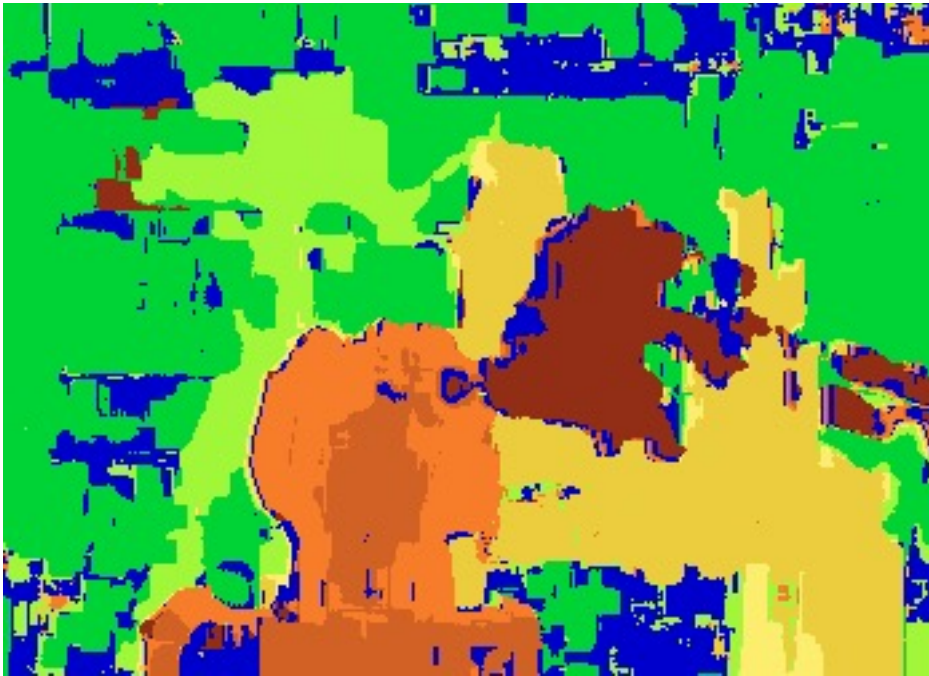


Scene



Ground truth

Results with Window Search



Window-based matching
(best window size)



Ground truth

Better methods exist...



Energy Minimization

Boykov et al., [Fast Approximate Energy Minimization via Graph Cuts](#),
International Conference on Computer Vision, September 1999.



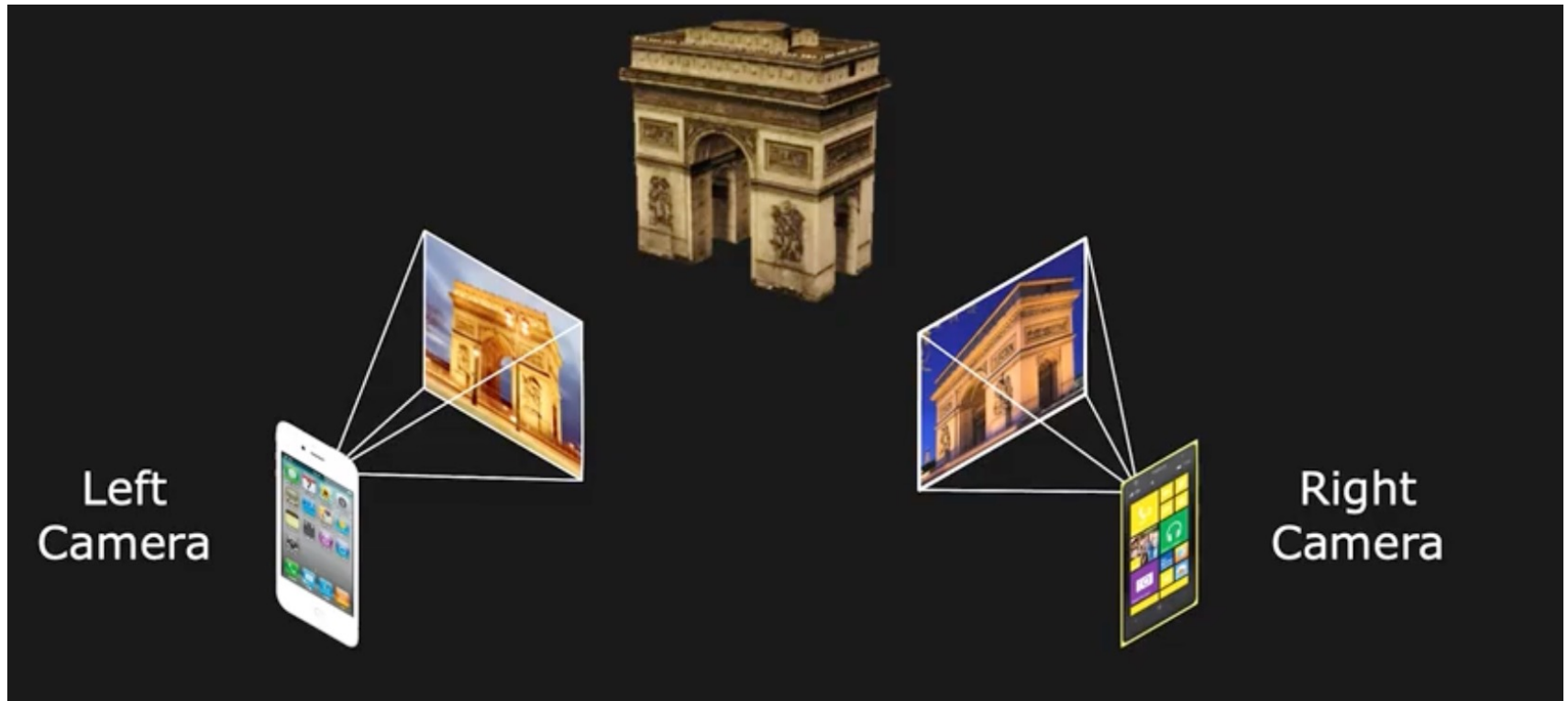
Ground truth

Summary

- With a simple stereo system, **how much pixels move, or “disparity”** give information about the depth
- Correspondences to measure the pixel disparity

Next: Uncalibrated Stereo

- From two arbitrary views



- Assume intrinsics are known (f_x , f_y , o_x , o_y)