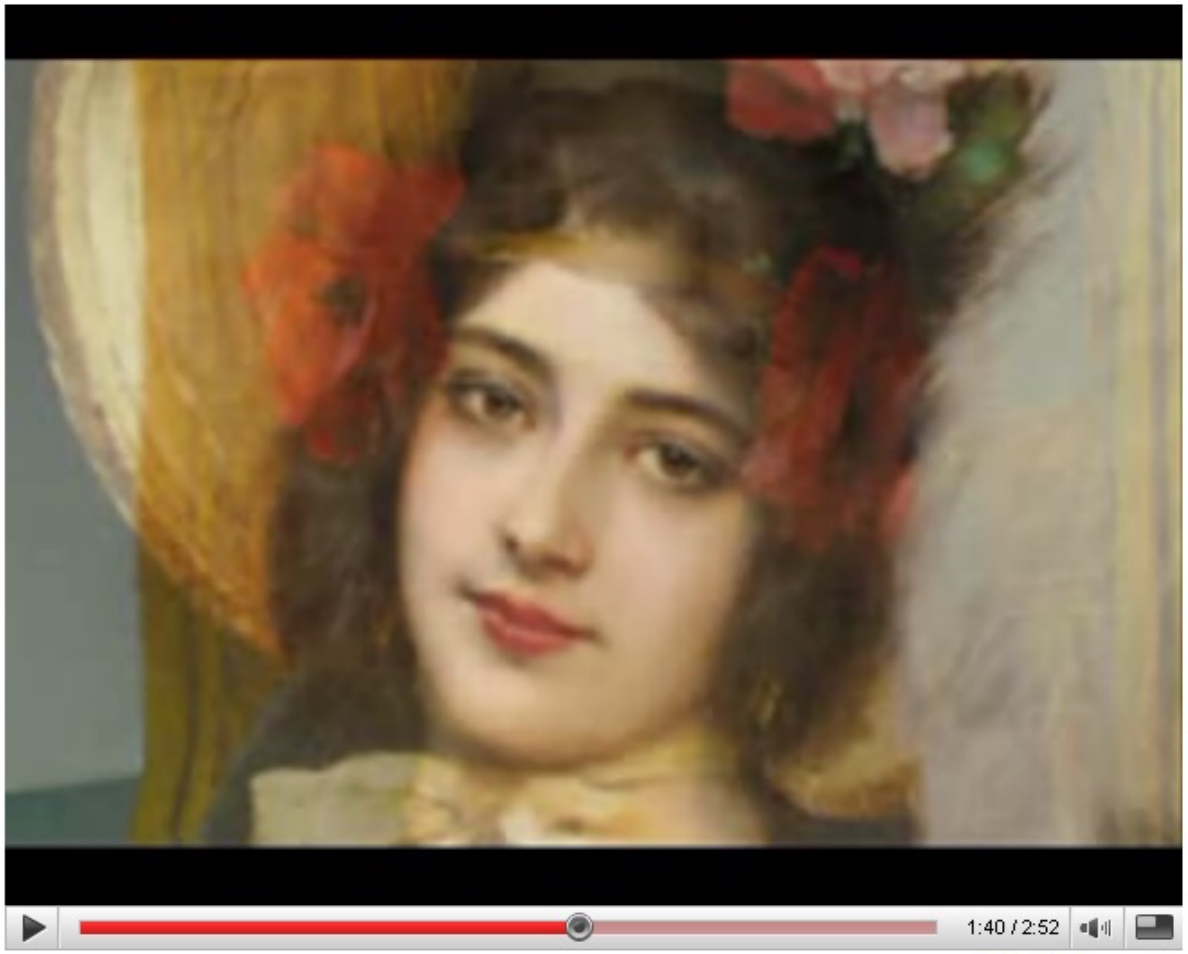
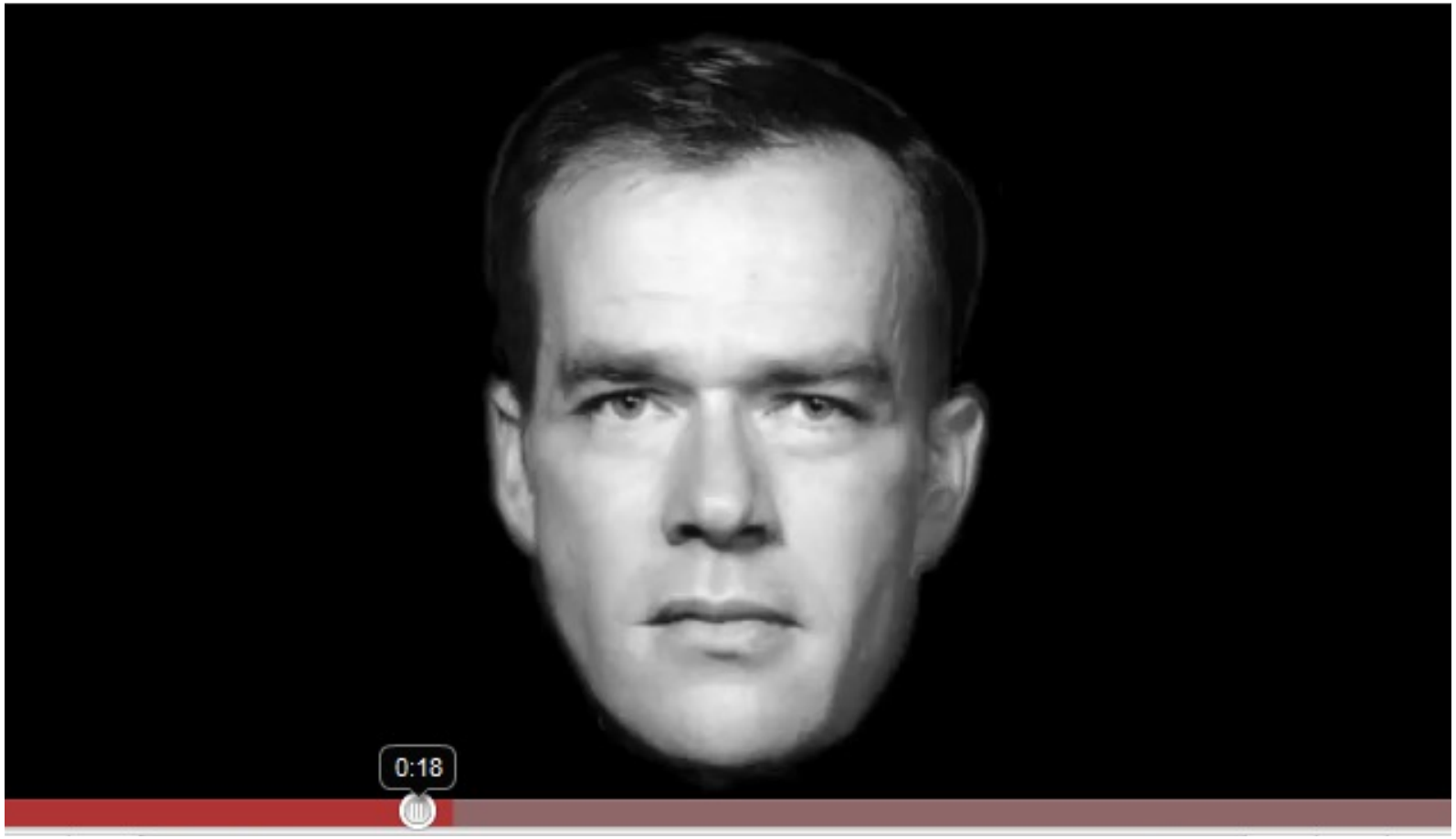
Amuse-bouche



http://youtube.com/watch?v=nUDIoN-_Hxs



http://www.youtube.com/watch?v=L0GKp-uvjO0

Image Warping and Morphing

© Alexey Tikhonov

CS180/280A: Intro to Computer Vision and Comp. Photo

Angjoo Kanazawa & Alexei Efros, UC Berkeley, Fall 2023

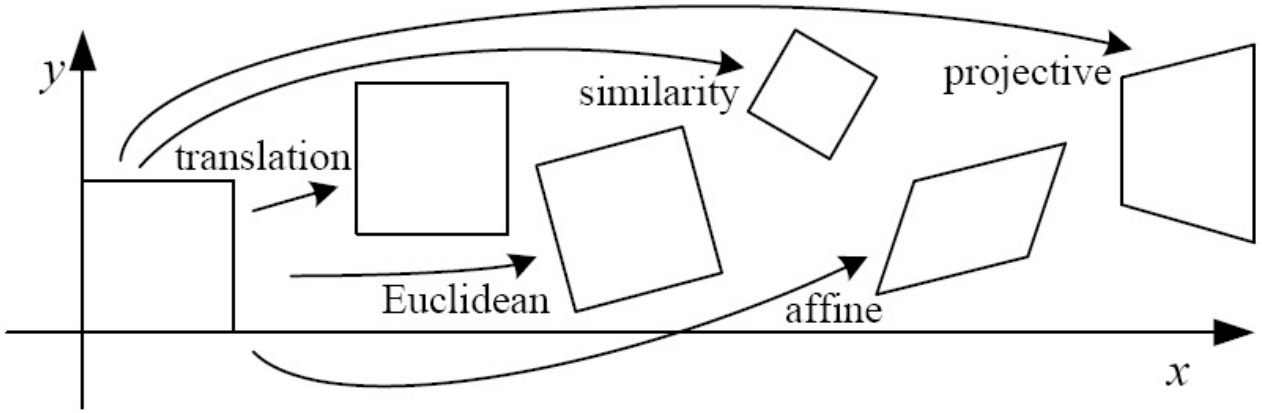
Project 3 out today!!

project 2 how did it go?

project 3 is harder!

Project 1 class choice award form is up (see ed), vote (1 ec!)

2D image transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$			
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$			
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$			
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times3}$			
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times3}$			

These transformations are a nested set of groups

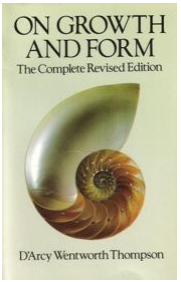
- Closed under composition and inverse is a member

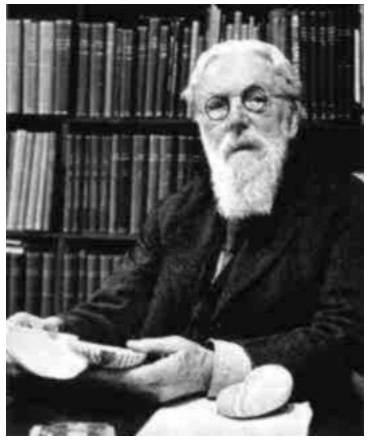
Image Warping in Biology

D'Arcy Thompson

http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/darcy.html

http://en.wikipedia.org/wiki/D'Arcy_Thompson





Importance of shape and structure in evolution

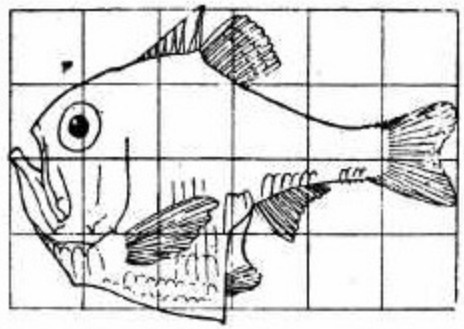


Fig. 517. *Argyropelecus Olfersi.*

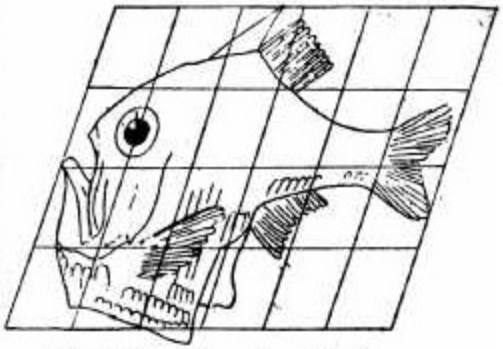
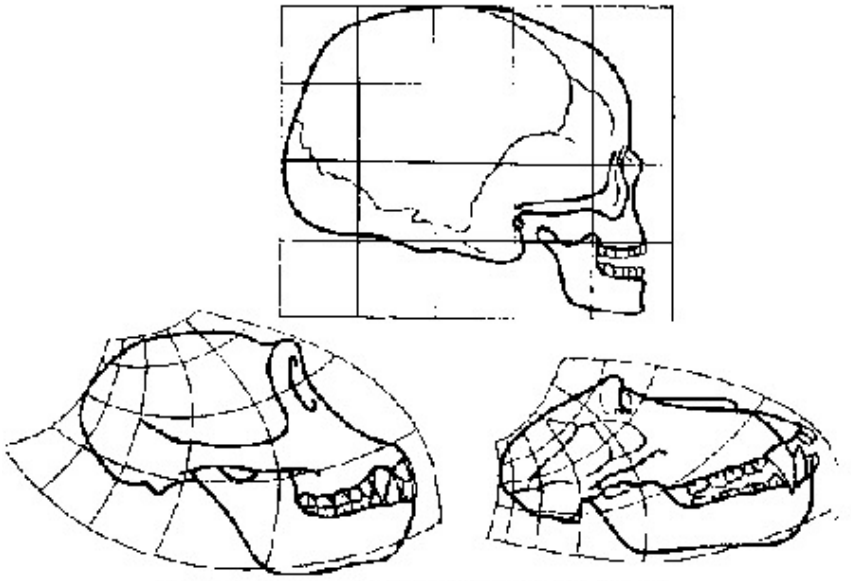


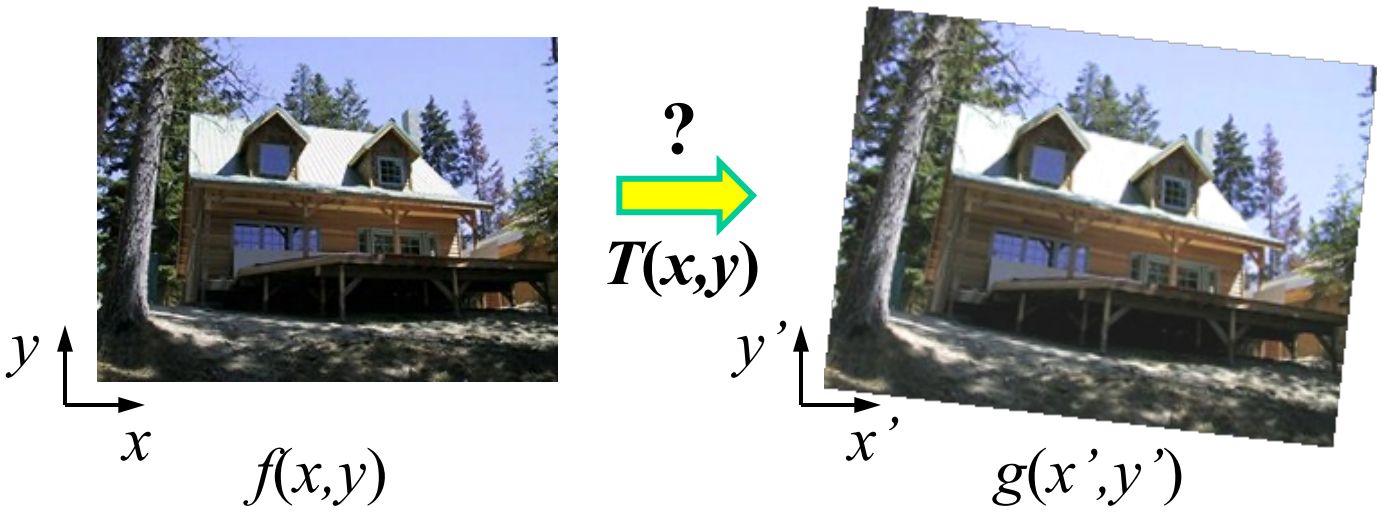
Fig. 518. *Sternoptyx diaphana.*



Skulls of a human, a chimpanzee and a baboon and transformations between them

Slide by Durand and Freeman

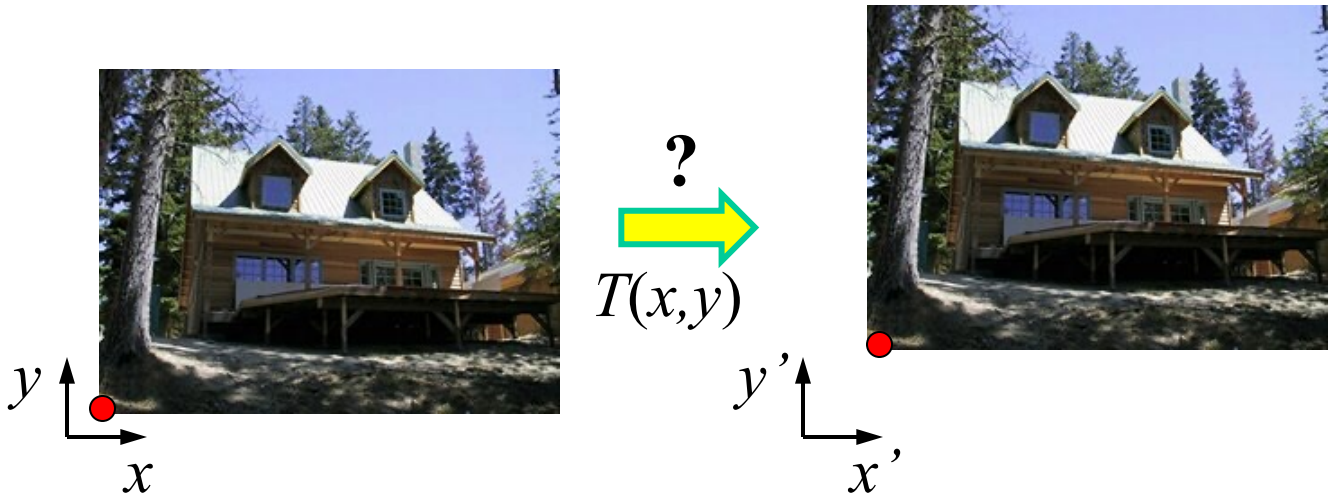
Recovering Transformations



What if we know *f* and *g* and want to recover the transform T?

- e.g. better align images from Project 1
- willing to let user provide correspondences
  - How many do we need?

Translation: # correspondences?



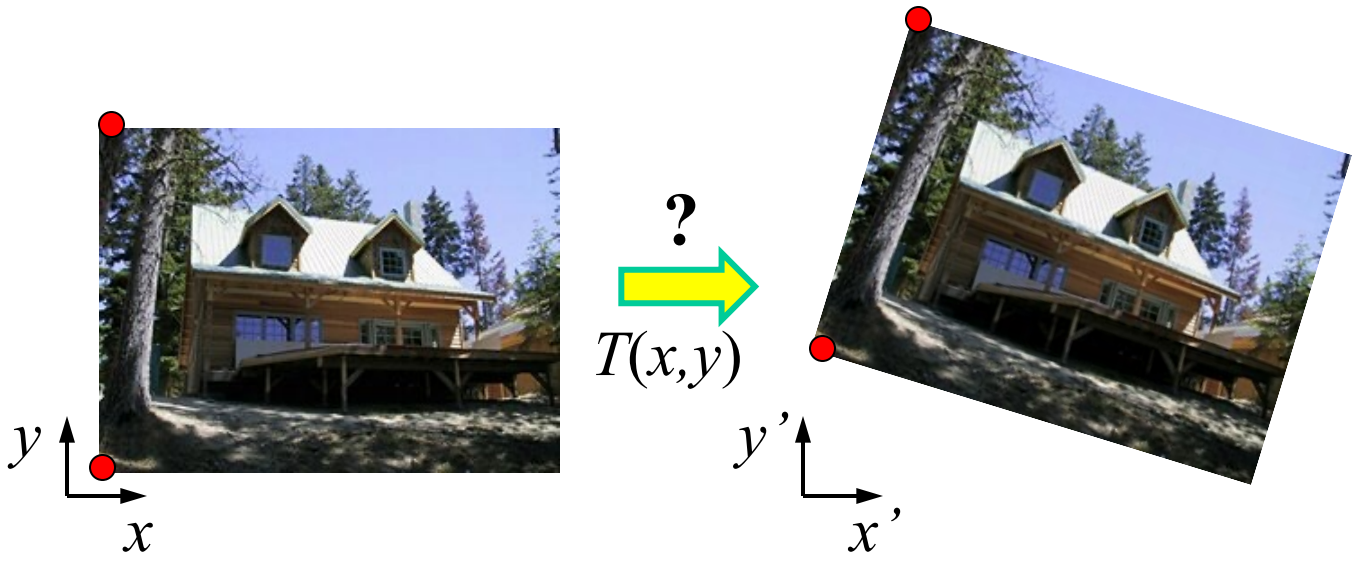
How many correspondences needed for translation?

How many Degrees of Freedom?

What is the transformation matrix?

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

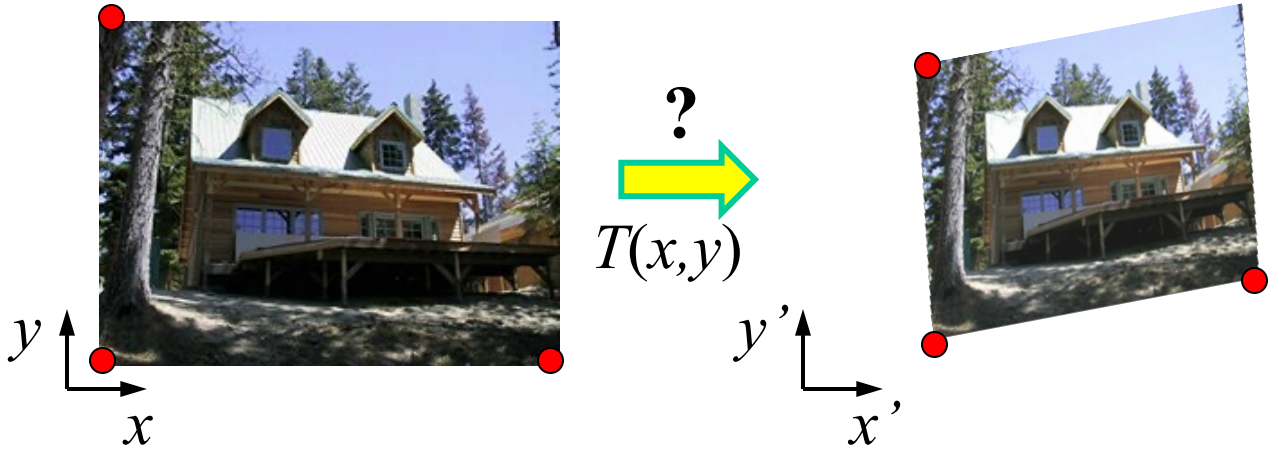
Euclidian: # correspondences?



How many correspondences needed for translation+rotation?

How many DOF?

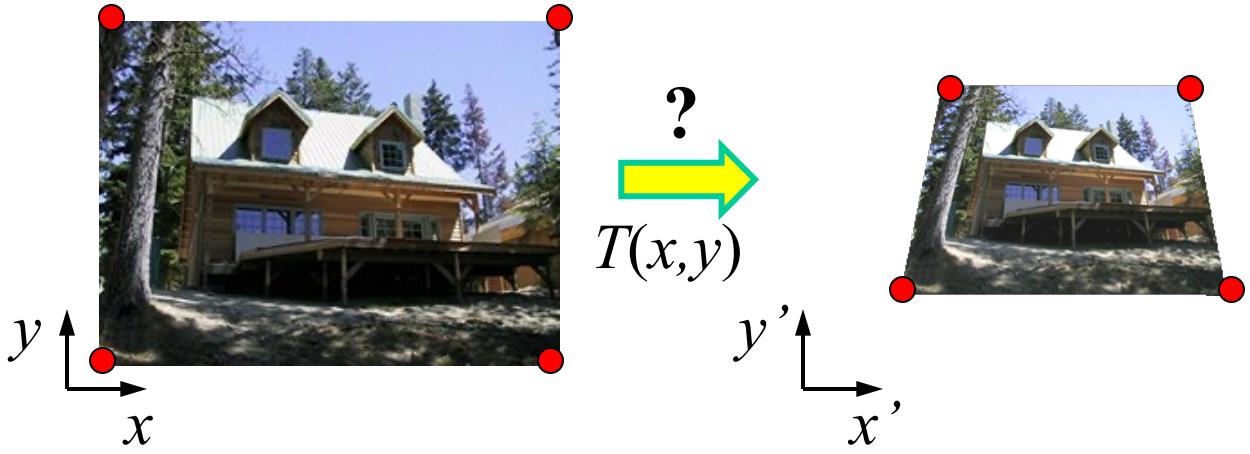
Affine: # correspondences?



How many correspondences needed for affine?

How many DOF?

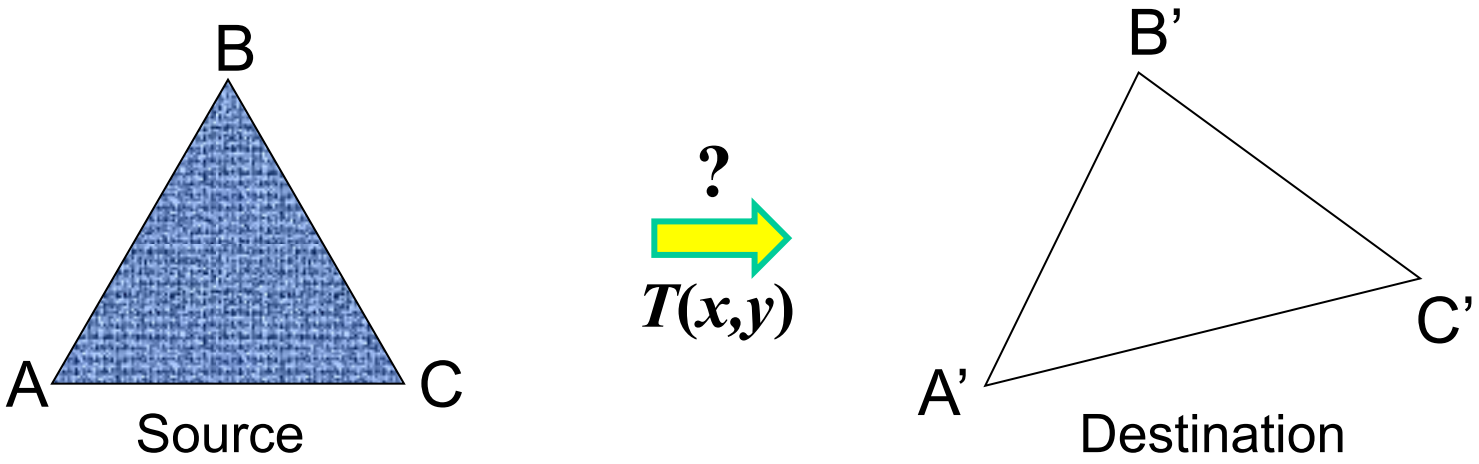
Projective: # correspondences?



How many correspondences needed for projective?

How many DOF?

Example: warping triangles



Given two triangles: ABC and A'B'C' in 2D (12 numbers)

Need to find transform T to transfer all pixels from one to the other.

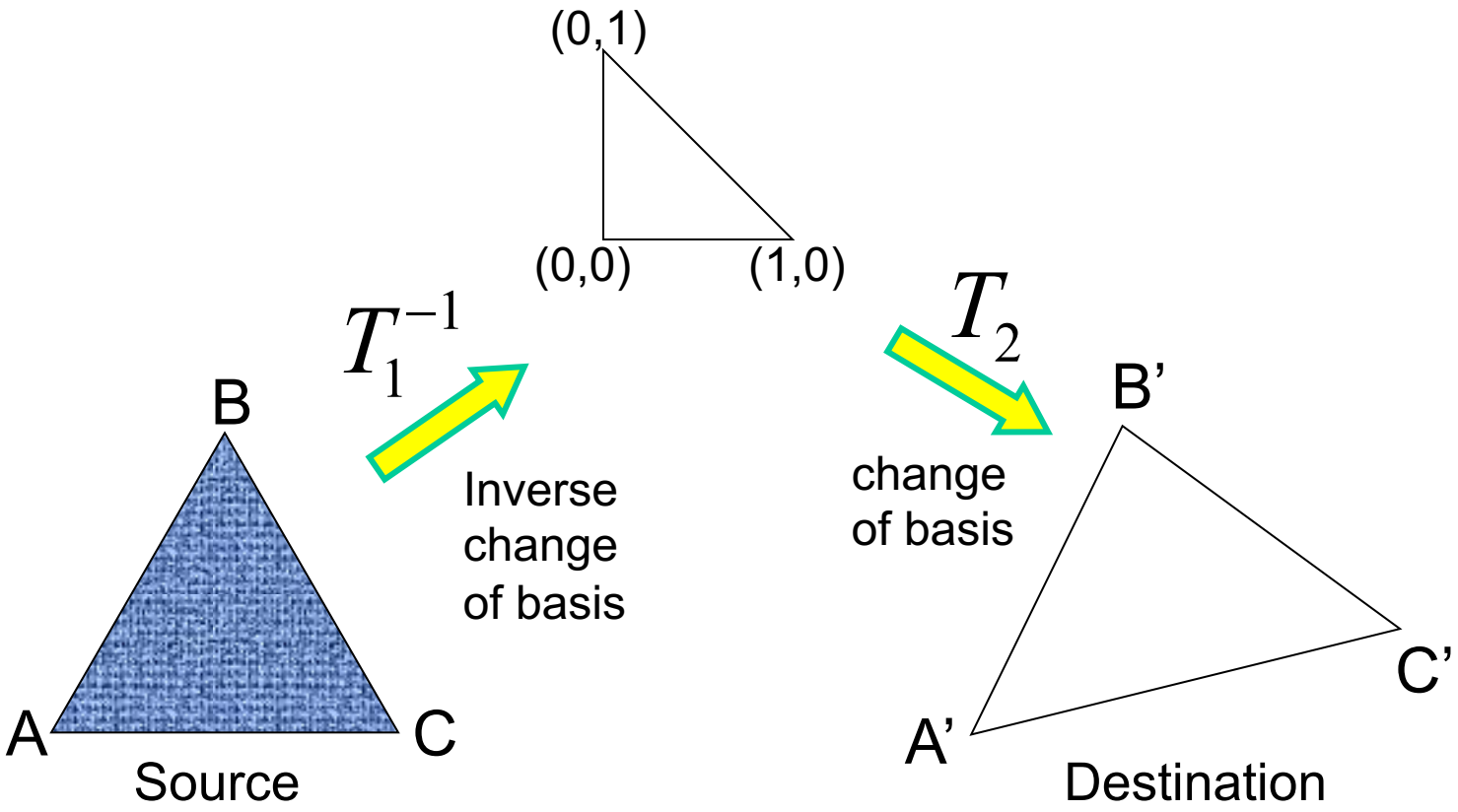
What kind of transformation is T?

How can we compute the transformation matrix:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Two ways: Algebraic and geometric

warping triangles (Barycentric Coordinates)



Don't forget to move the origin too!

Very useful for Project 3… (hint,hint,nudge,nudge)

Morphing = Object Averaging



The aim is to find “an average” between two objects

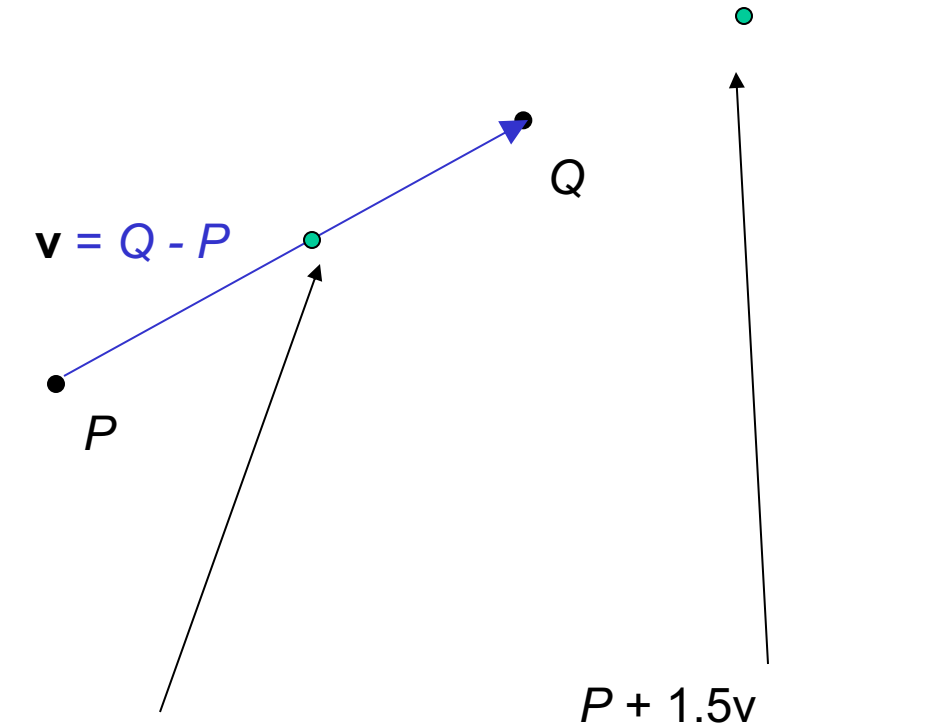
- Not an average of two images of objects…
- …but an image of the average object!
- How can we make a smooth transition in time?
  - Do a “weighted average” over time t

How do we know what the average object looks like?

- We haven’t a clue!
- But we can often fake something reasonable
  - Usually required user/artist input

Averaging Points

What's the average of P and Q?



Linear Interpolation (Affine Combination):
New point *aP* + *bQ*,
defined only when a+*b* = 1
So aP+bQ = aP+(1-a)Q

P and Q can be anything:

- points on a plane (2D) or in space (3D)
- Colors in RGB or HSV (3D)
- Whole images (m-by-n D)… etc.

Idea #1: Cross-Dissolve

Interpolate whole images:

$$Image_{halfway} = (1-t)*Image_1 + t*image_2$$

This is called **cross-dissolve** in film industry

But what is the images are not aligned?

Idea #2: Align, then cross-disolve

Align first, then cross-dissolve

- Alignment using global warp – picture still valid

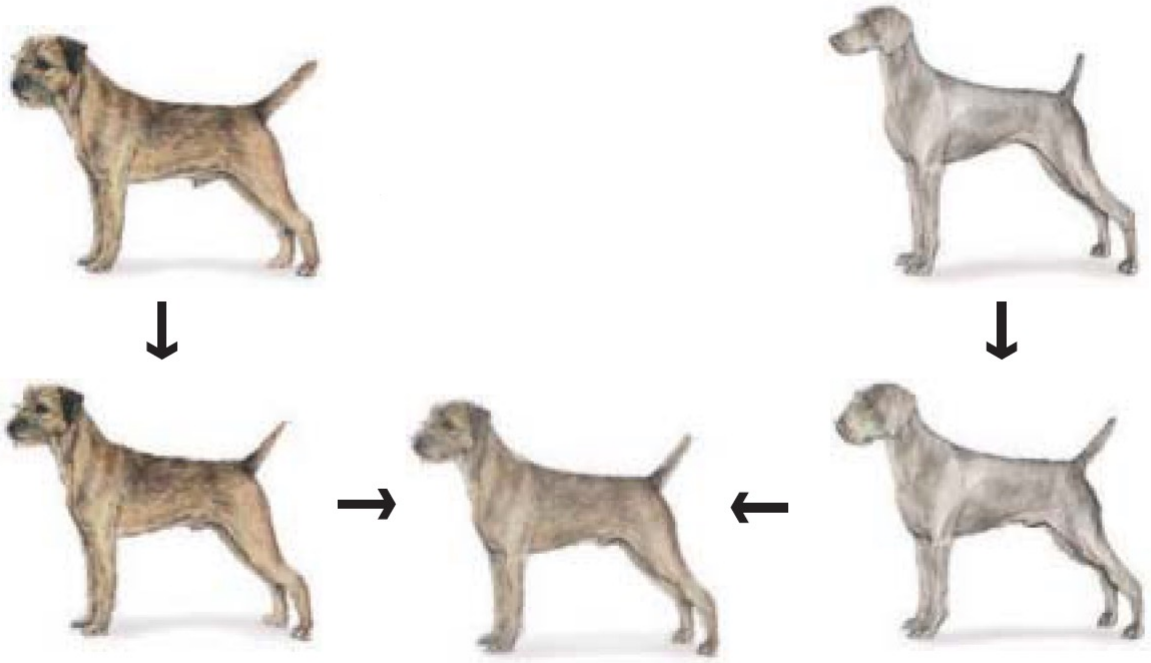
Image Morphing

Morphing = warping + cross-dissolving

shape (geometric)

color (photometric)

Two-stage Morphing Procedure



Morphing procedure:

for every t,

1. Find the average shape (the “mean dog”☺)
   - warping
2. Find the average color
   - Cross-dissolve the warped images

BUT: global warp not always enough!

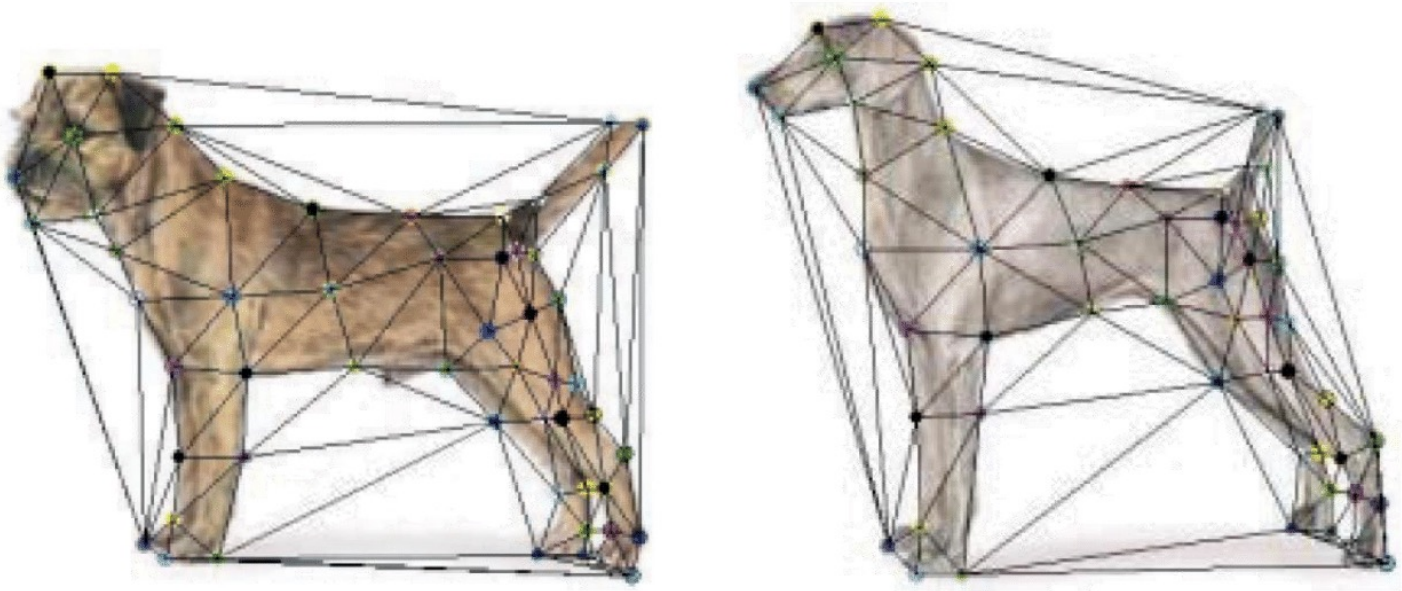
What to do?

- Cross-dissolve doesn't work
- Global alignment doesn't work
  - Cannot be done with a global transformation (e.g. affine)
- Any ideas?

Feature matching!

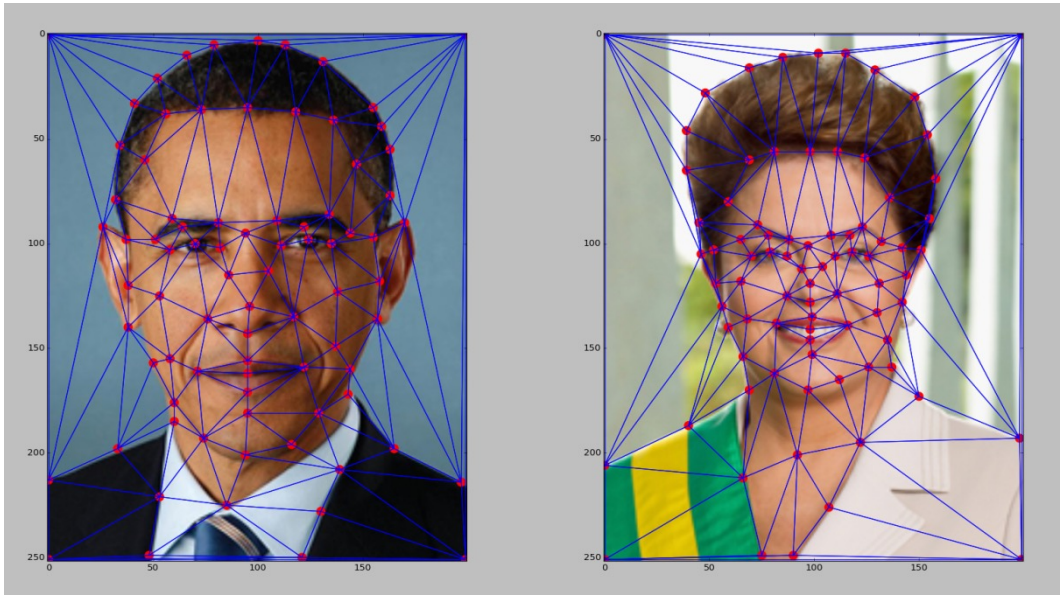
- Nose to nose, tail to tail, etc.
- But what to do with all the intermediate pixels?

Triangular Mesh

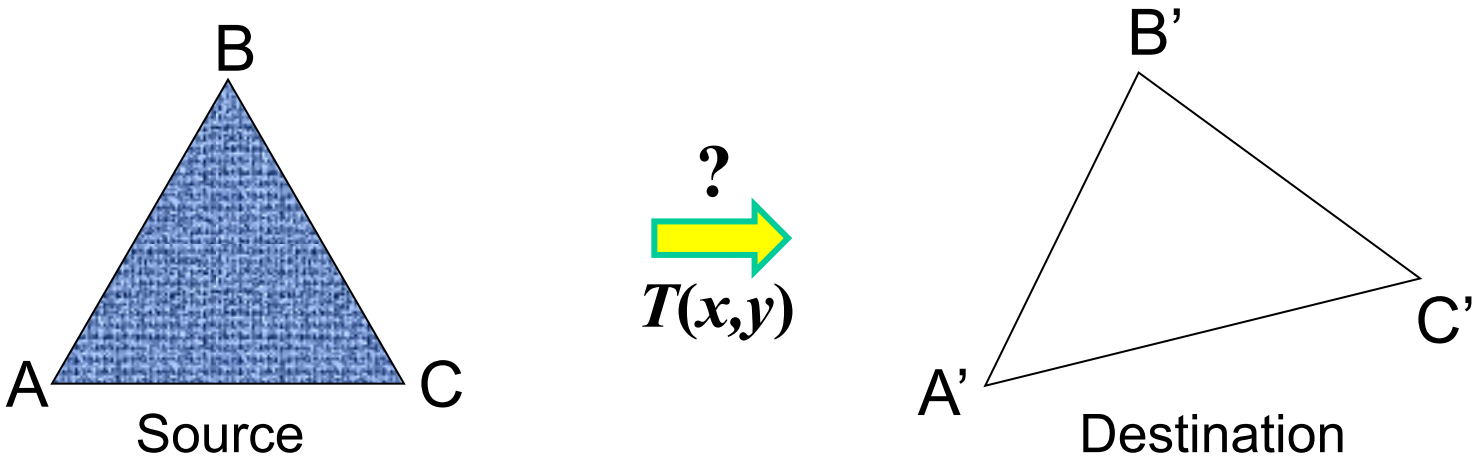


1. Input correspondences at key feature points
2. Define a triangular mesh over the points
   - Same mesh in both images!
   - Now we have triangle-to-triangle correspondences
3. Warp each triangle separately from source to destination
   - How do we warp a triangle?

Full morphing result



Warping triangles



Given two triangles: ABC and A'B'C' in 2D (12 numbers)

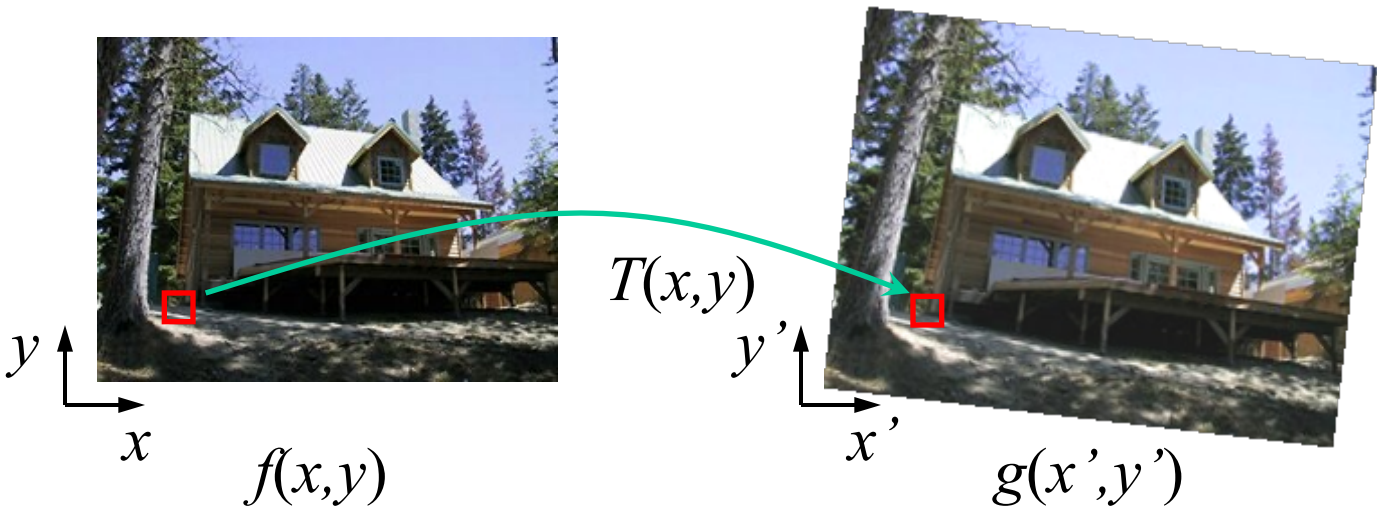
Need to find transform T to transfer all pixels from one to the other.

What kind of transformation is T?

How can we compute the transformation matrix:

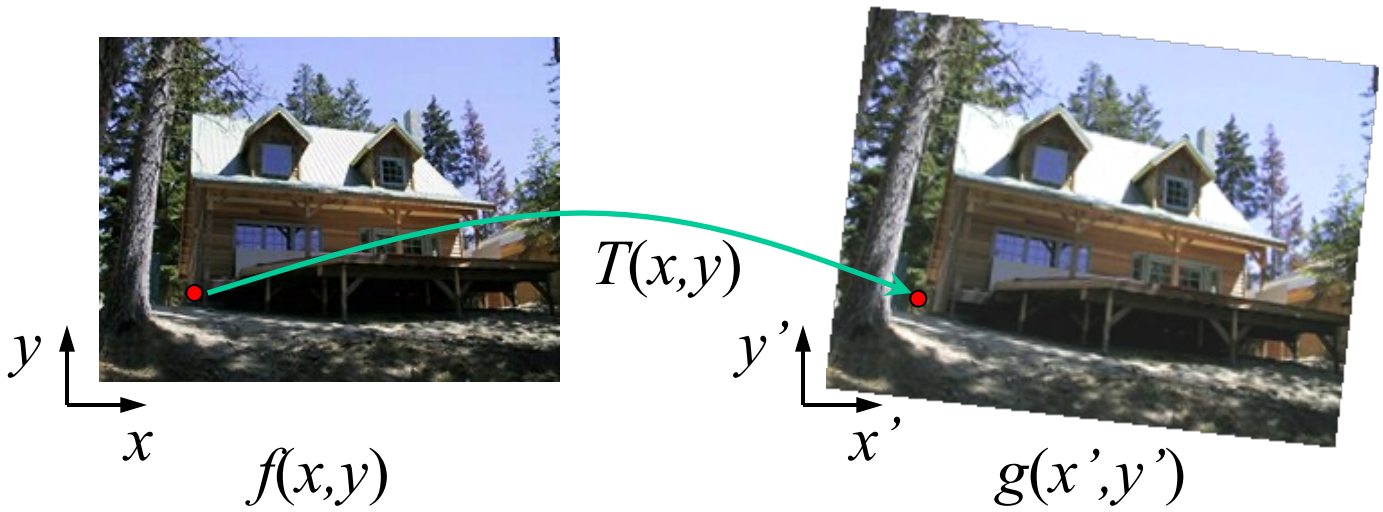
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Warping Pixels



Given a coordinate transform *(x',y')* = *T*(*x*,*y*) and a source image *f*(*x*,*y*), how do we compute a transformed image *g(x',y')* = *f*(*T*(*x*,*y*))?

Forward warping

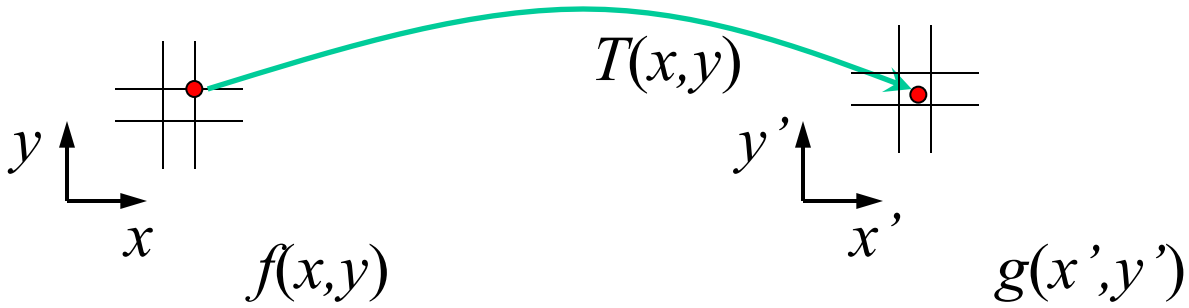


Send each pixel $f(x,y)$ to its corresponding location

$(x',y') = T(x,y)$ in the second image

Q: what if pixel lands "between" two pixels?

Forward warping



Send each pixel $f(x,y)$ to its corresponding location

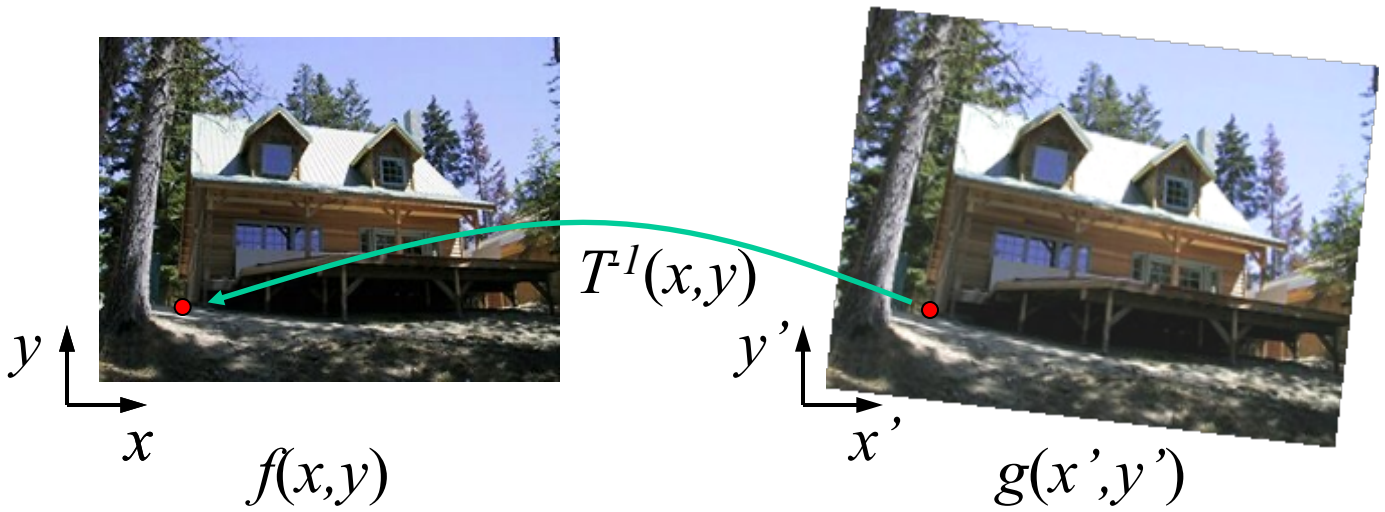
$(x',y') = T(x,y)$ in the second image

Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels (x',y')

- Known as "splatting"
- Check out `griddata` in Matlab

Inverse warping

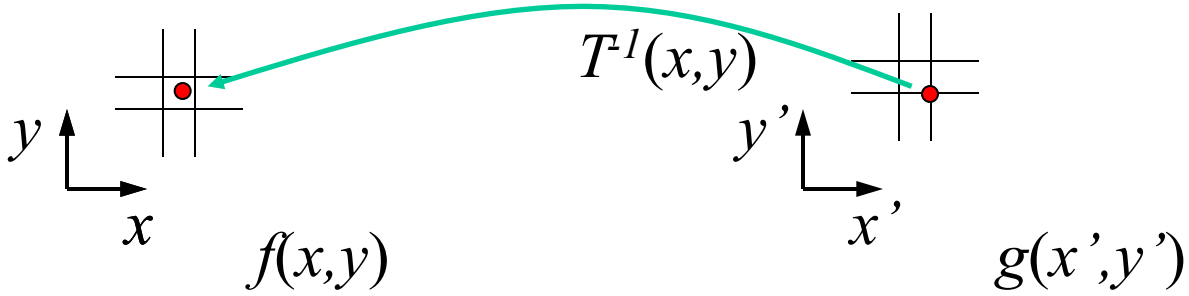


Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from "between" two pixels?

Inverse warping



Get each pixel $g(x',y')$ from its corresponding location

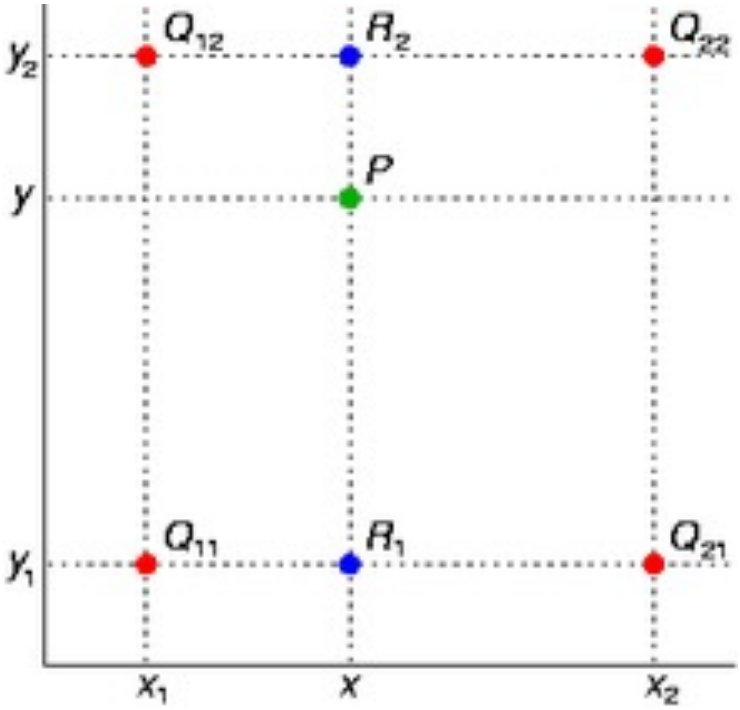
$(x,y) = T^{-1}(x',y')$ in the first image

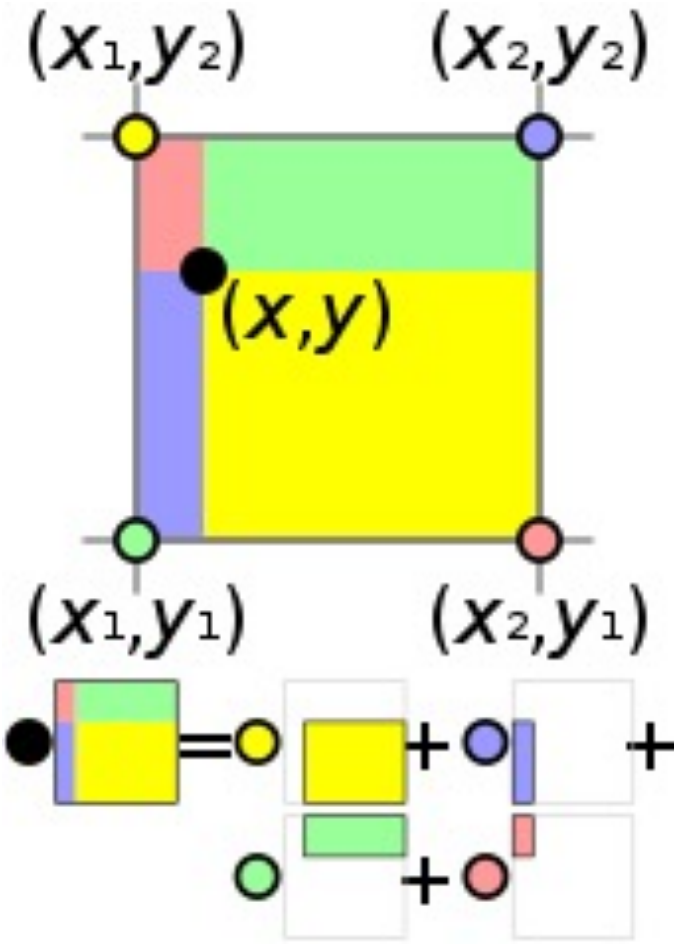
Q: what if pixel comes from "between" two pixels?

A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- Check out `interp2` in Matlab / Python

Bilinear Interpolation





http://en.wikipedia.org/wiki/Bilinear_interpolation

```
Help interp2
```

Forward vs. inverse warping

Q: which is better?

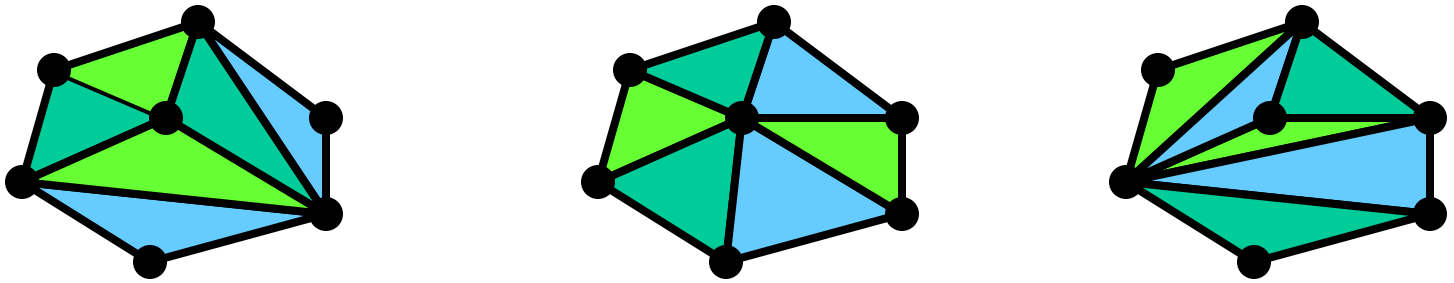
A: usually inverse—eliminates holes

- however, it requires an invertible warp function—not always possible...

Triangulations

A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.

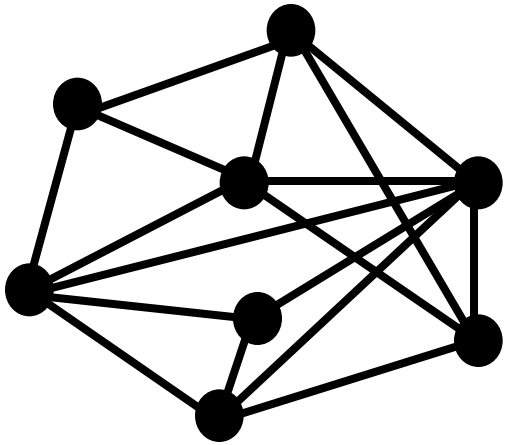
There are an exponential number of triangulations of a point set.



An O(n^3) Triangulation Algorithm

Repeat until impossible:

- Select two sites.
- If the edge connecting them does not intersect previous edges, keep it.



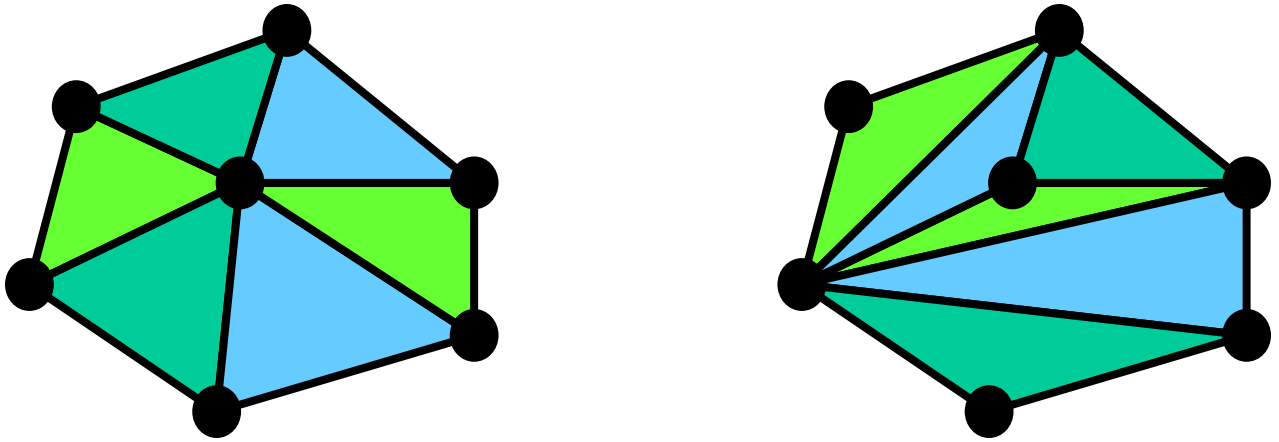
“Quality” Triangulations

Let $\alpha(T) = (\alpha_1, \alpha_2, ..., \alpha_{3t})$ be the vector of angles in the triangulation T in increasing order.

A triangulation T_1 will be “better” than T_2 if $\alpha(T_1) > \alpha(T_2)$ lexicographically.

The Delaunay triangulation is the “best”

- Maximizes smallest angles

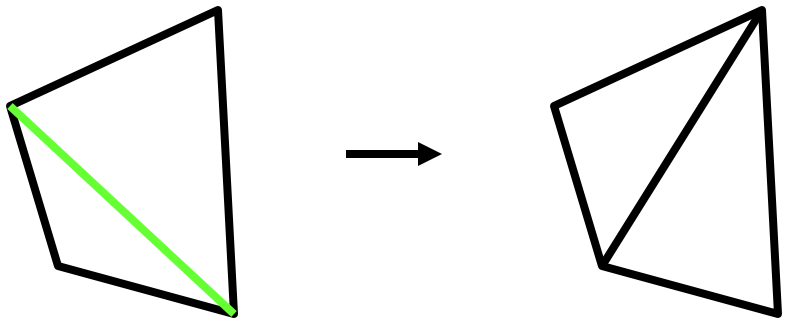


good

bad

Improving a Triangulation

In any convex quadrangle, an *edge flip* is possible. If this flip *improves* the triangulation locally, it also improves the global triangulation.

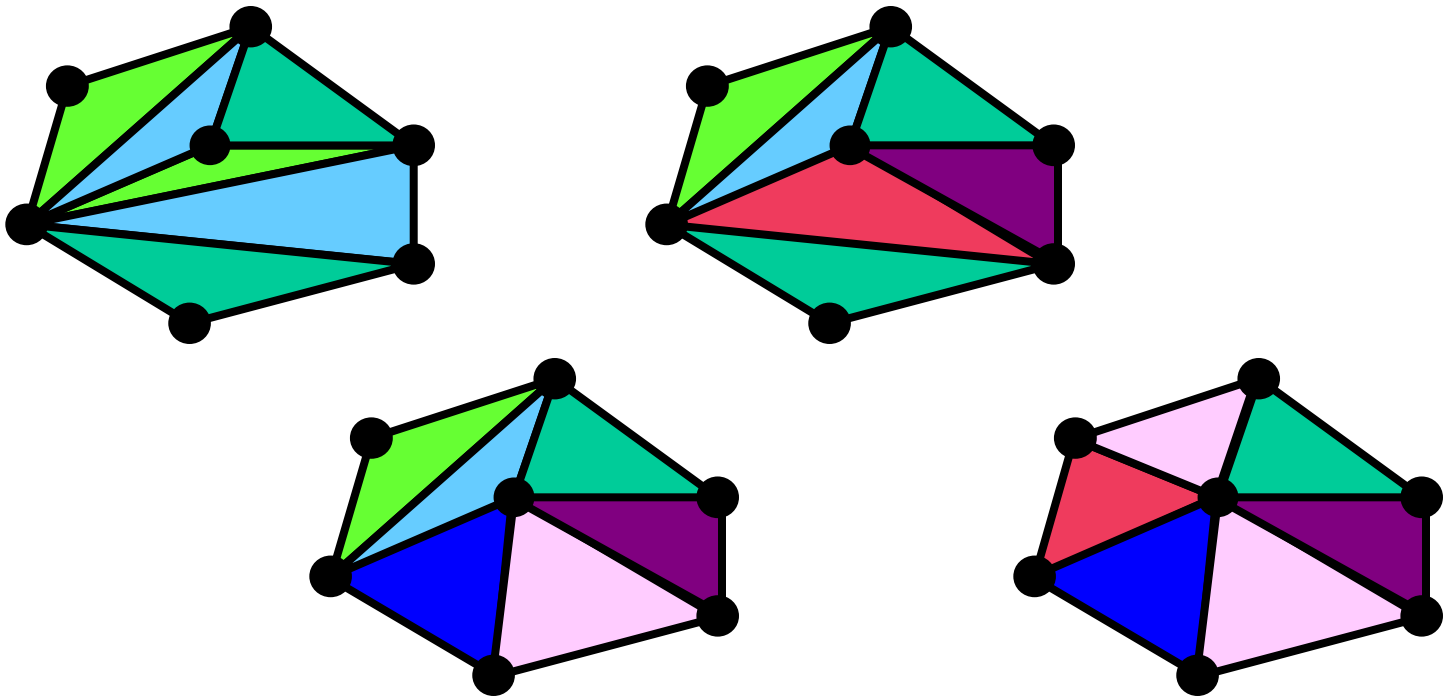


If an edge flip improves the triangulation, the first edge is called *illegal*.

Naïve Delaunay Algorithm

Start with an arbitrary triangulation. Flip any illegal edge until no more exist.

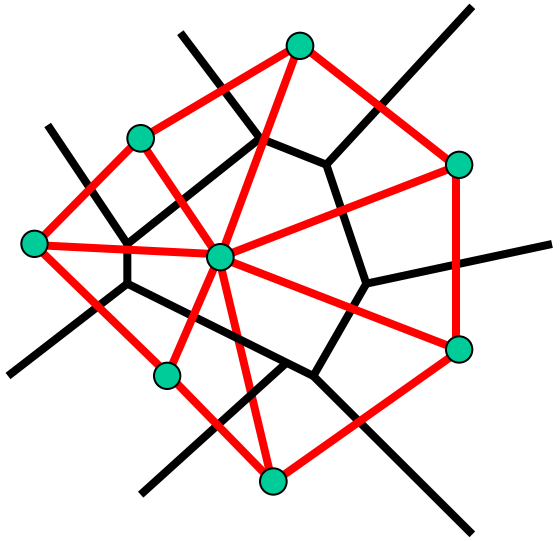
Could take a long time to terminate.



Delaunay Triangulation by Duality

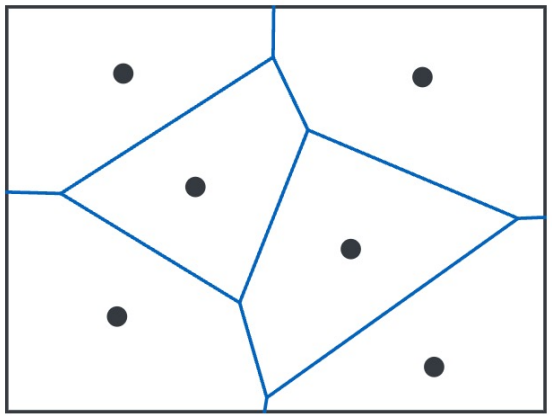
General position assumption: There are no four co-circular points.

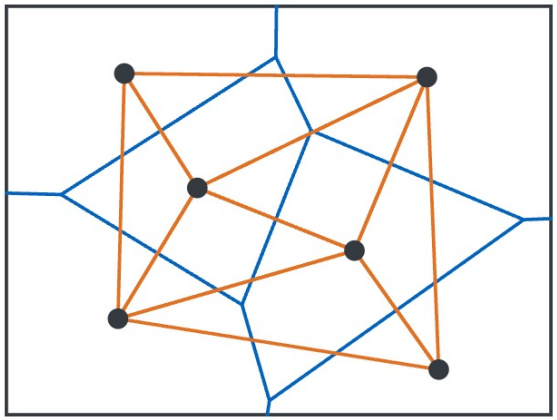
Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.



Corollary: The DT may be constructed in O(*n*log*n*) time.

This is what Matlab's `delaunay` function uses.



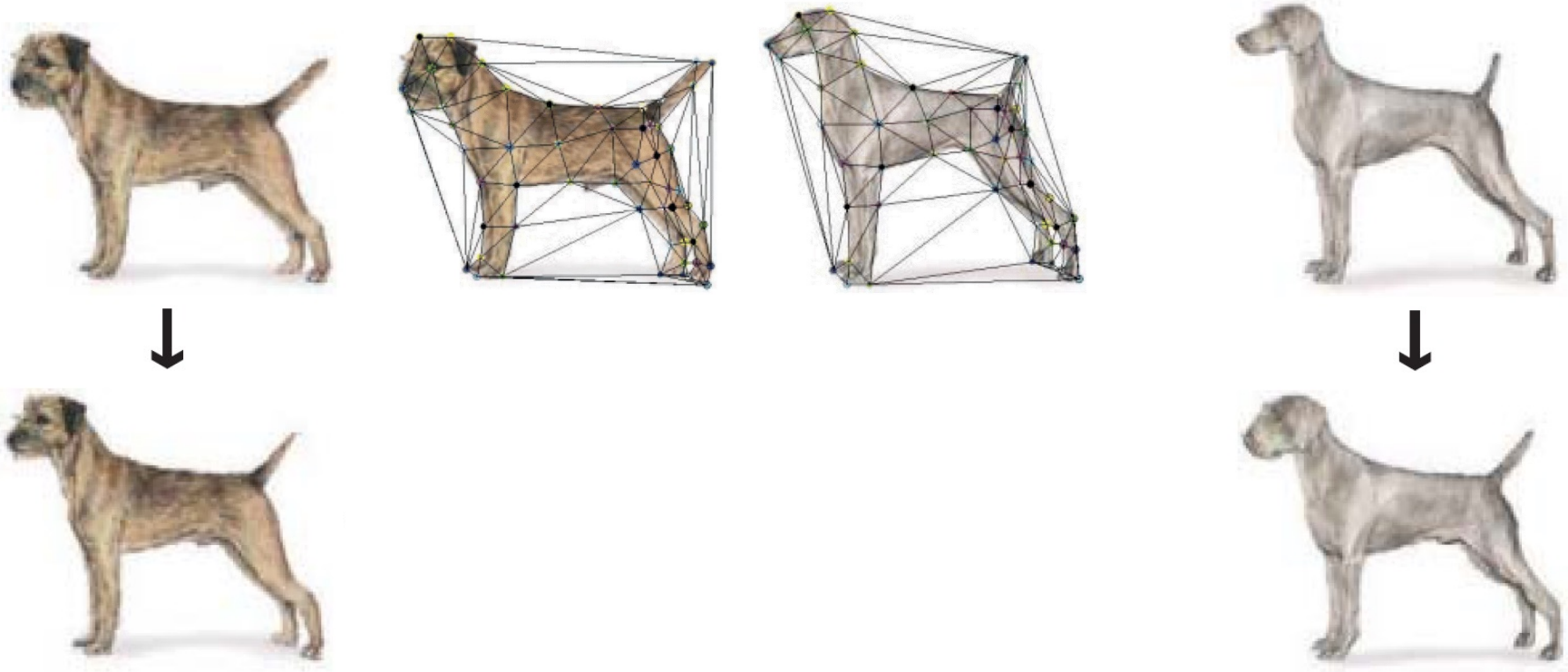


1. Create Average Shape

How do we create an intermediate warp at time t?

- Assume t = [0,1]
- Simple linear interpolation of each feature pair

  p=(x,y) -> p'(x,y)
- (1-t)*p+t*p' for corresponding features p and p'



2. Create Average Color

Interpolate whole images:

$Image_{halfway} = (1-t)*Image + t*image'$

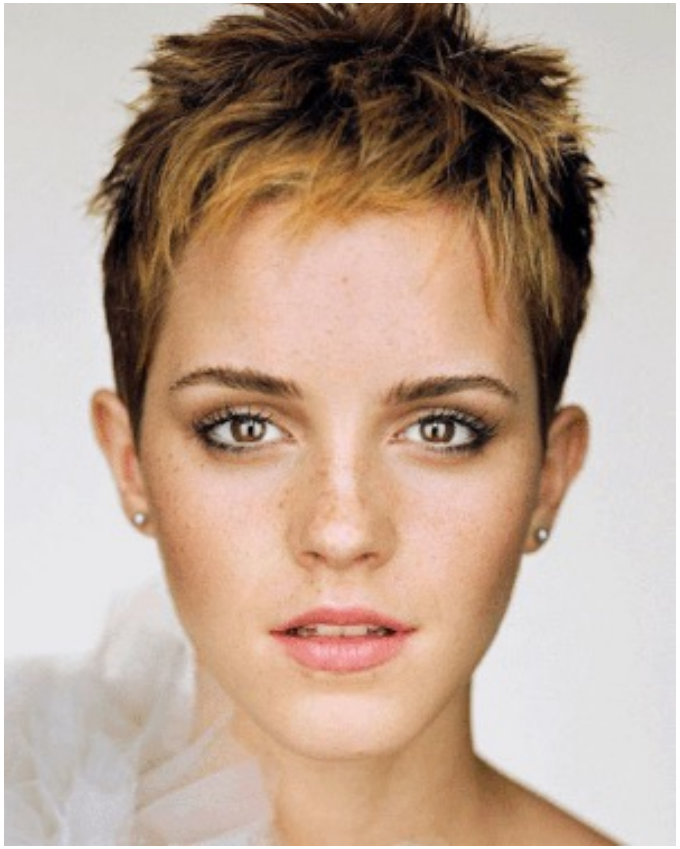
cross-dissolve!

Project #3: morphing

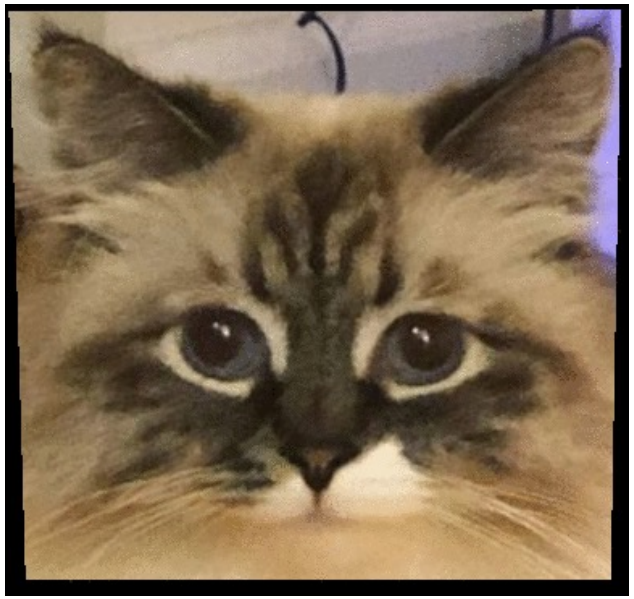
1. Define corresponding points
2. Define triangulation on points
   - Use same triangulation for both images
3. For each t = 0:step:1
   a. Compute the average shape at t (weighted average of points)
   b. For each triangle in the average shape
      – Get the affine projection to the corresponding triangles in each image
      – For each pixel in the triangle, find the corresponding points in each image and set value to weighted average (cross-dissolve each triangle)
   c. Save the image as the next frame of the sequence

Life-hack: can be done with just two nested loops (for t, and for each triangle). Hint: compute warps for all pixels first, then use `interp2`

Examples



Examples from last year



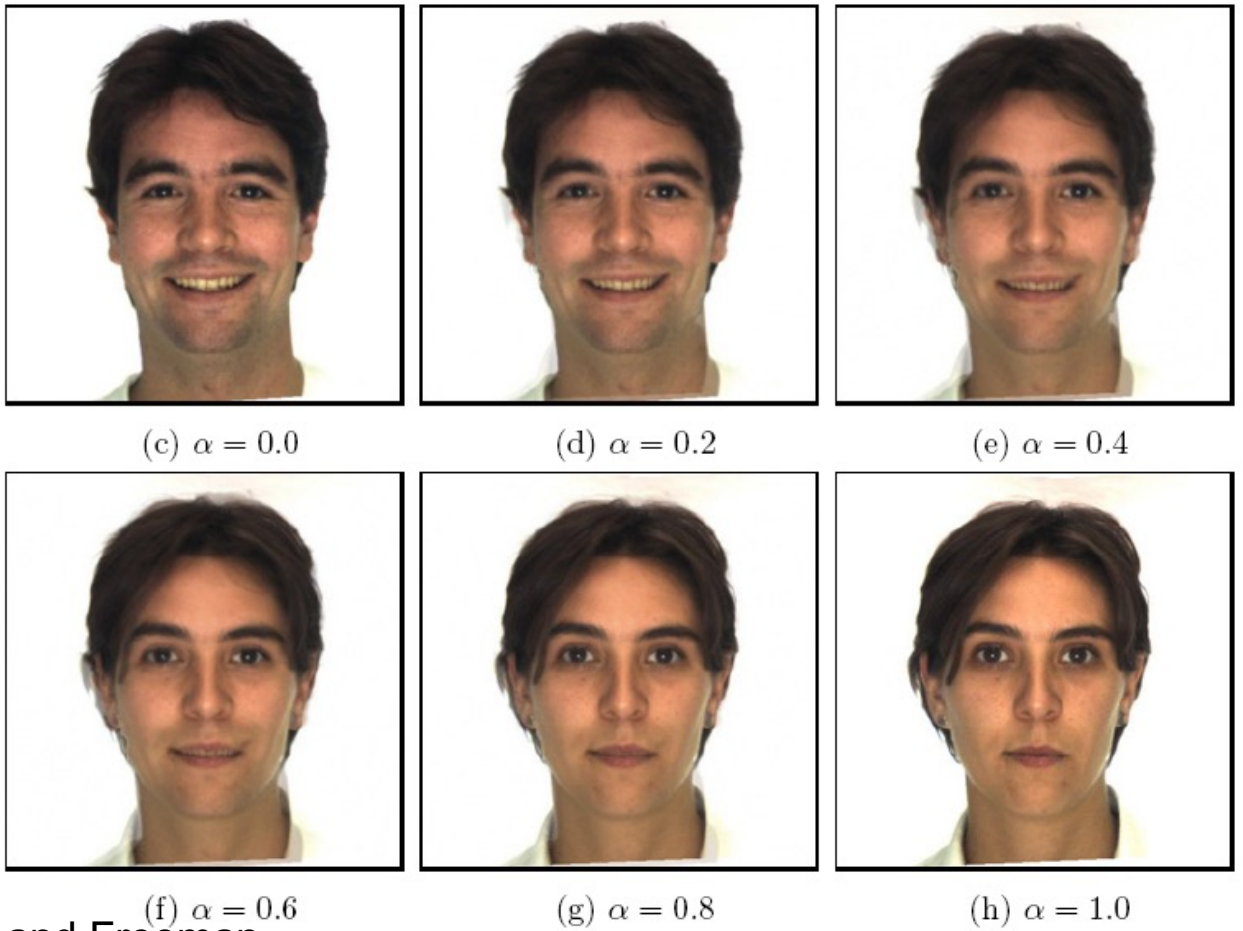
@Michael Jayasuriya

@Varun Saran

What's the difference?

Morphing & matting

Extract foreground first to avoid artifacts in the background

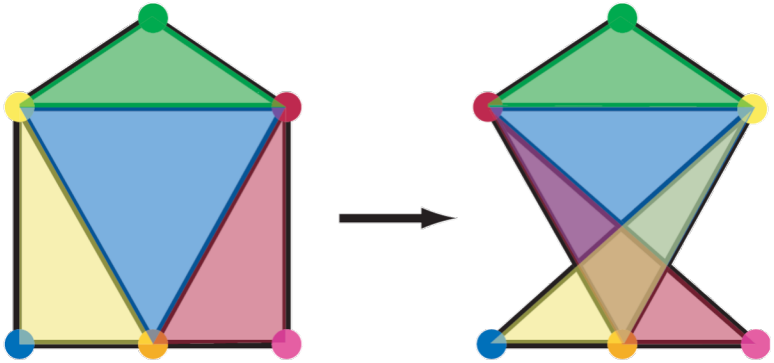


(c) $\alpha = 0.0$ (d) $\alpha = 0.2$ (e) $\alpha = 0.4$

(f) $\alpha = 0.6$ (g) $\alpha = 0.8$ (h) $\alpha = 1.0$

Slide by Durand and Freeman

Other Issues



Beware of folding

- You are probably trying to do something 3D-ish

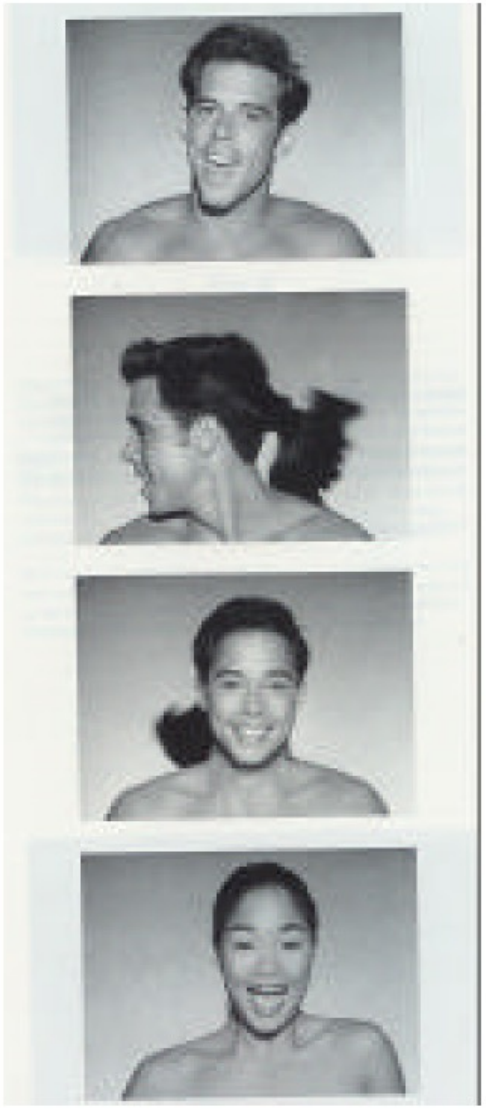
Morphing can be generalized into 3D

- If you have 3D data, that is!

Extrapolation can sometimes produce interesting effects

- Caricatures

Dynamic Scene (“Black or White”, MJ)



http://www.youtube.com/watch?v=R4kLKv5gtxc