Neural Radiance Fields pt 2

Video from the original ECCV'20 paper

CS180/280A: Intro to Computer Vision and Computational Photography

Angjoo Kanazawa and Alexei Efros

UC Berkeley Fall 2023

Lots of content from Noah Snavely and [ECCV 2022 Tutorial on Neural Volumetric Rendering for Computer Vision]

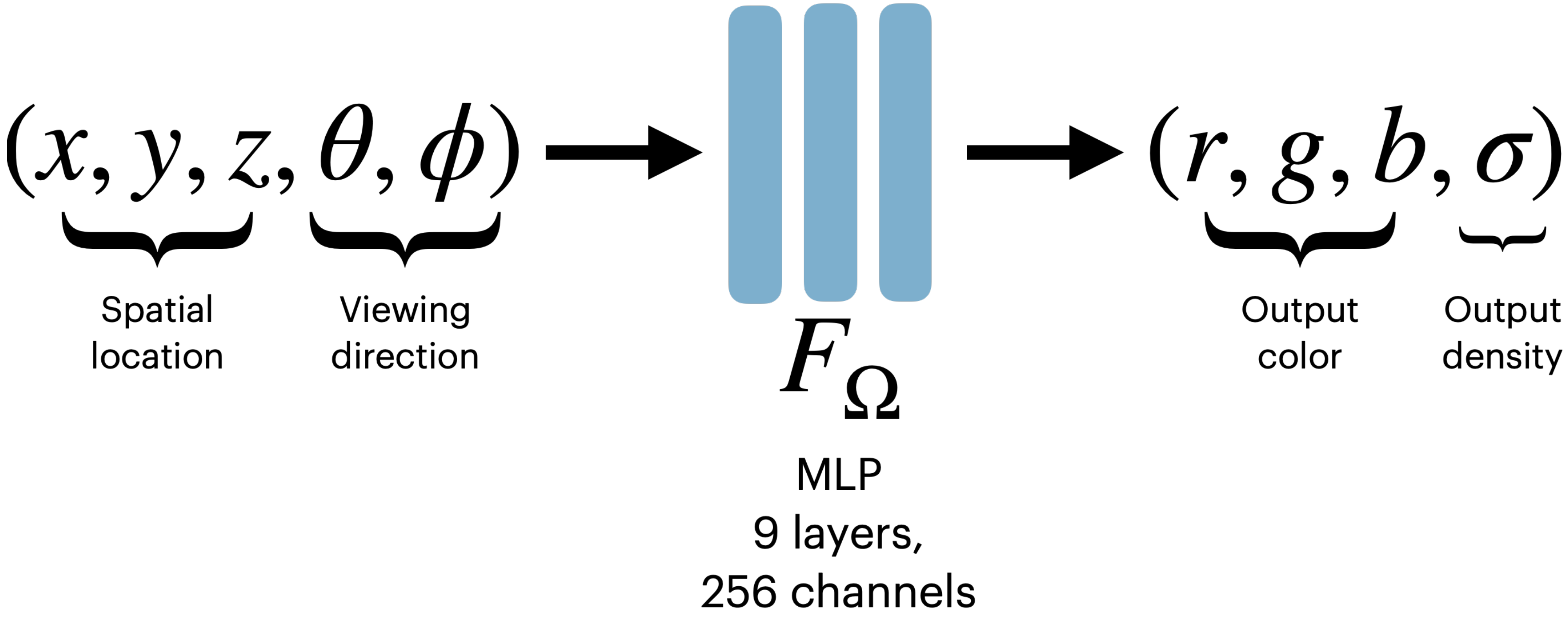
Logistics

- Project 5 out today!!

Last lecture

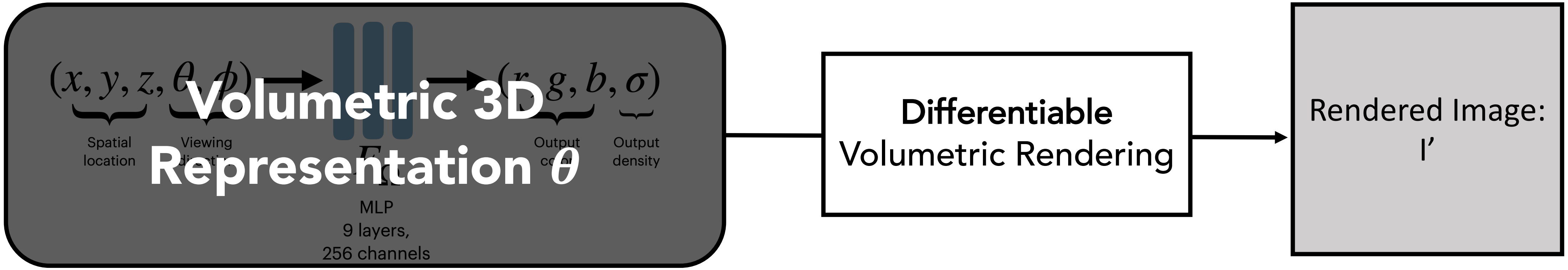
- Big picture of what NeRF does
  - what does this view direction mean?
- How is it different from multi-view stereo (photogrammetry)?
- How is it different from lightfields?

"Neural Radiance Fields"



“Neural Radiance Fields”

How an image is made (“Inference”)

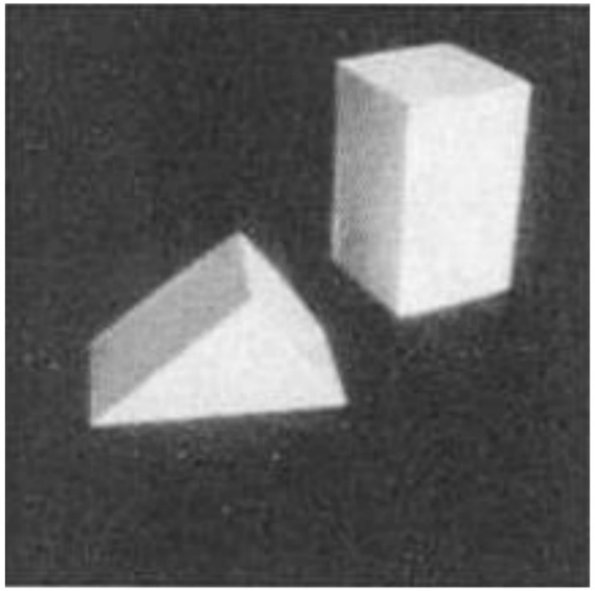


“Training” Objective (aka Analysis-by-Synthesis):

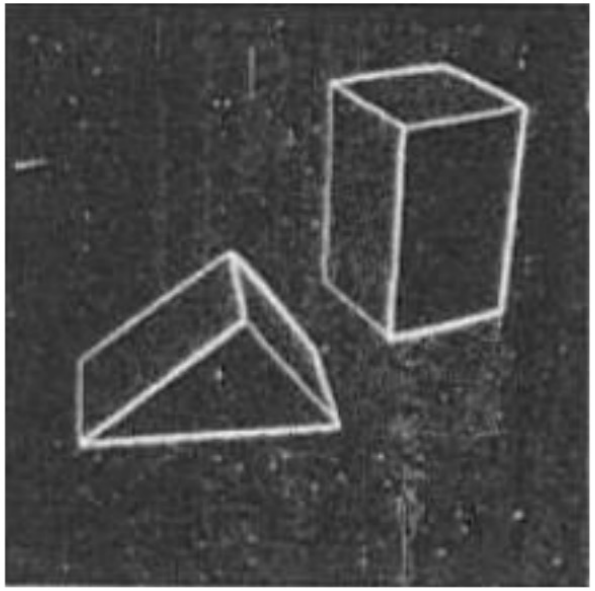
$$\min_{\theta} \| \text{Rendered Image: I'} - \text{Observed Image: I} \|_2$$

Analysis-by-Synthesis

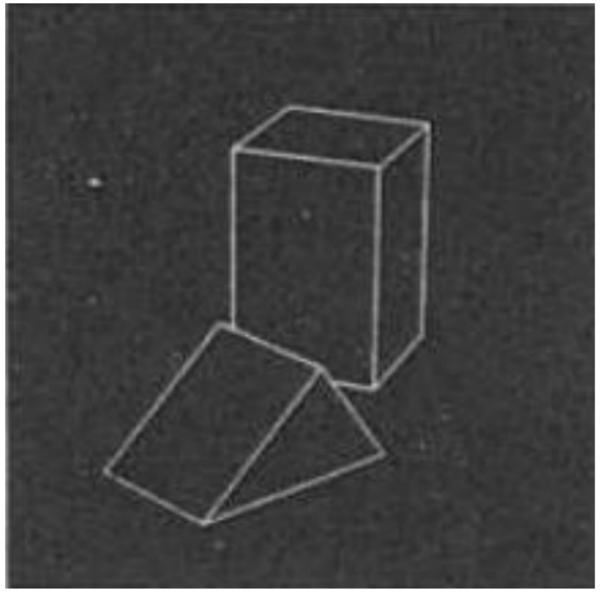
Larry Roberts
“Father of Computer Vision”



Input image



2x2 gradient operator

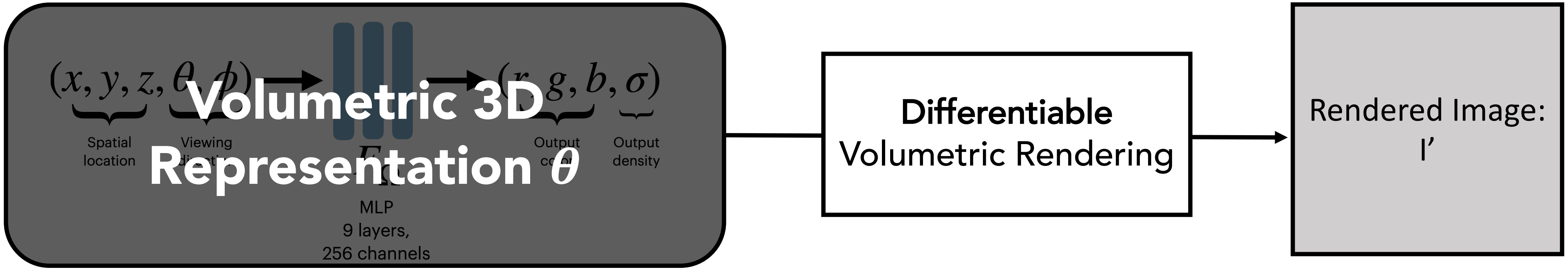


computed 3D model
rendered from new viewpoint

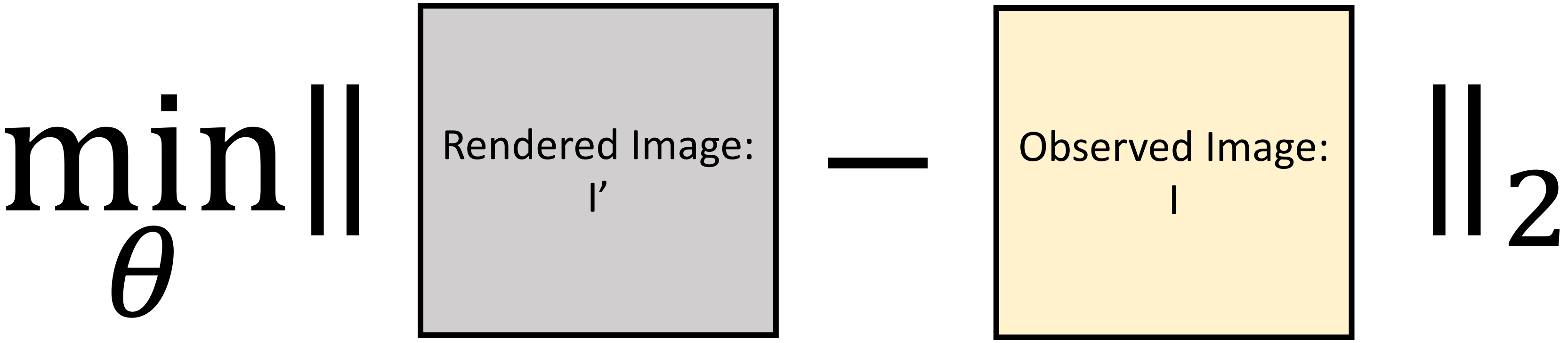
- History goes way back to the **first** Computer Vision paper!
  Roberts: Machine Perception of Three-Dimensional Solids, MIT, 1963

“Neural Radiance Fields”

Forward Function: How an image is made (Inference)

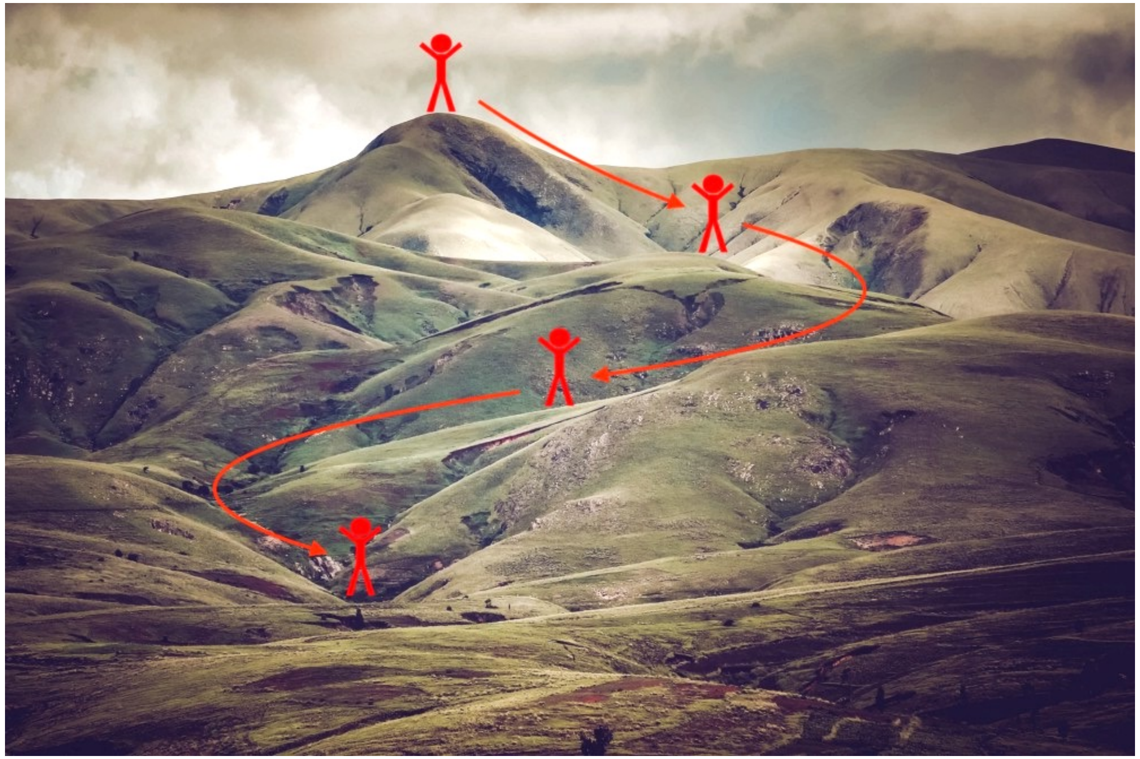


“Training” Objective (aka Analysis-by-Synthesis):



Differentiable Rendering

- How to change θ (network parameter) so that we get the final image?



- Gradient Descent "Hiking"

Same idea here, "hiking" now means you're going to change the network parameter little by little.

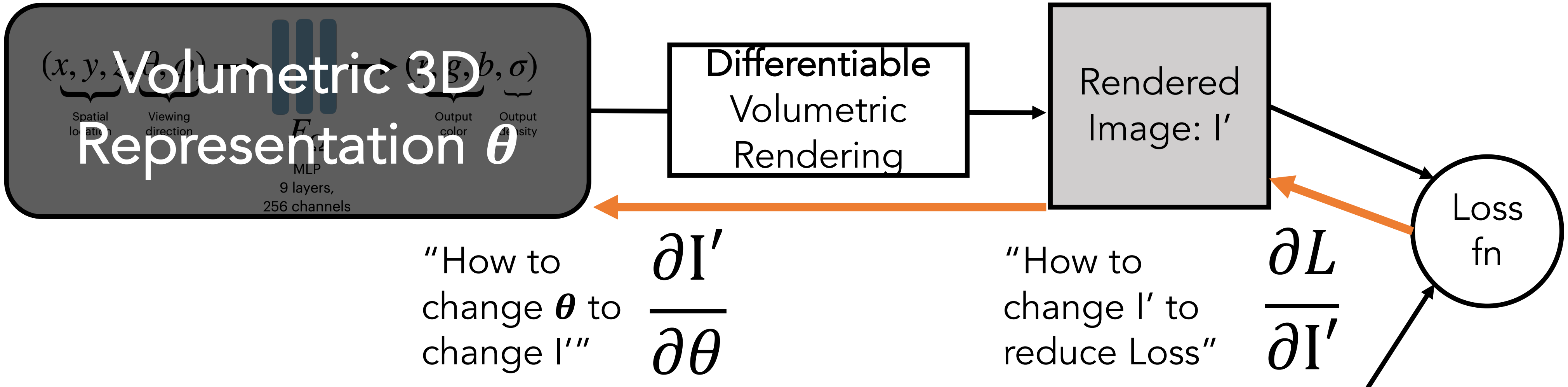
The "Mountain" or the "Loss" comes from the reconstruction loss.

$$L = \|I' - I\|$$
$$I' = f(x\,;\theta)$$

$$\frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial \mathrm{I}'}\frac{\partial \mathrm{I}'}{\partial \theta}$$

Chain rule, aka Back propagation

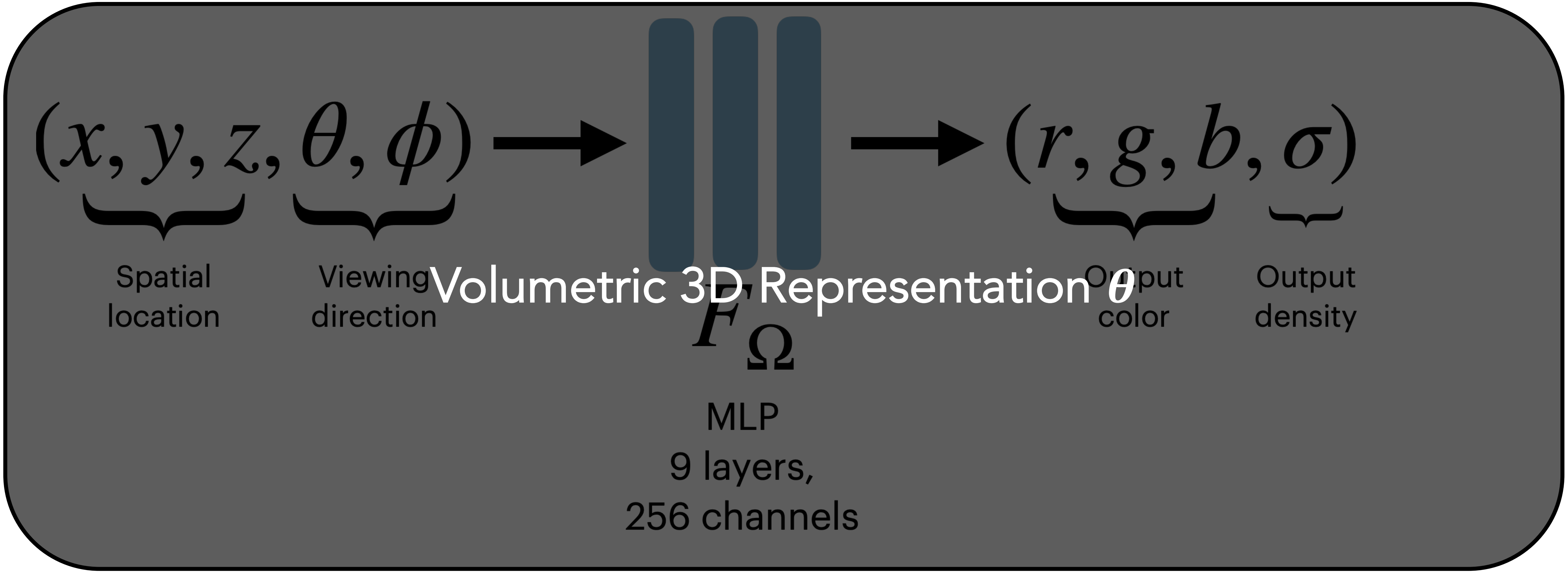
"Neural Radiance Fields"



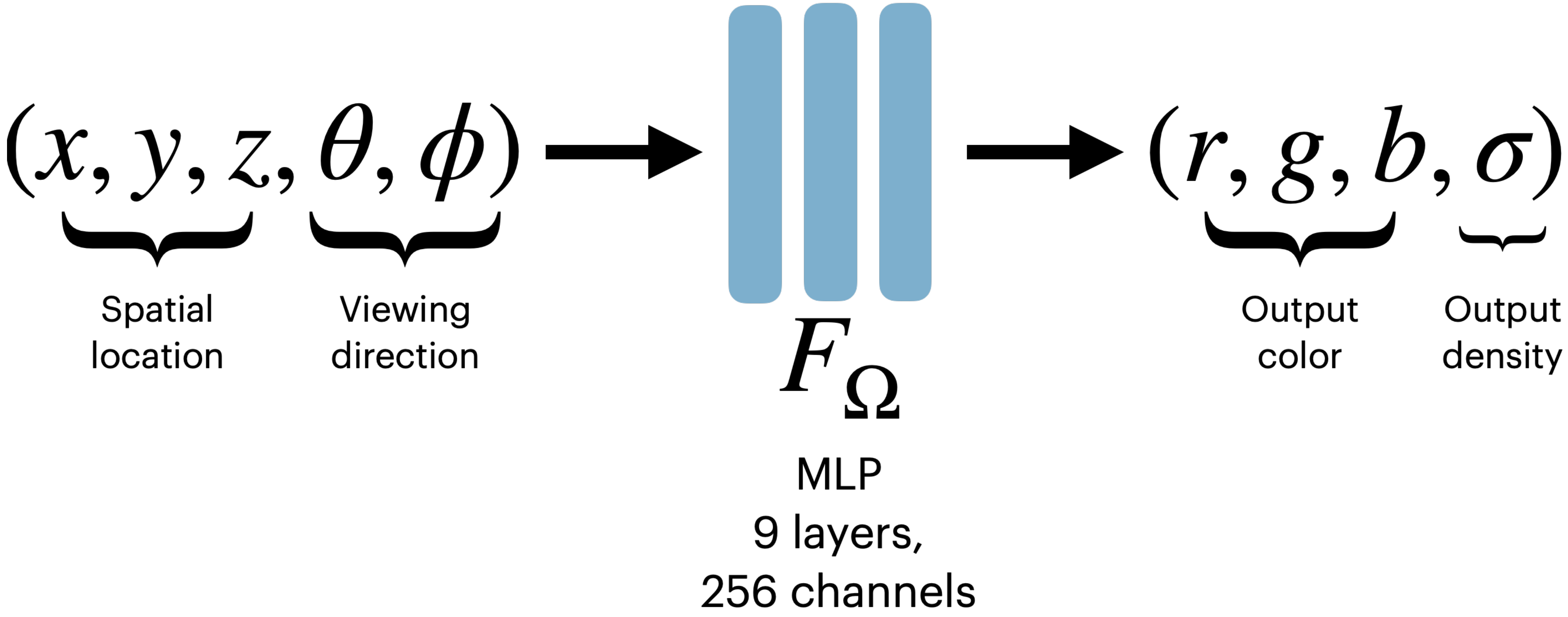
Multiplied together

"How to change $\boldsymbol{\theta}$ to reduce Loss" $\frac{\partial L}{\partial \theta}$

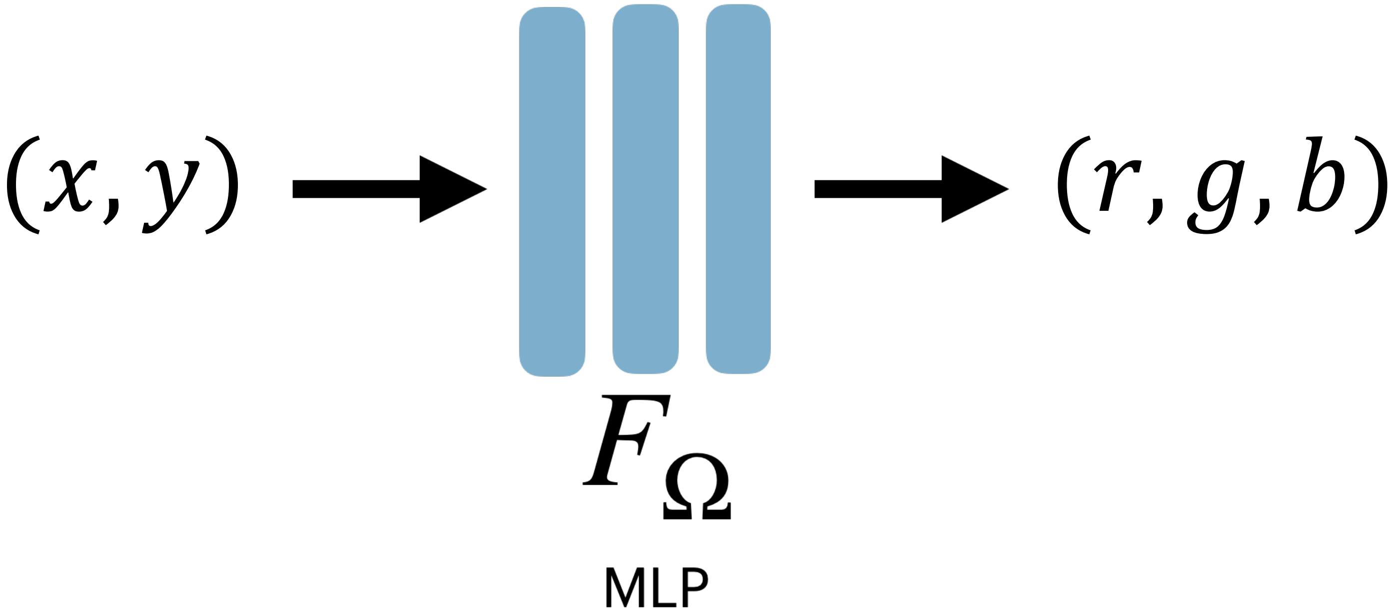
“Neural Radiance Fields”



"Neural Radiance Fields"



Let's simplify, do this in 2D:



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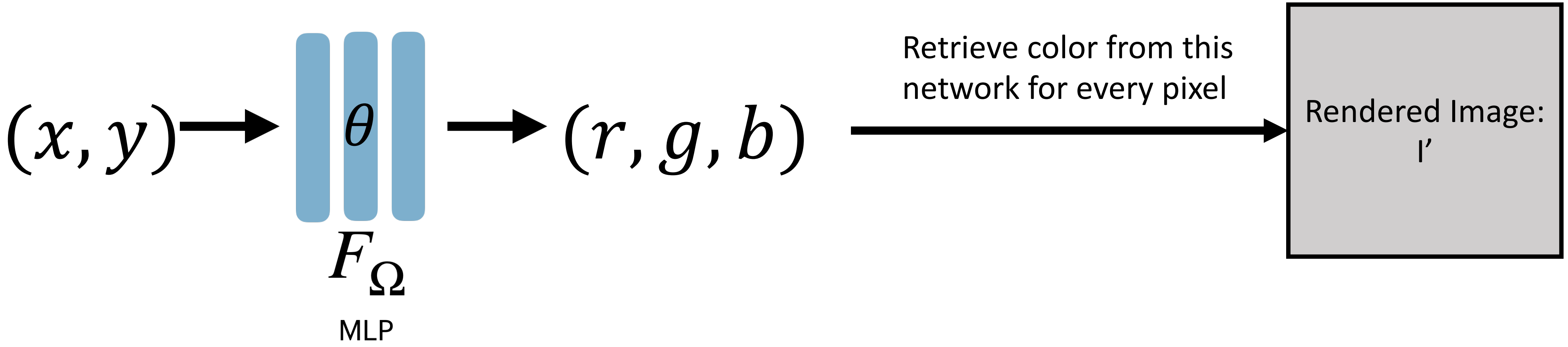


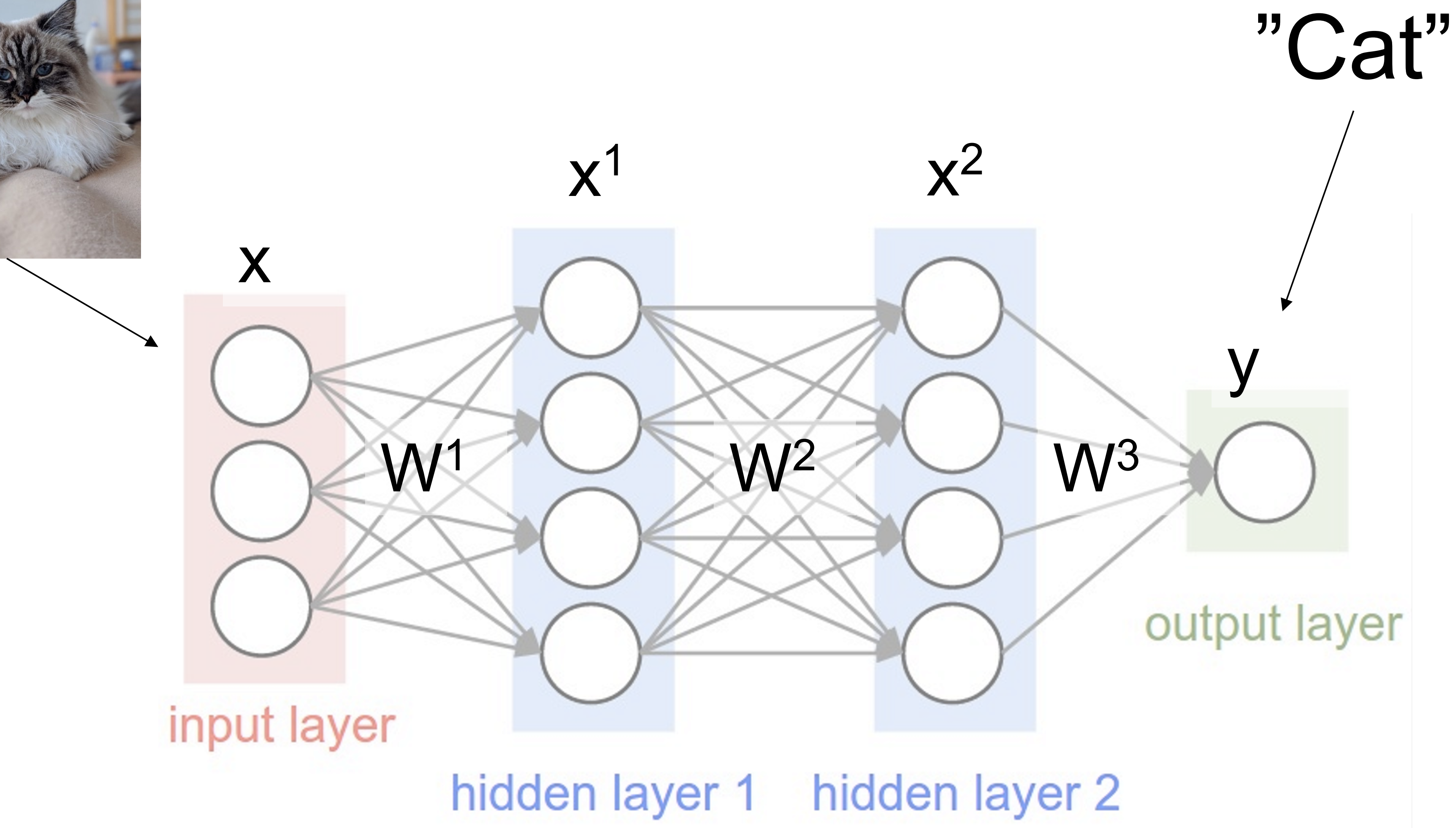
Optimize with "Training" Objective (aka Analysis-by-Synthesis):

$$\min_{\theta} \| \text{Rendered Image: I'} - \text{Observed Image: I} \|_2$$

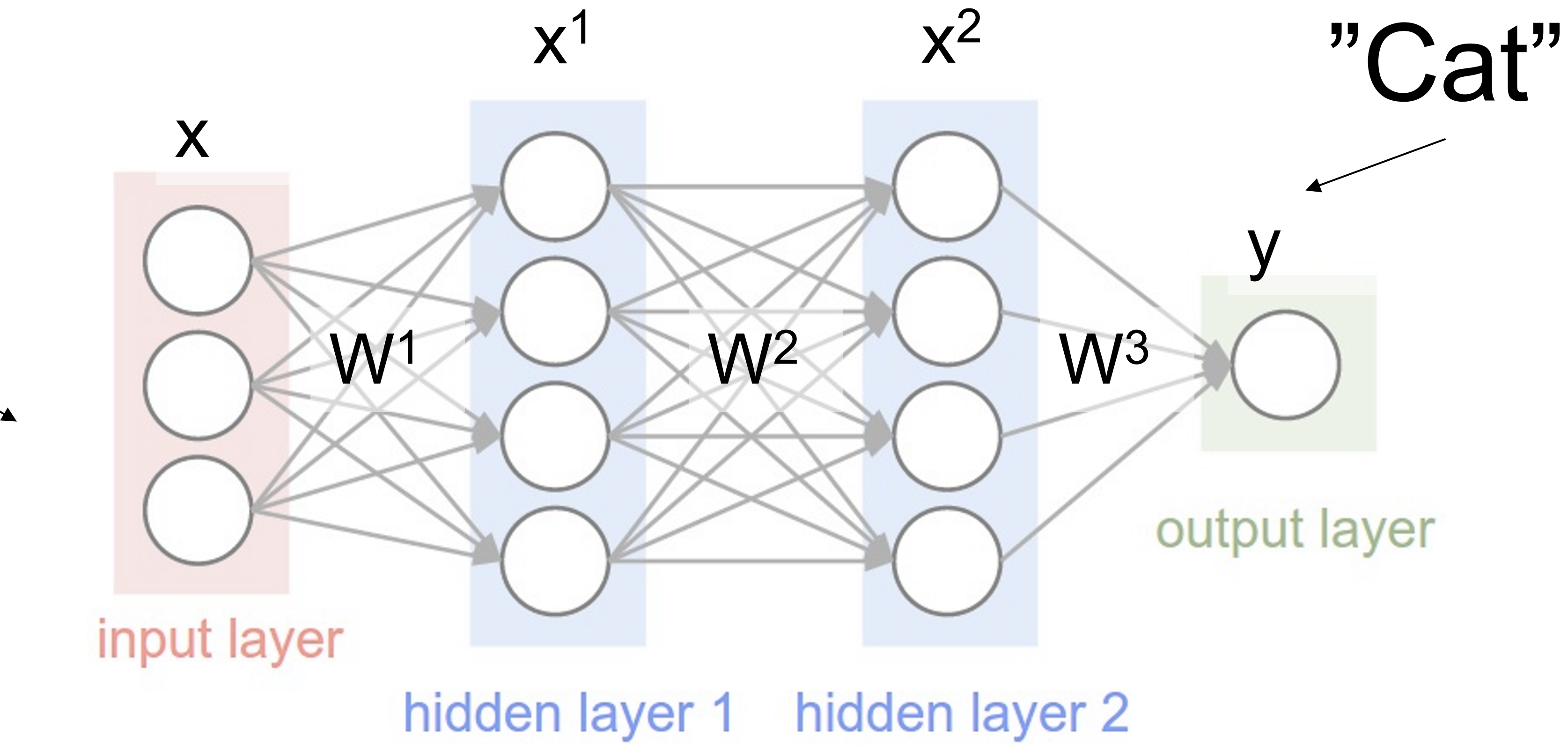
$$\frac{\partial L}{\partial \theta} = \frac{\partial (rgb \; - rgb')}{\partial \theta}$$

Straight forward to implement with Pytorch

ML Recap: Multi-layer perceptrons / Fully-Connected Layer



Multi-layer perceptrons / Fully-Connected Layer



In each layer:

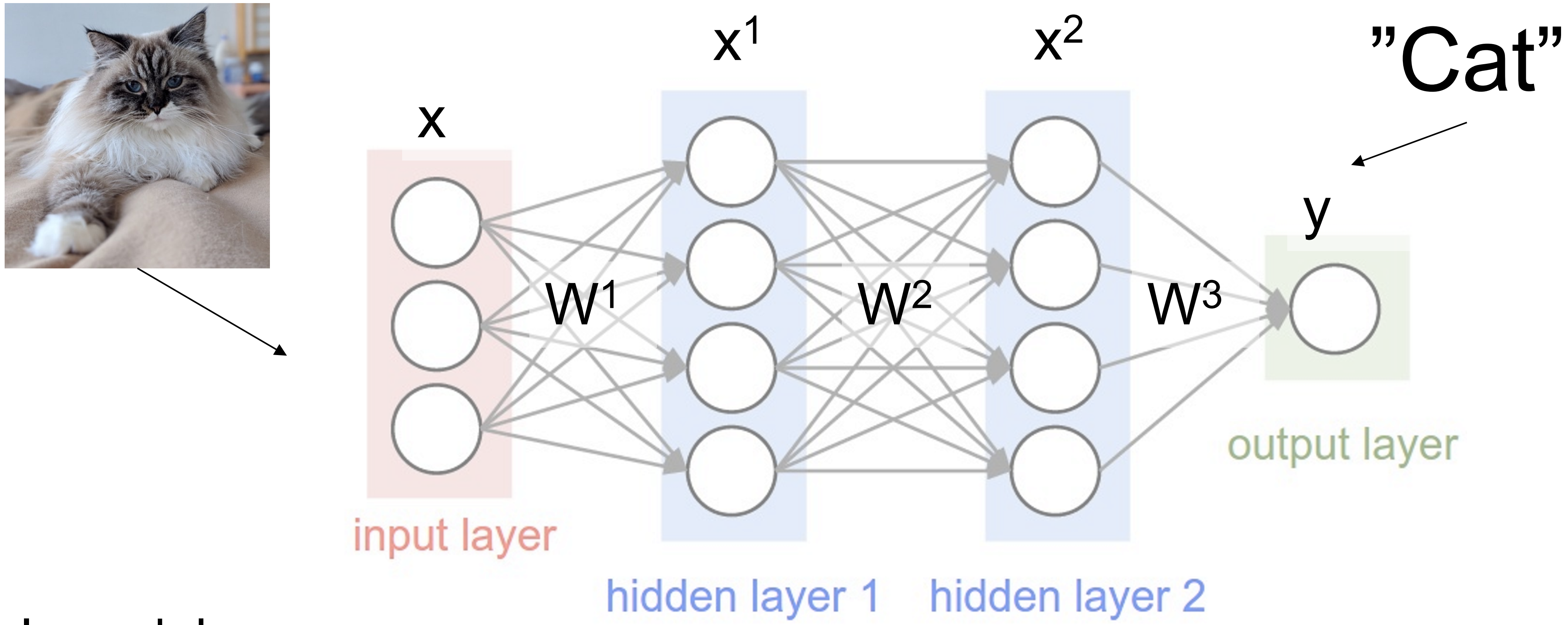
1. Linear Transform $z = W^l x^{l-1} + b$
2. Apply Non-Linearity $x^l = f(z)$

Usually

$$f = RELU(z) = \max(0, z)$$

what happens if f is identity?

Multi-layer perceptrons / Fully-Connected Layer



In each layer:

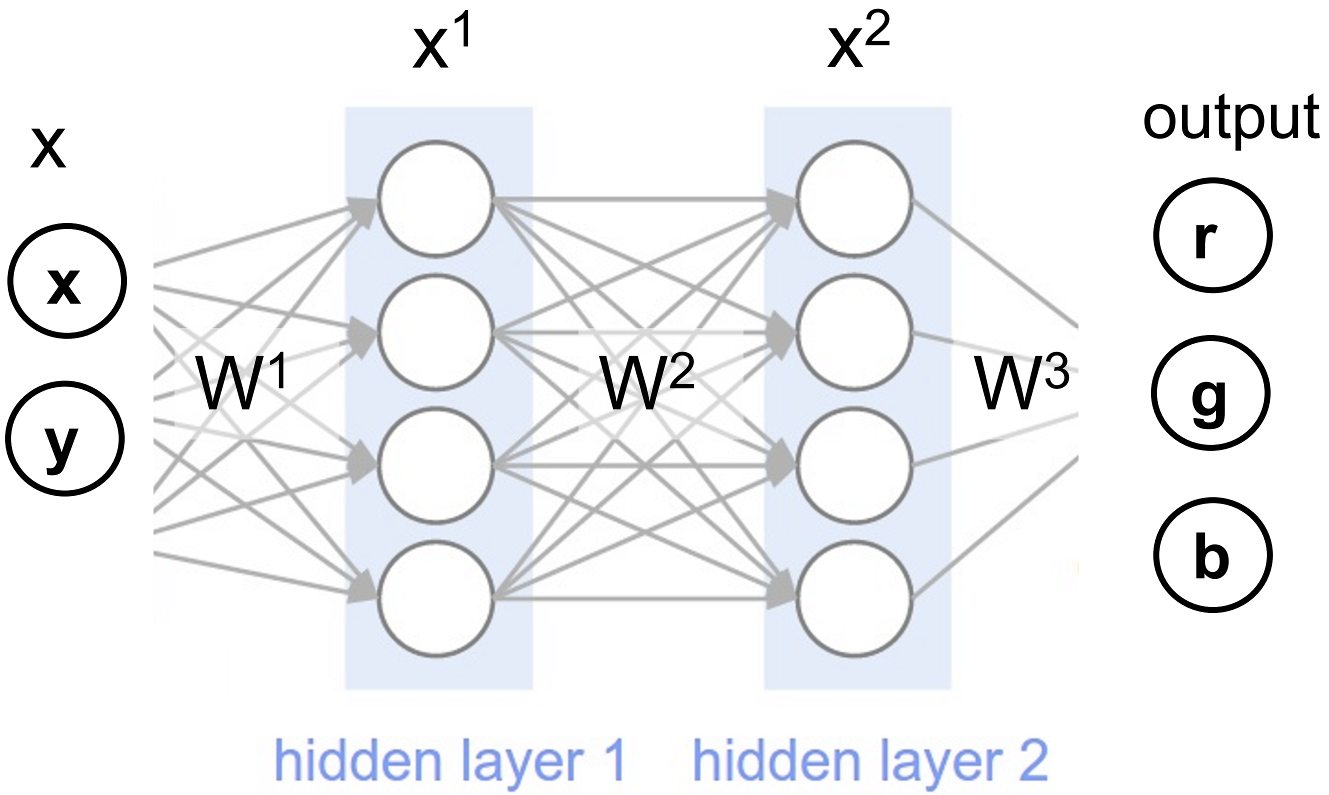
1. Linear Transform $z = W^l x^{l-1} + b$
2. Apply Non-Linearity $x^l = f(z)$

Usually

$$f = RELU(z)$$
$$= \max(0, z)$$

What are the learnable parameters?

In our 2D case:



In each layer:

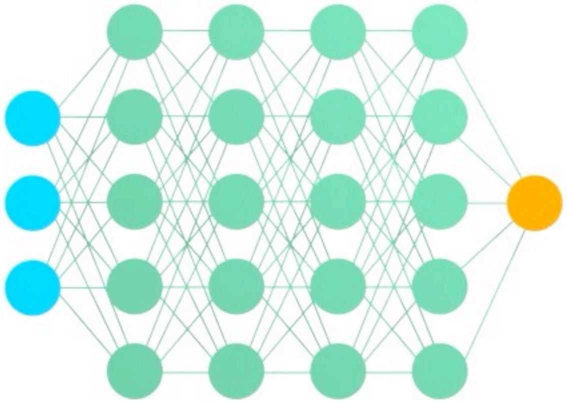
1. Linear Transform $z = W^l x^{l-1} + b$
2. Apply Non-Linearity $x^l = f(z)$

Usually
$$f = RELU(z) = \max(0, z)$$

What are the learnable parameters?

Coordinate Based Neural Network

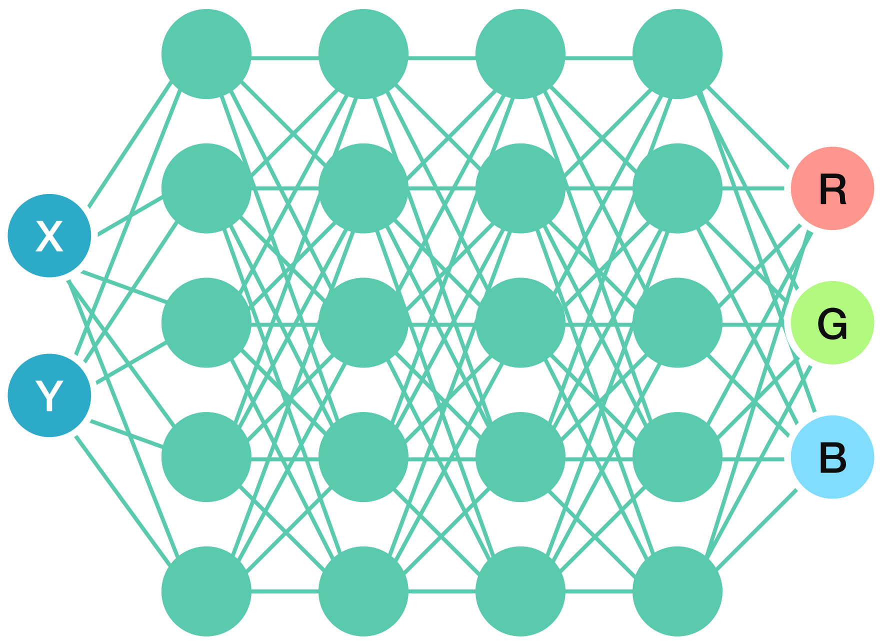
Input
Coordinate



Value at
Coordinate

Multi Layer Perceptron
MLP

Image Representation



Challenge:

How to get MLPs to represent higher frequency functions?

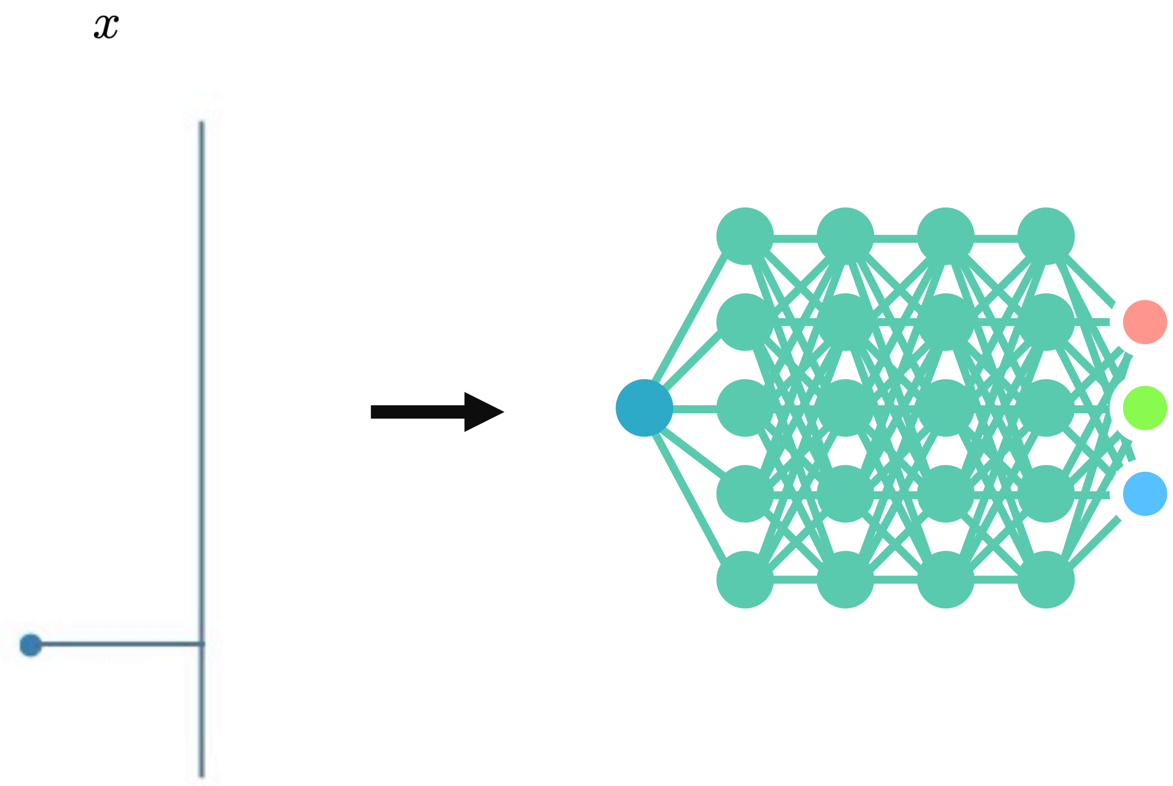
Rahaman et al. 2019, Basri et al. 2020

what happens if you naively optimize this network

MLP output

Supervision image

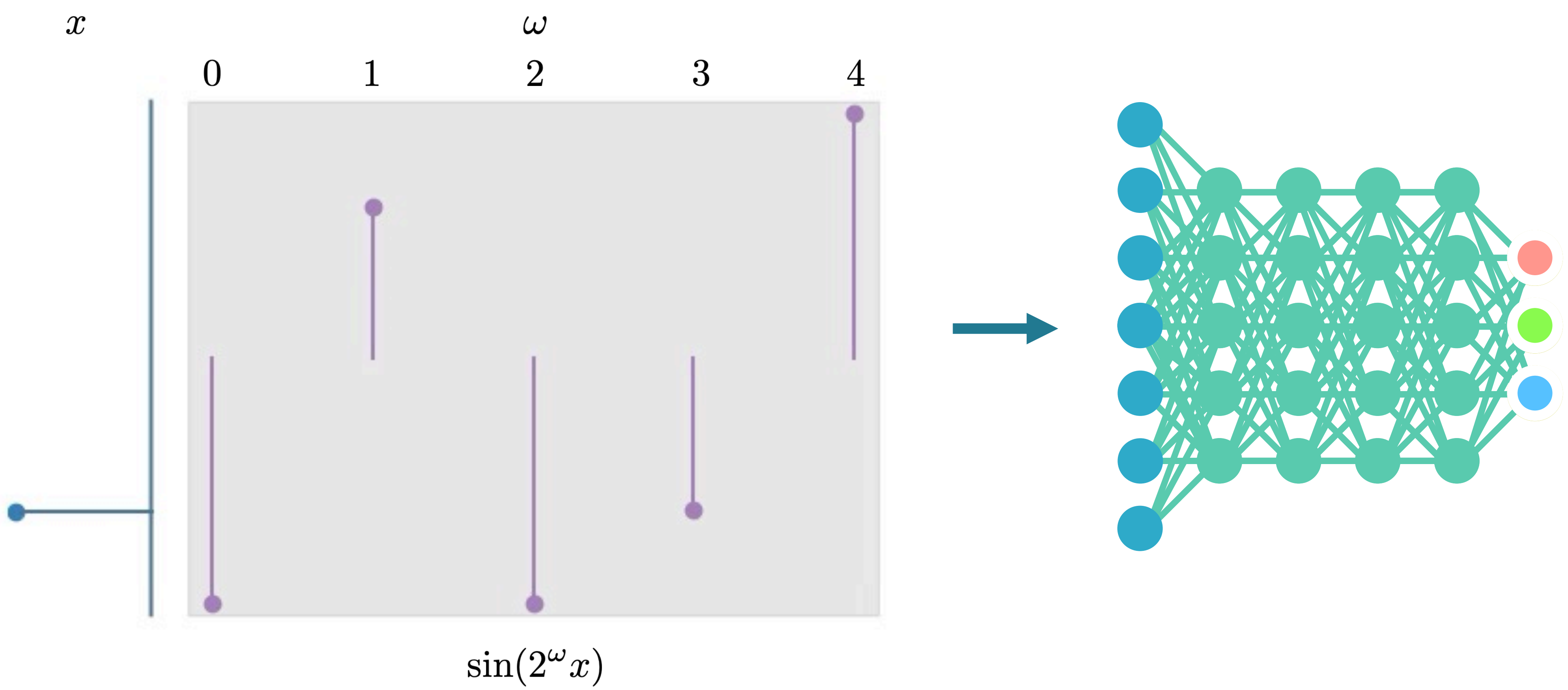
Standard input



Positional Encoding

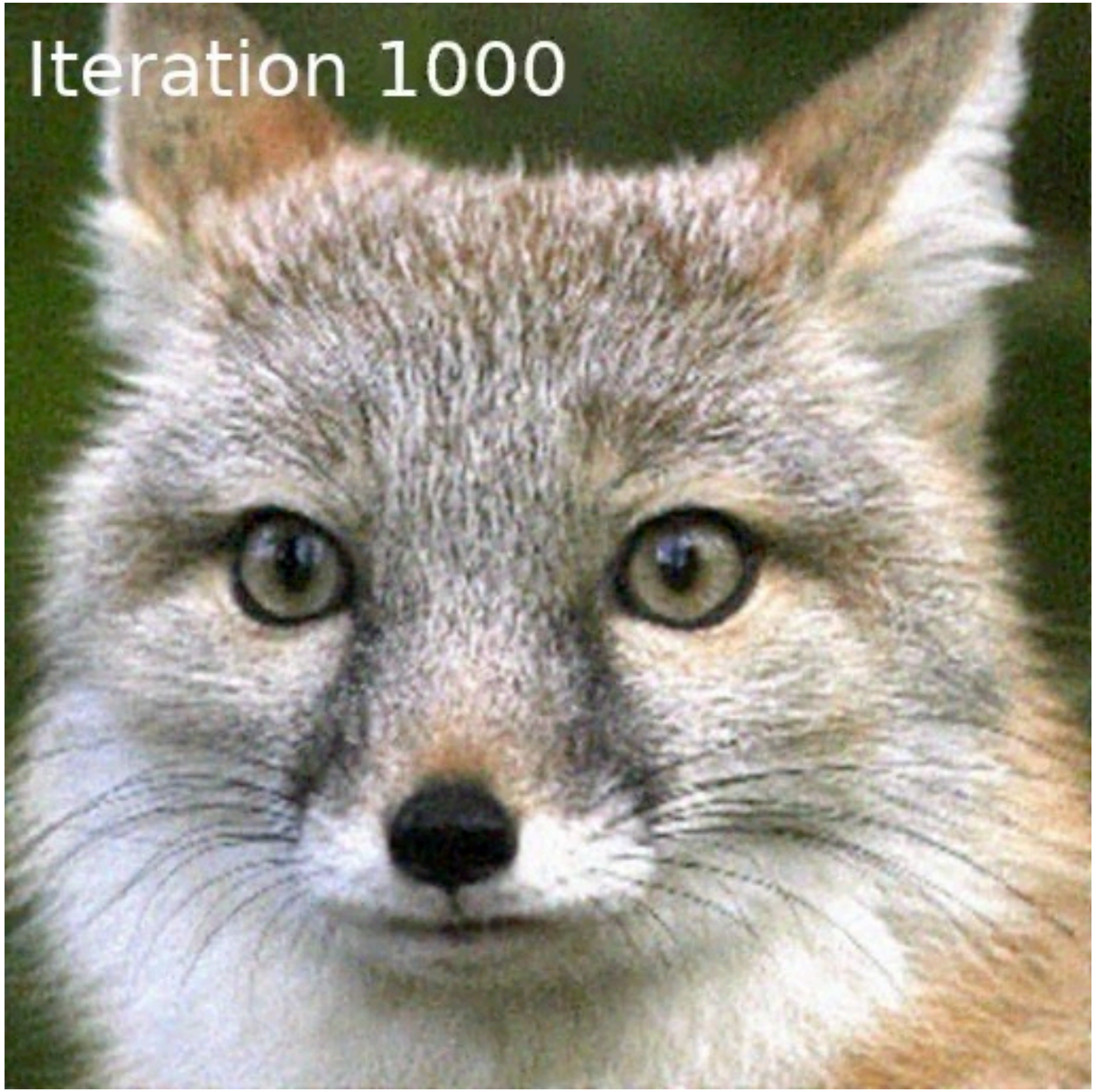
Standard input

Positionally Encoded input



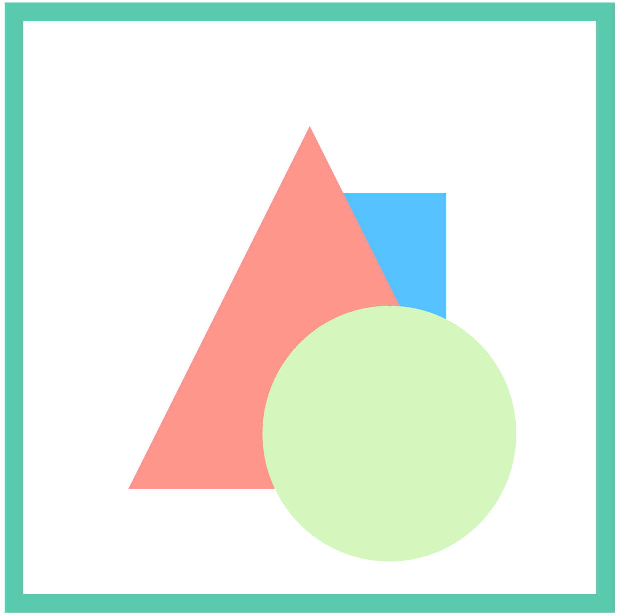
Fourier Features $\gamma(p) = \left(\sin(2^0 \pi p), \cos(2^0 \pi p), \cdots, \sin(2^{L-1} \pi p), \cos(2^{L-1} \pi p) \right)$

Standard MLP



MLP with Fourier features

Why does positional encoding help?

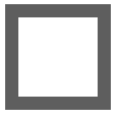


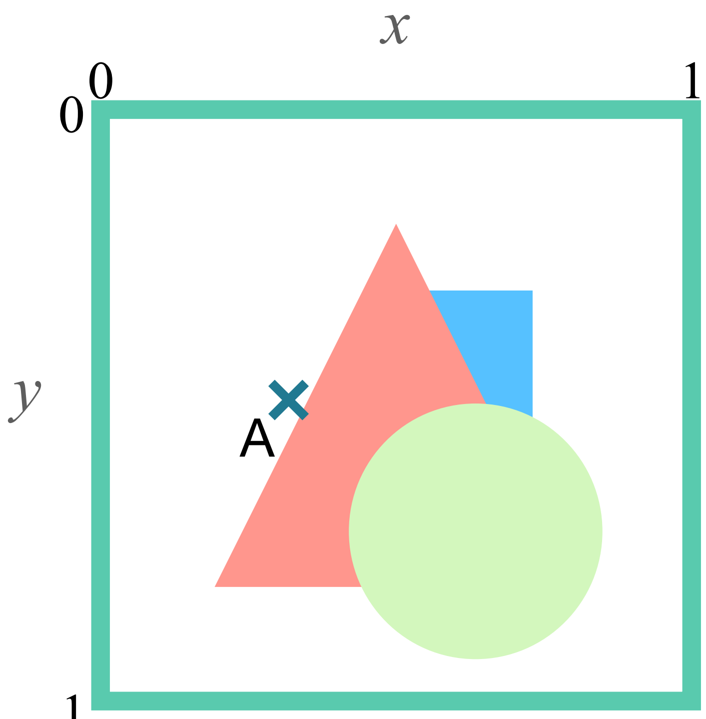
Target Image

Why does positional encoding help?

Input		
	x	y
A	.36	.5

Target





Target Image

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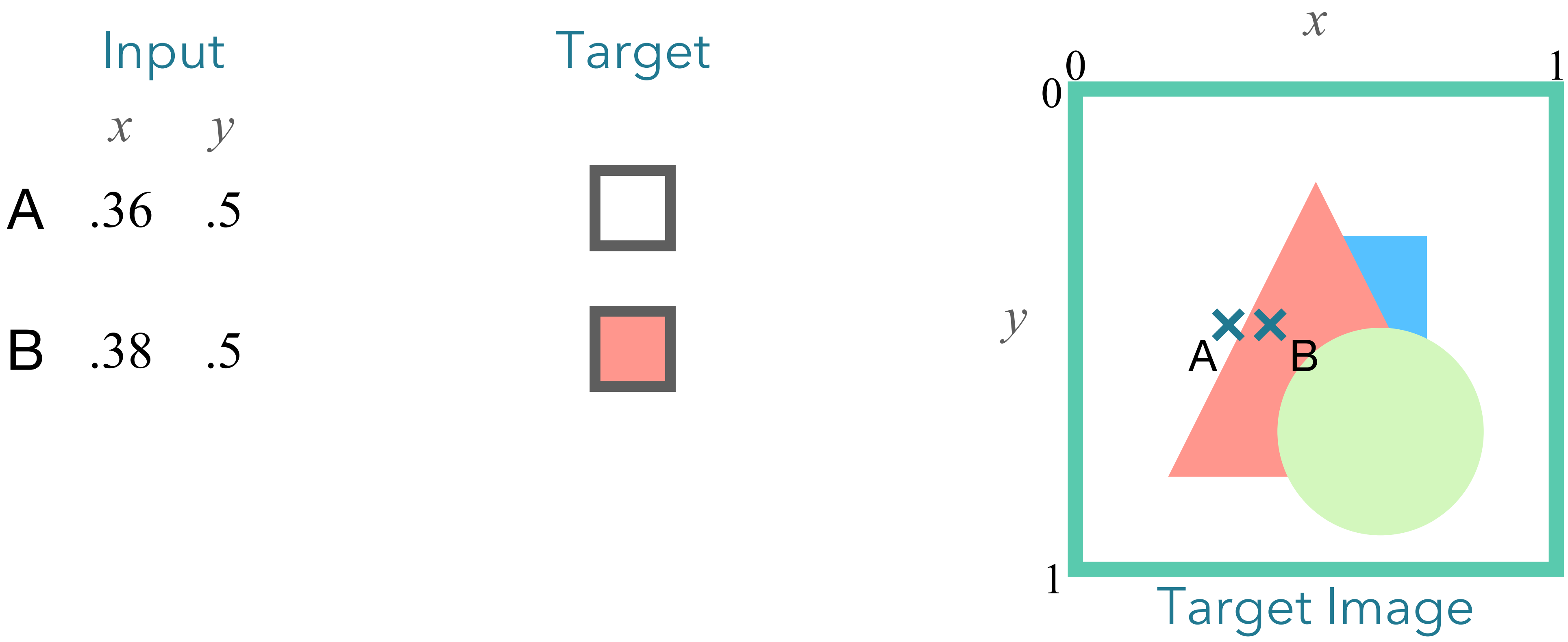

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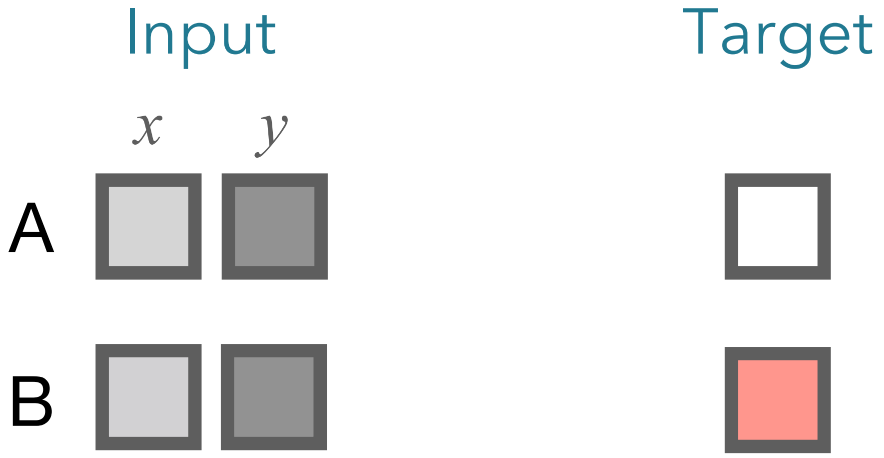
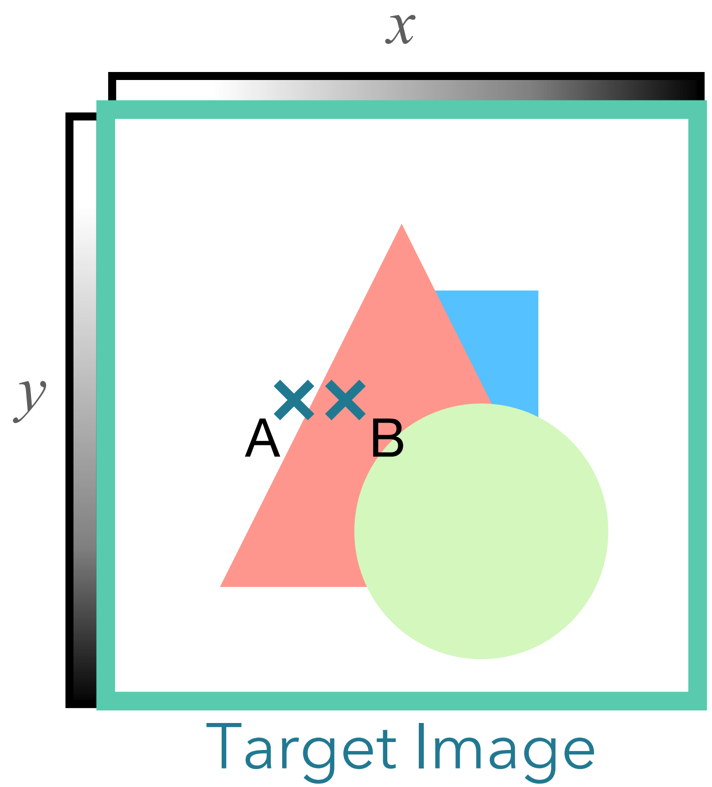
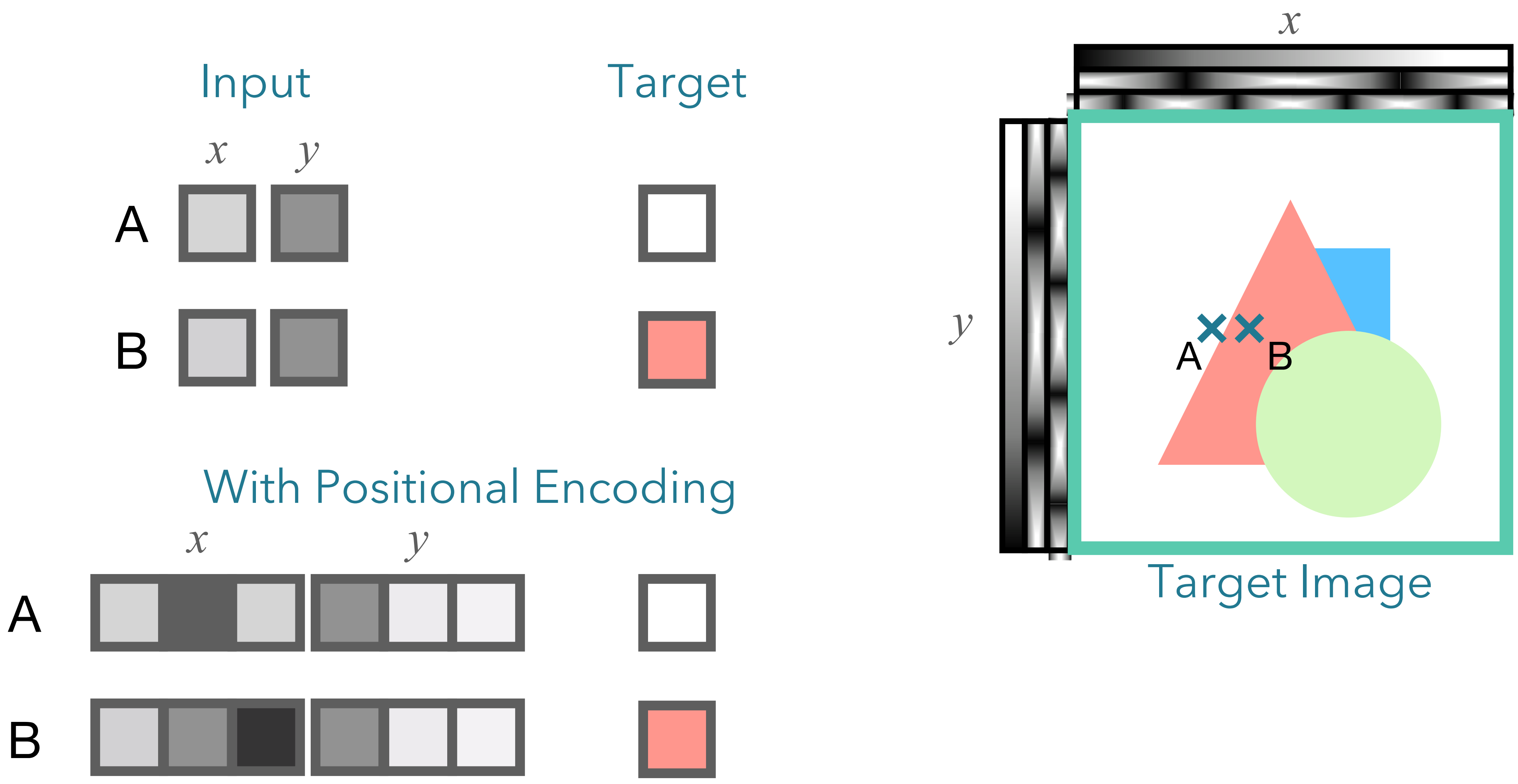


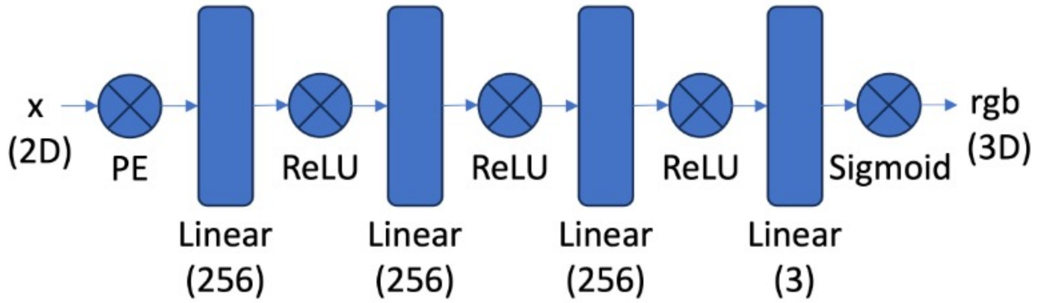


Why does positional encoding help?



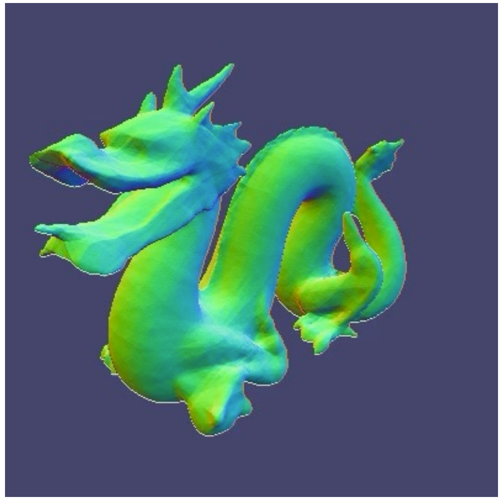
Project 5 Part 1

- Fit a Neural Network to a single image
- Implement this network, and Positional Embedding (PE) and reconstruct an image:

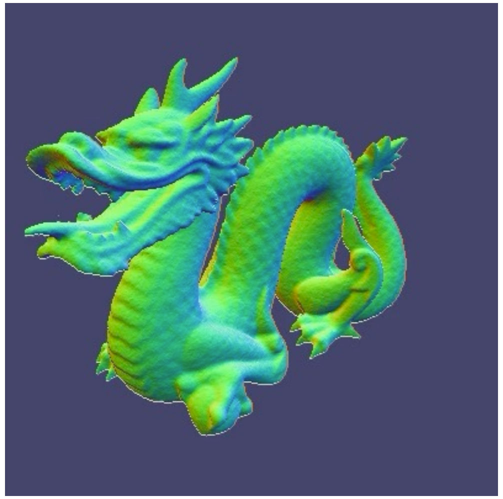


Coordinate-based MLPs can replace any low-dimensional array

Without Encoding



With Encoding



3D Shape

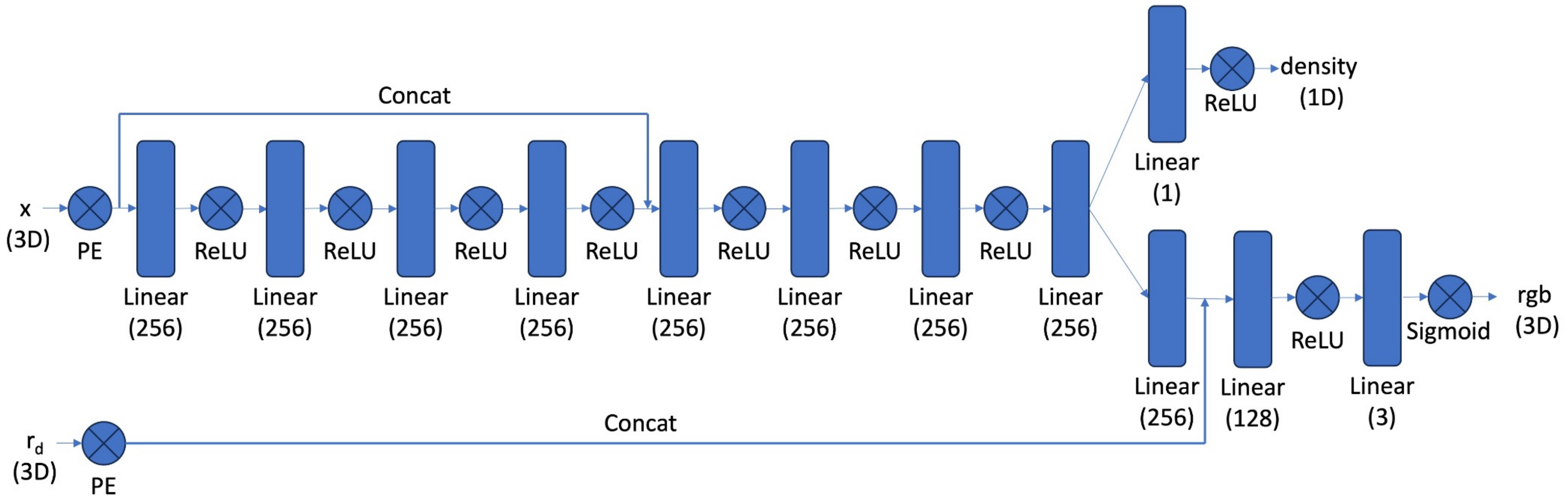
NeRF with and without positional encoding

NeRF (Naive)

NeRF (with positional encoding)

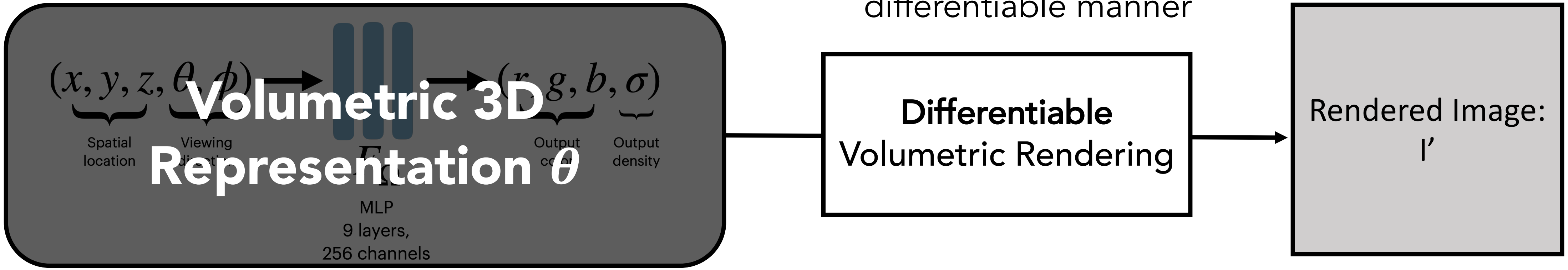
NeRF Network Architecture

Next section you will implement this:

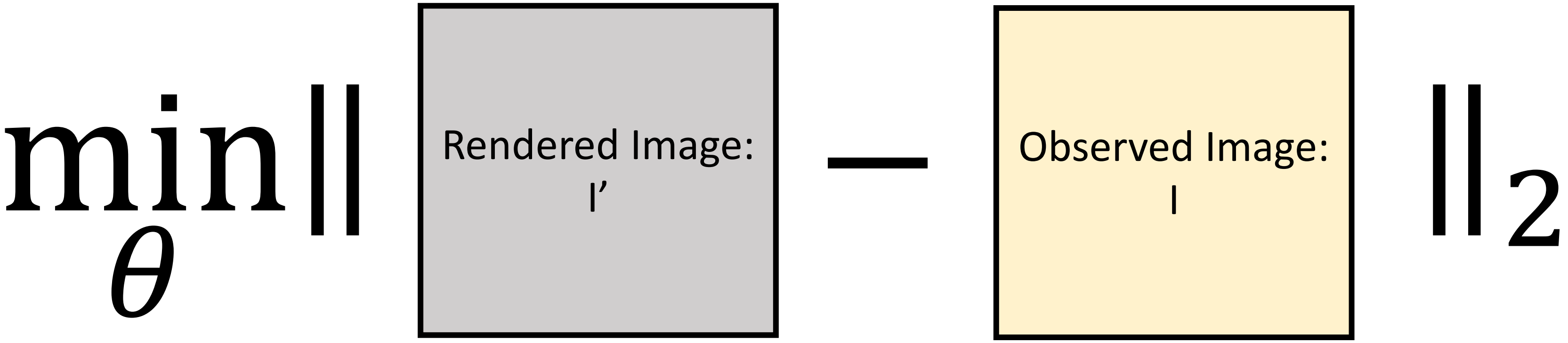


Let's go back to 3D

Now we need to render an image from this 3D representation in a differentiable manner

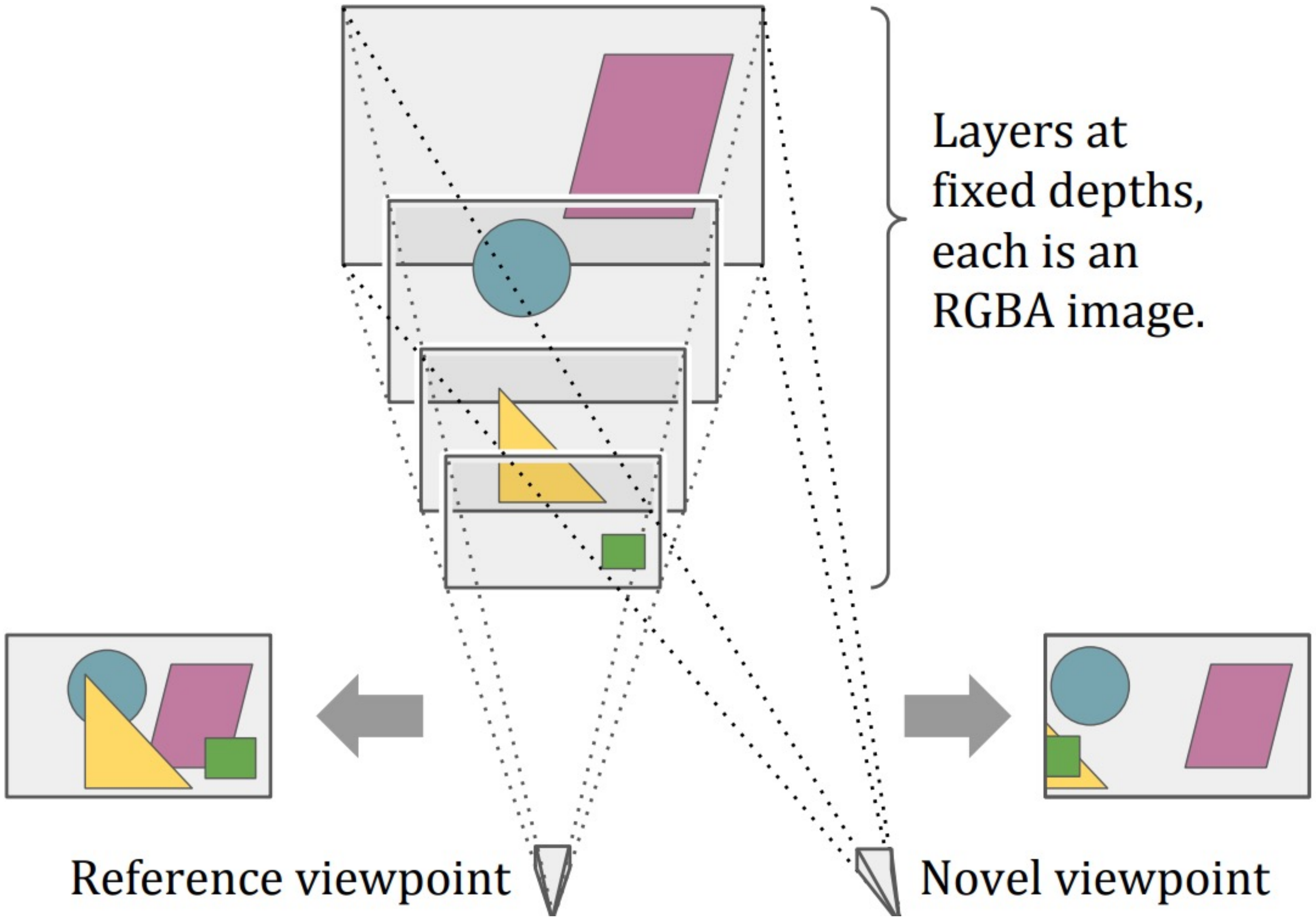


"Training" Objective (aka Analysis-by-Synthesis):

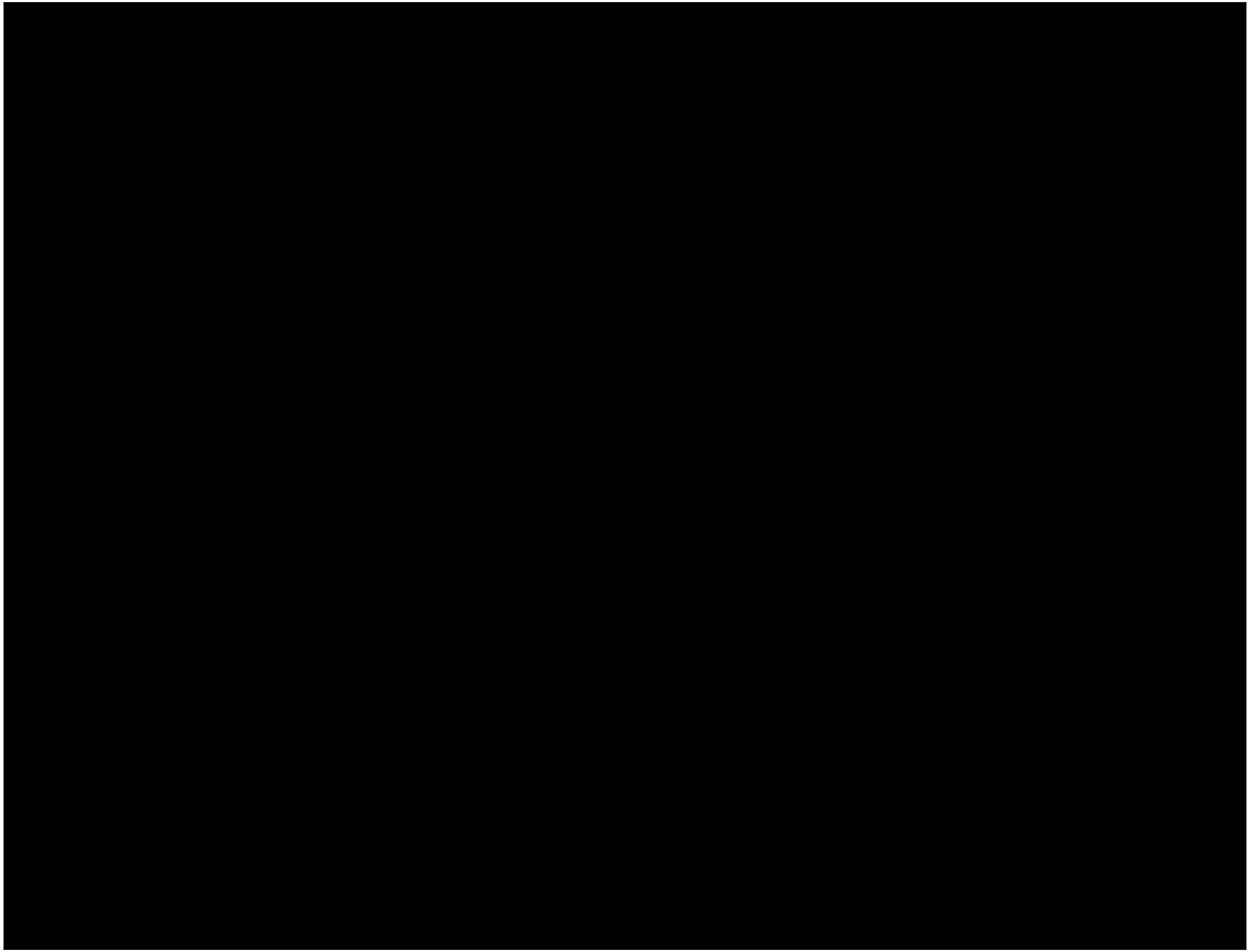


Differentiable Volumetric Rendering

A Precursor: Multi-plane Images



Zou et al. Stereo Magnification, SIGGRAPH 2018



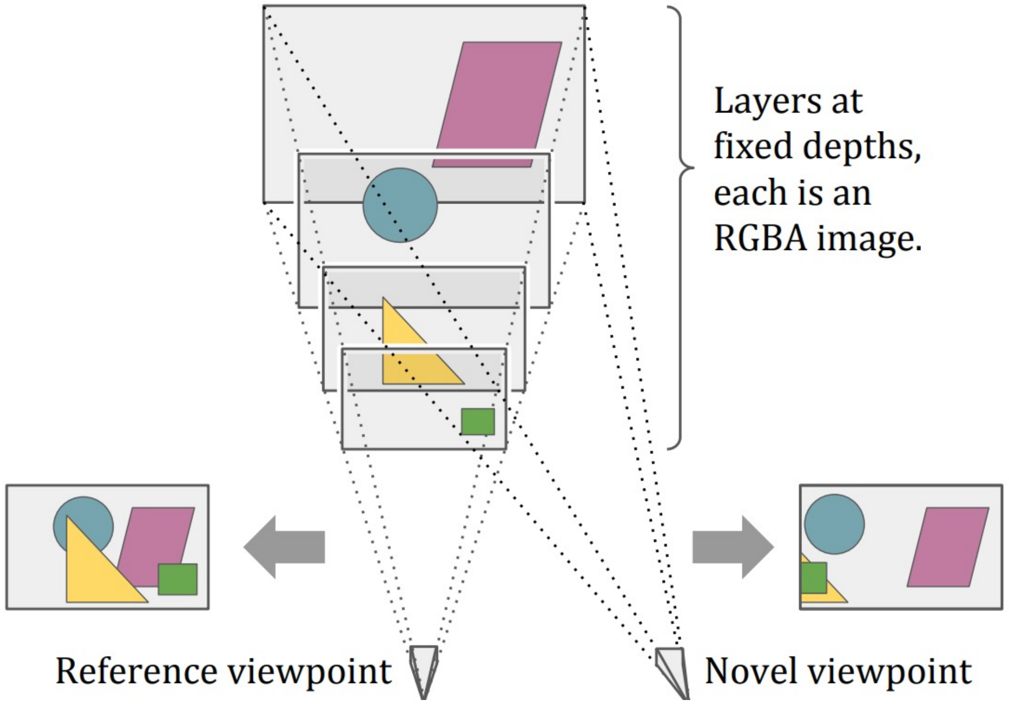
Multi-plane Camera at Disney

https://www.youtube.com/watch?v=YdHTlUGN1zw

Generating an Image MPI

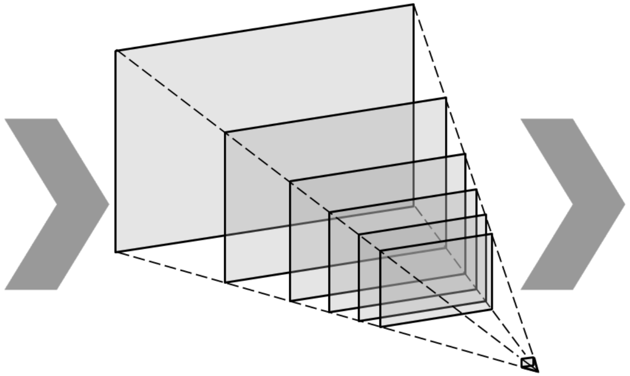
To render a novel view:

1. Homography warp the image from the new viewpoint
2. Alpha Blend each layer



Zou et al. Stereo Magnification, SIGGRAPH 2018

Sample Novel View Synthesis with a MPI

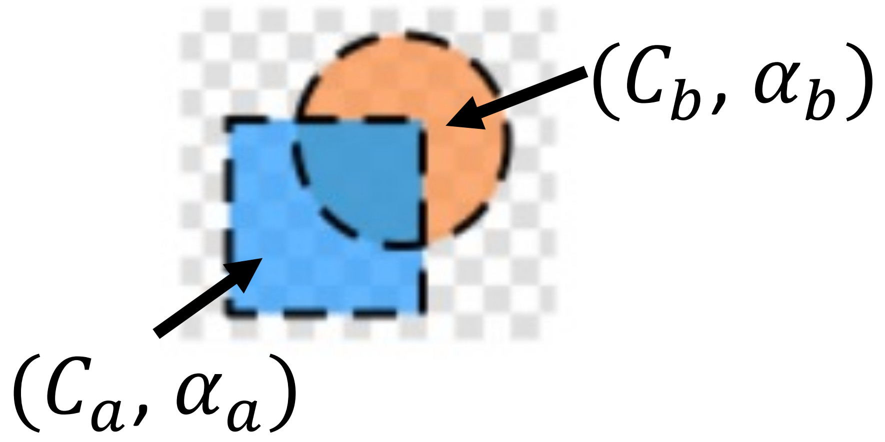


Single-view view synthesis with multiplane images, Tucker and Snavely CVPR 2020

Also called front-to-back compositing or "over" operation

Alpha Blending

for two image case, A and B,
both partially transparent:

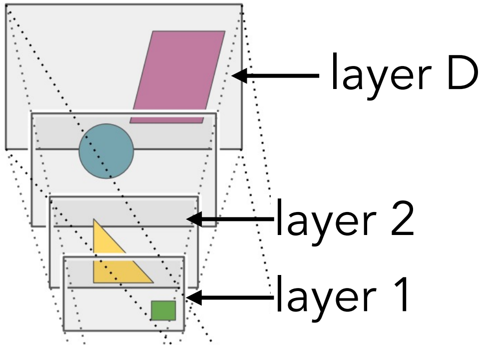


$$I = C_a\,\alpha_a + C_b\,\alpha_b\underbrace{(1 - \alpha_a)}$$

How much light is the previous laver letting through?

General D layer case:

$$I = \sum_{i=1}^{D} C_i \alpha_i \prod_{j=1}^{i-1} (1 - \alpha_j)$$



What is missing in MPIs?

- Look at it from the side??
- You'll see all the edges!!

➔ Limited camera mobility

NeRF overcomes this problem, because it's defined everywhere
Volumetric Rendering behaves similarly to alpha compositing

Back to NeRFs

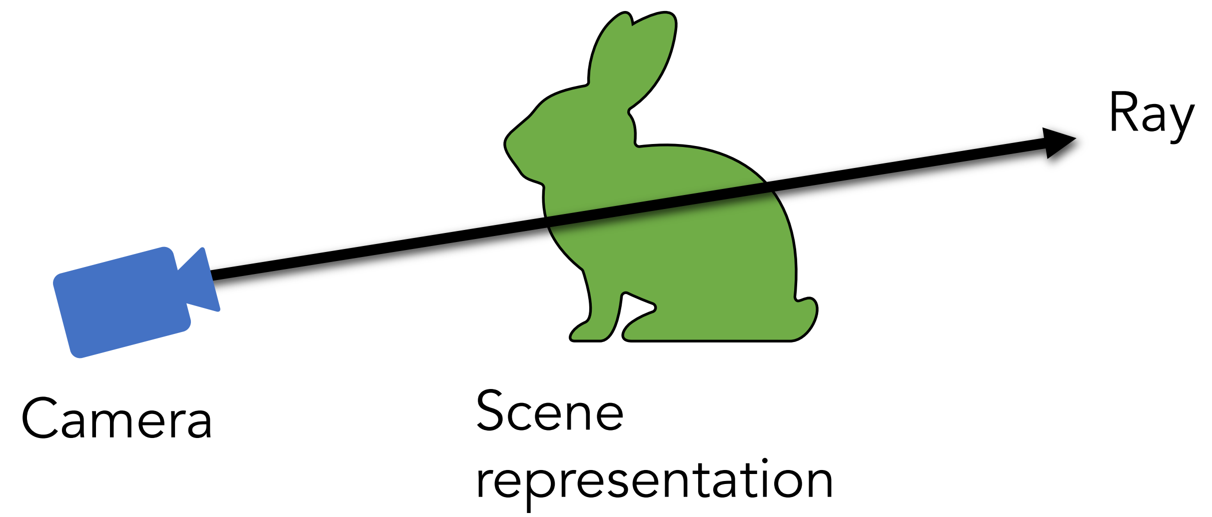
Neural Volumetric Rendering

Through Volumetric Representation (No surfaces)!

computing color along rays through 3D space

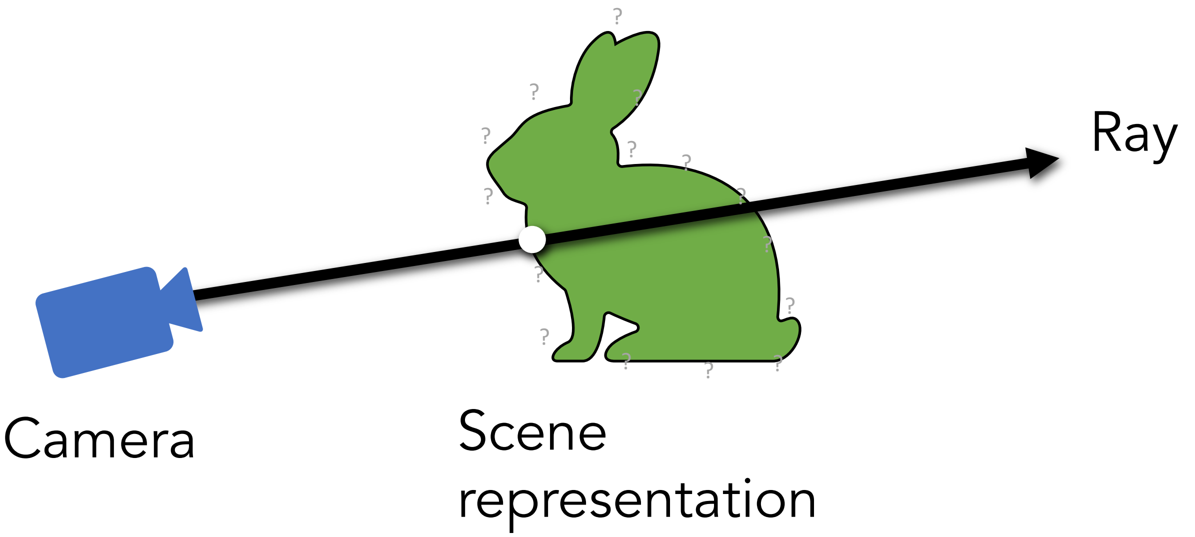
What color is this pixel?

Surface vs. volume rendering



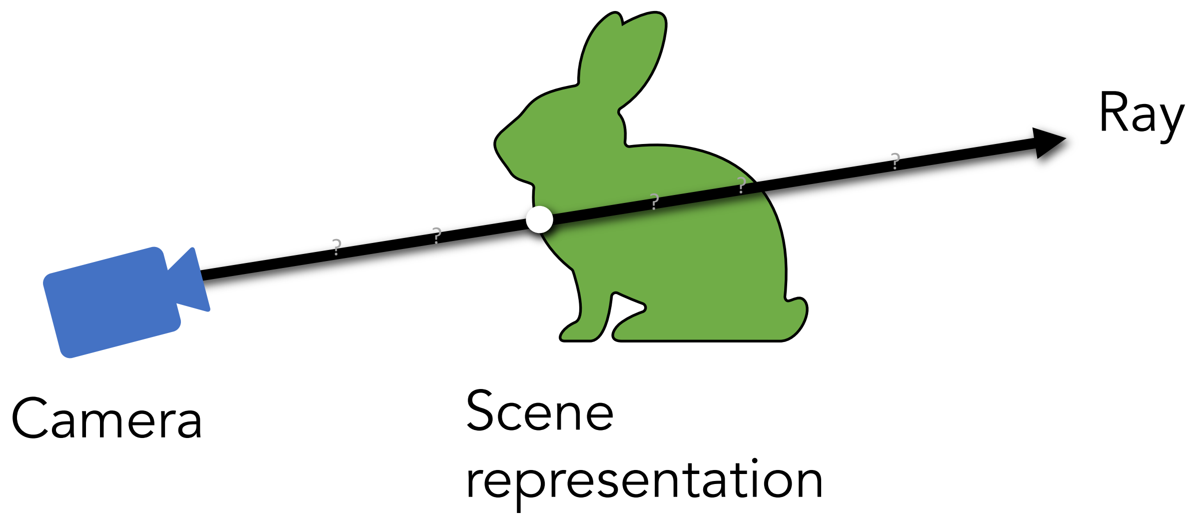
Want to know how ray interacts with scene

Surface vs. volume rendering



Surface rendering — loop over geometry, check for ray hits

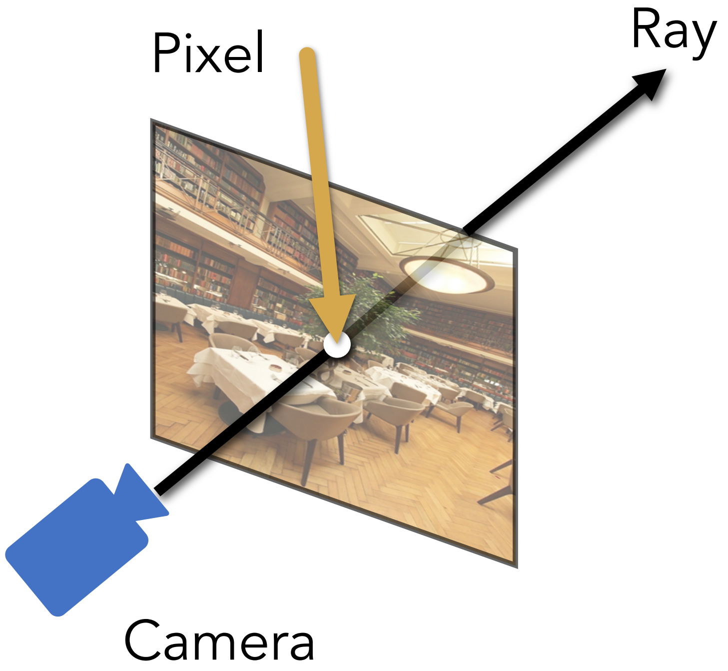
Surface vs. volume rendering



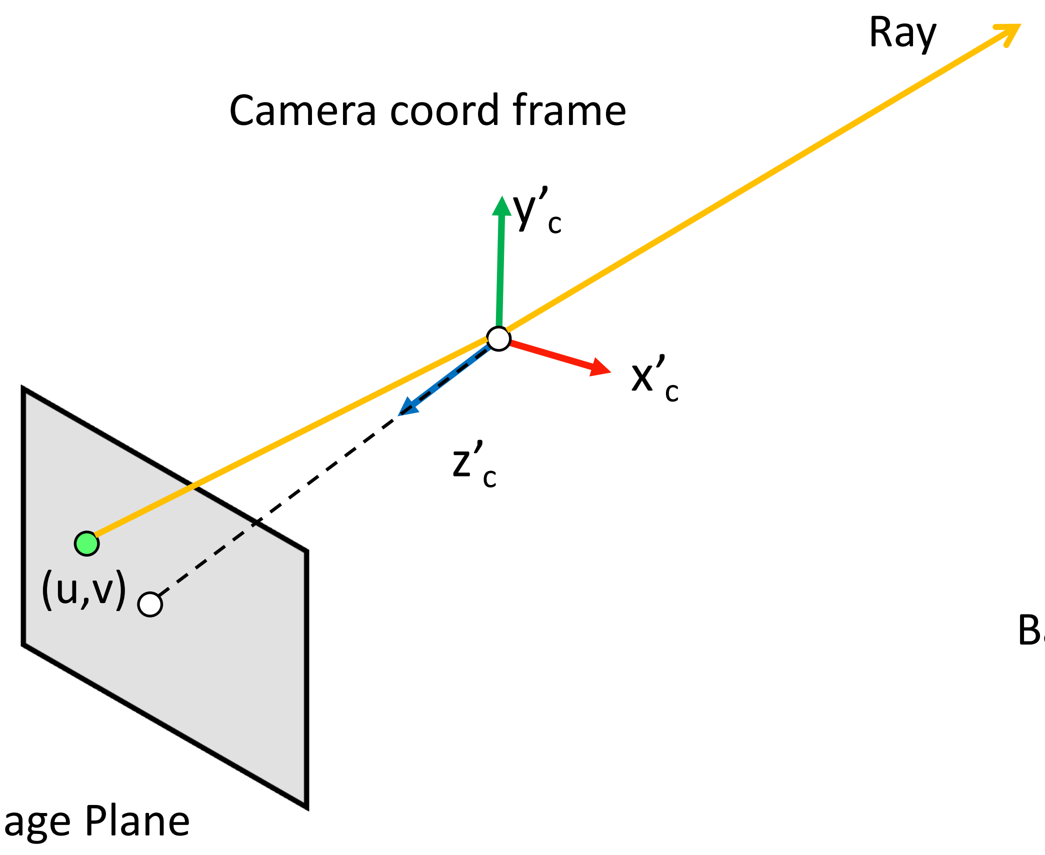
Volume rendering — loop over ray points, query geometry

Recap: Cameras and rays

- We need the mathematical mapping from (*camera*, *pixel*) → *ray*
- Then can abstract underlying problem as learning the function *ray* → *color*



Compute the Ray



3D to 2D:
(point)

$$u = f_x \frac{x_c}{z_c} + o_x$$

$$v = f_y \frac{y_c}{z_c} + o_y$$

2D to 3D:
(ray)
Back projection

$$x = \frac{z}{f_x}(u - o_x)$$

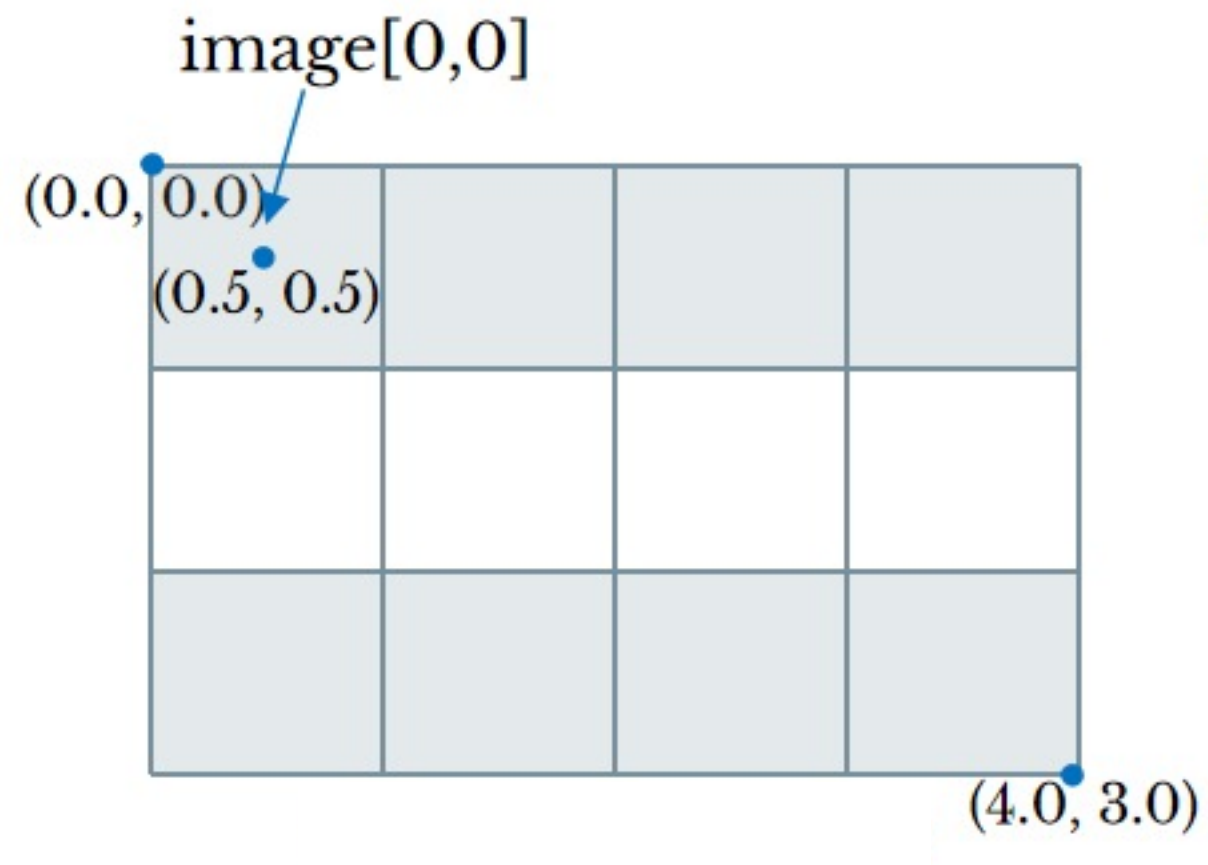
$$y = \frac{z}{f_y}(v - o_y)$$

$$z > 0$$

Slide credit: Shree Nayar

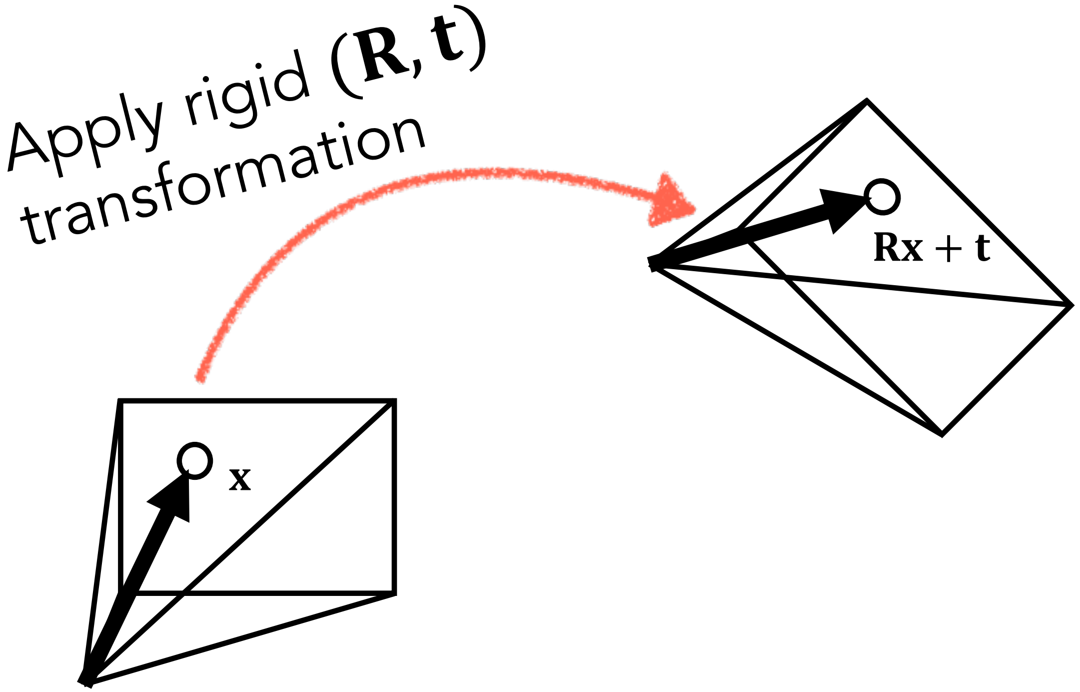
Details:

A half-pixel offset — add 0.5 to *i* and *j* so ray precisely hits pixel center



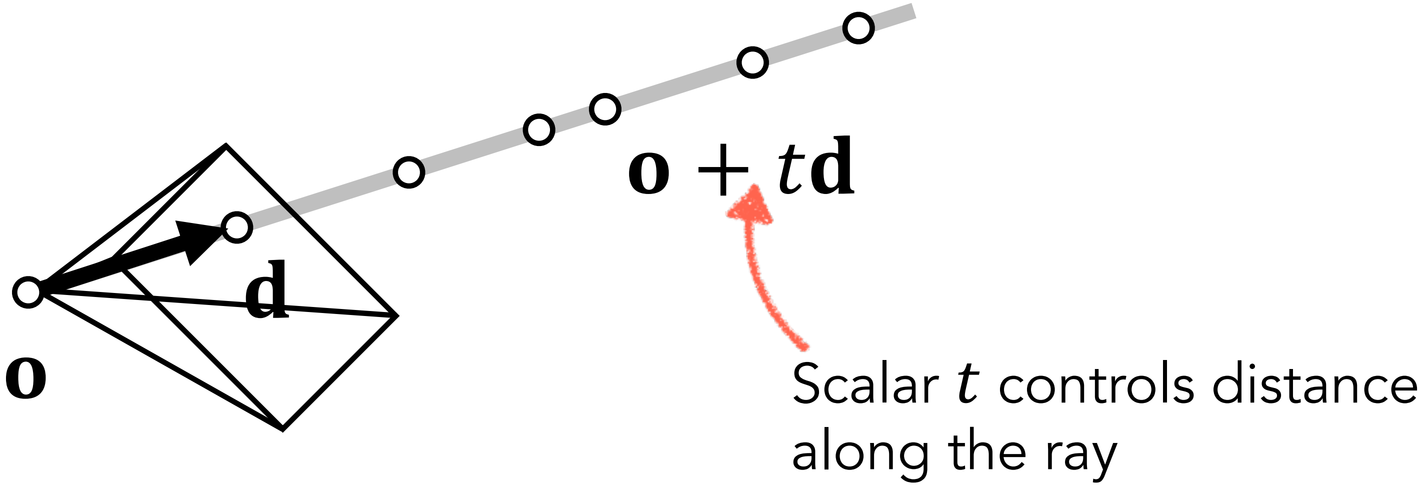
Want: Ray in the World

- What coordinate space is the current ray in?
- Convert it to World!



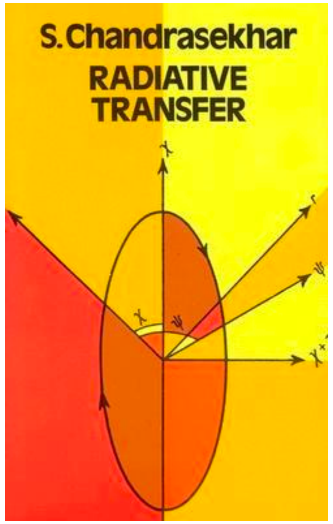
Calculating points along a ray

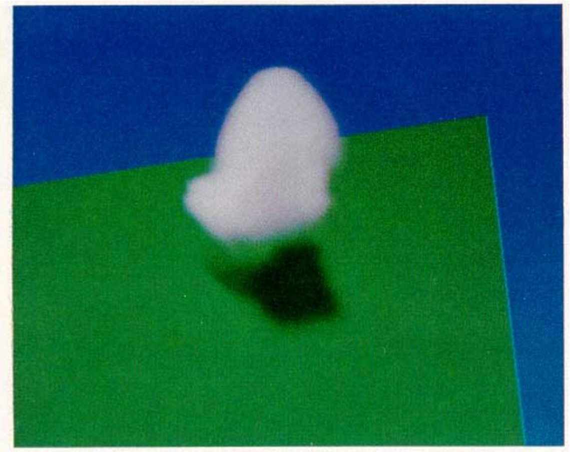
In the world coordinate frame:



History of volume rendering

In Early computer graphics





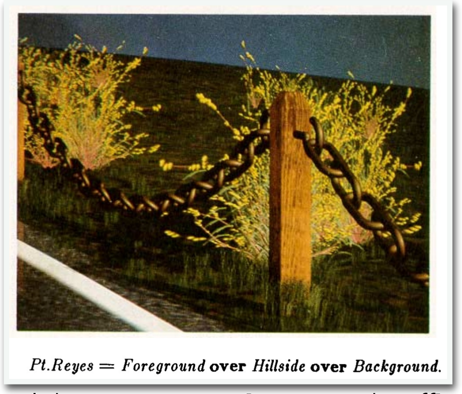
Ray tracing simulated cumulus cloud [Kajiya]

- Theory of volume rendering co-opted from physics in the 1980s: absorption, emission, out-scattering/in-scattering
- Adapted for visualising medical data and linked with alpha compositing
- Modern path tracers use sophisticated Monte Carlo methods to render volumetric effects

Chandrasekhar 1950, *Radiative Transfer*
Kajiya 1984, *Ray Tracing Volume Densities*

Alpha compositing

- Theory of volume rendering co-opted from physics in the 1980s: absorption, emission, out-scattering/in-scattering

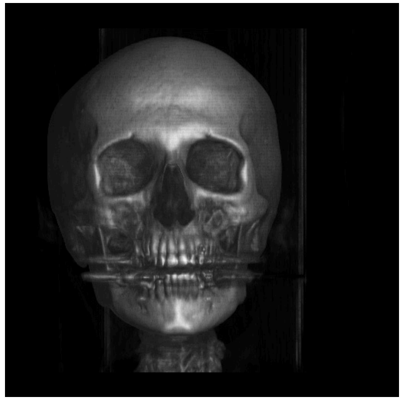


Alpha compositing [Porter and Duff]

- Alpha rendering developed for digital compositing in VFX movie production

Porter and Duff 1984, *Compositing Digital Images*

Volume rendering for visualization

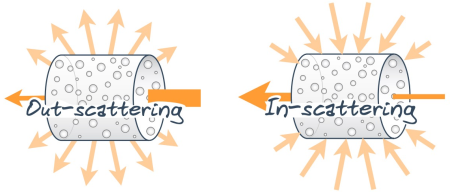


Medical data visualisation [Levoy]

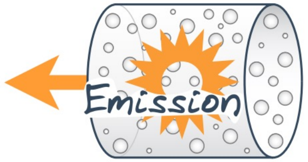
- Theory of volume rendering co-opted from physics in the 1980s: absorption, emission, out-scattering/in-scattering
- Alpha rendering developed for digital compositing in VFX movie production
- Volume rendering applied to visualise 3D medical scan data in 1990s

Chandrasekhar 1950, *Radiative Transfer*
Kajiya 1984, *Ray Tracing Volume Densities*
Porter and Duff 1984, *Compositing Digital Images*
Levoy 1988, *Display of Surfaces from Volume Data*
Max 1995, *Optical Models for Direct Volume Rendering*

Absorption

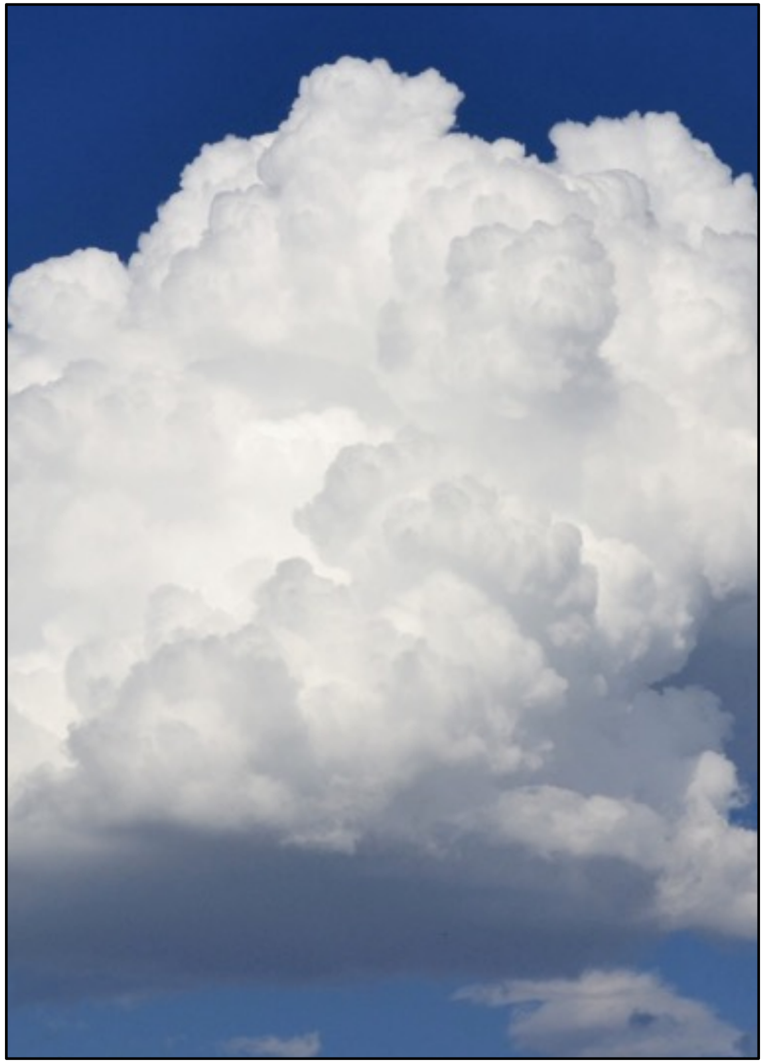


Scattering



Emission

http://commons.wikimedia.org



http://wikipedia.org

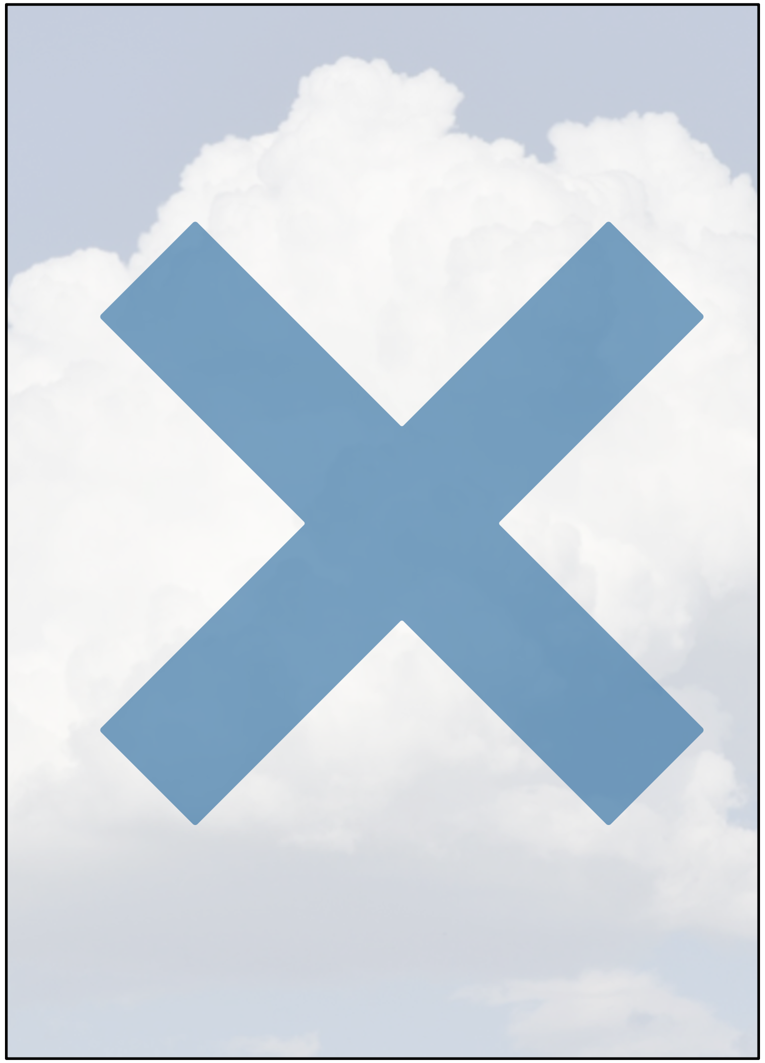
Slide credit: Novak et al 2018, *Monte Carlo methods for physically based volume rendering*

Simplify

Absorption

http://commons.wikimedia.org

Scattering



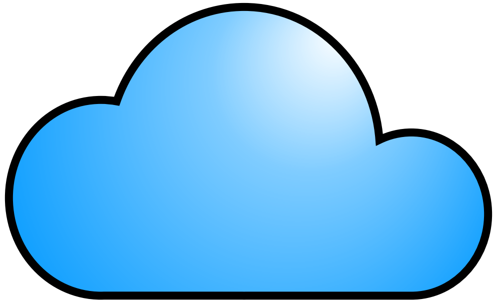
Emission

http://wikipedia.org

Slide credit: Novak et al 2018, *Monte Carlo methods for physically based volume rendering*

Volume rendering derivations

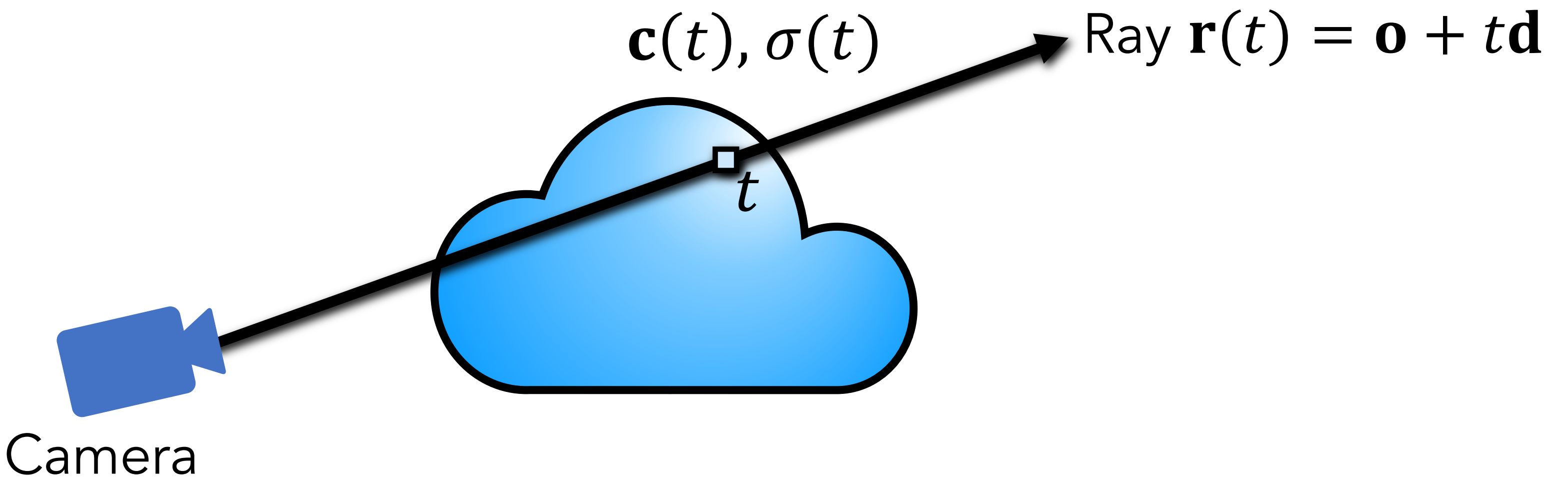
Volumetric formulation for NeRF



Scene is a cloud of tiny colored particles

Max and Chen 2010, *Local and Global Illumination in the Volume Rendering Integral*

Volumetric formulation for NeRF



at a point on the ray $\mathbf{r}(t)$, we can query color $\boldsymbol{c}(t)$ and density $\sigma(t)$

How to integrate all the info along the ray to get a color per ray?

Idea: Expected Color

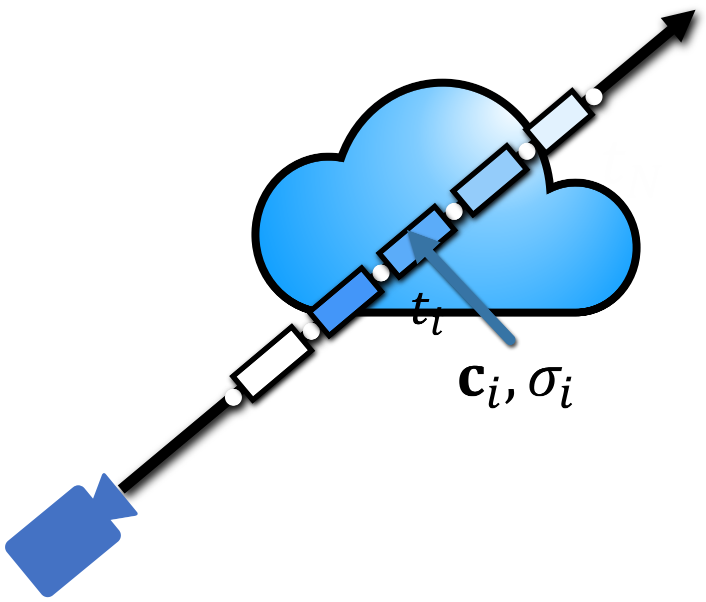
- Pose probabilistically.
- Each point on the ray has a probability to be the first "hit" : $P[first\ hit\ at\ t]$
- Color per ray = Expected value of color with this probability of first "hit"

for a ray $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$:

$$\boldsymbol{c}(\boldsymbol{r}) = \int_{t_0}^{t_1} P[first\ hit\ at\ t]\boldsymbol{c}(t)dt$$

$$\approx \sum_{t=0}^{T} P[first\ hit\ at\ t]\boldsymbol{c}(t)$$

$$\approx \sum_{t=0}^{T} w_t \boldsymbol{c}(t)$$



Differentiable Volumetric Rendering Formula

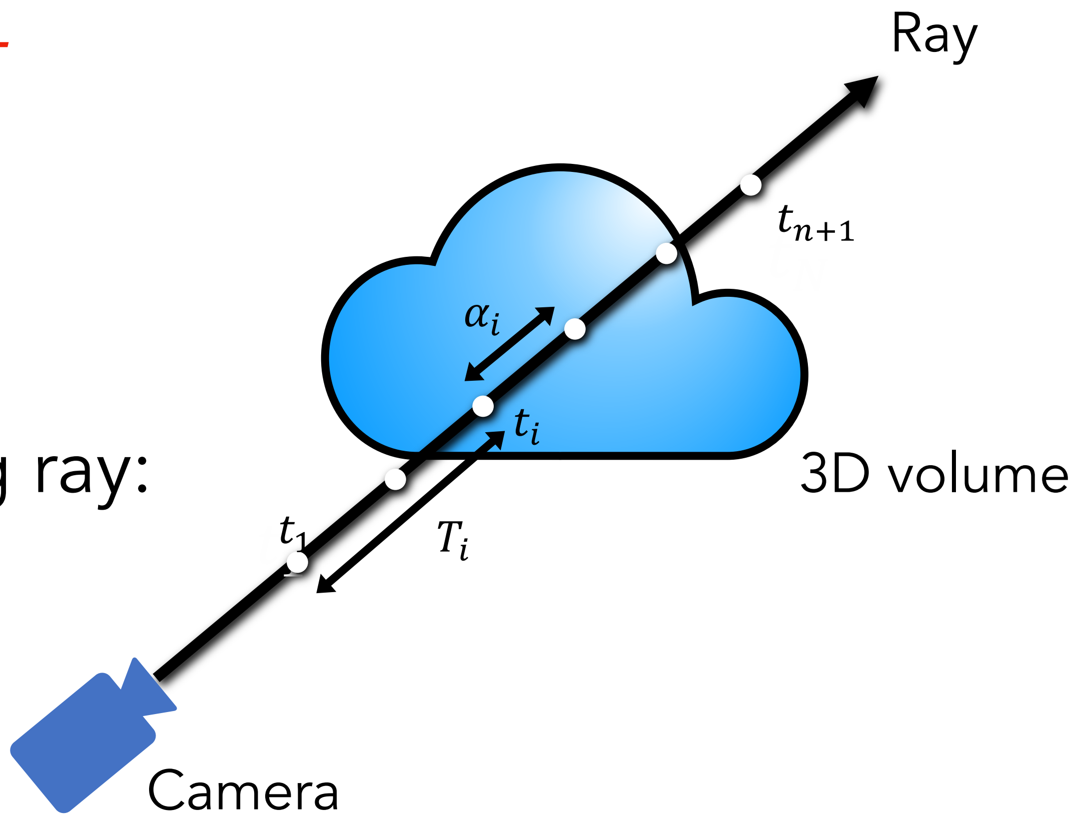
for a ray $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$:

$$\mathbf{c} \approx \sum_{i=1}^{n} w_i \mathbf{c}_i$$

differentiable w.r.t. $\mathbf{c}, \sigma$

weights

colors



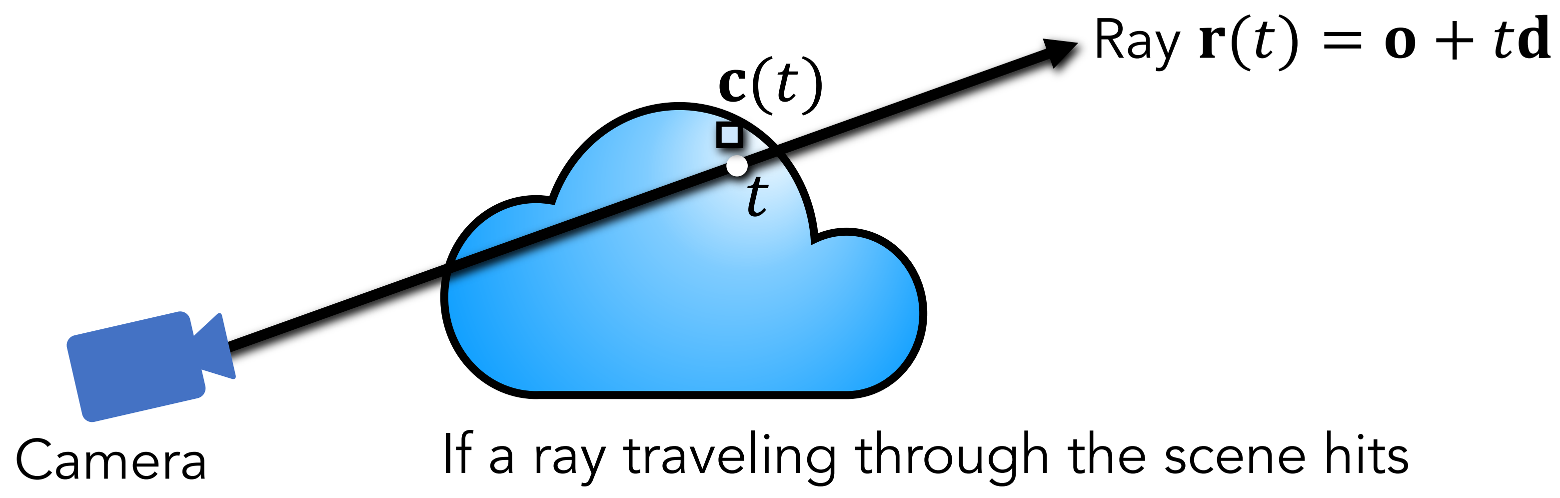
How much light is blocked earlier along ray:

$$T_i = \prod_{j=1}^{i-1} (1 - \alpha_j)$$

How much light is contributed by ray segment *i*:

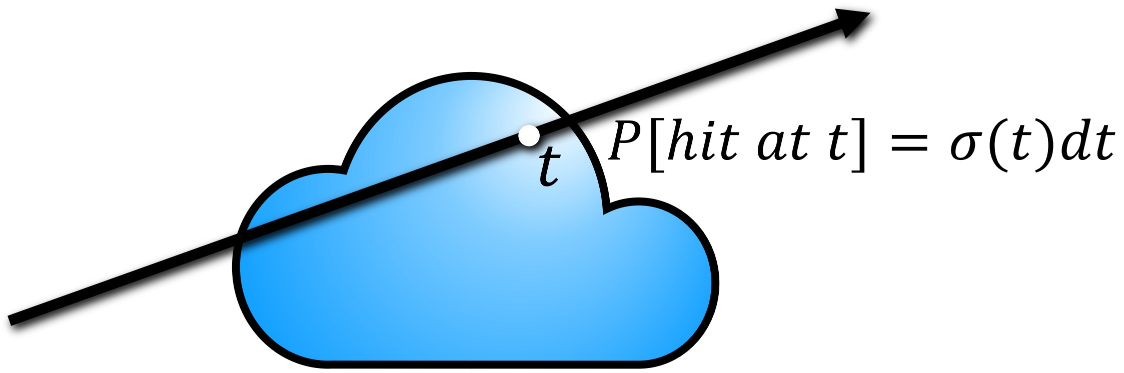
$$\alpha_i = 1 - \exp(-\sigma_i \delta_i)$$

Let's derive this:



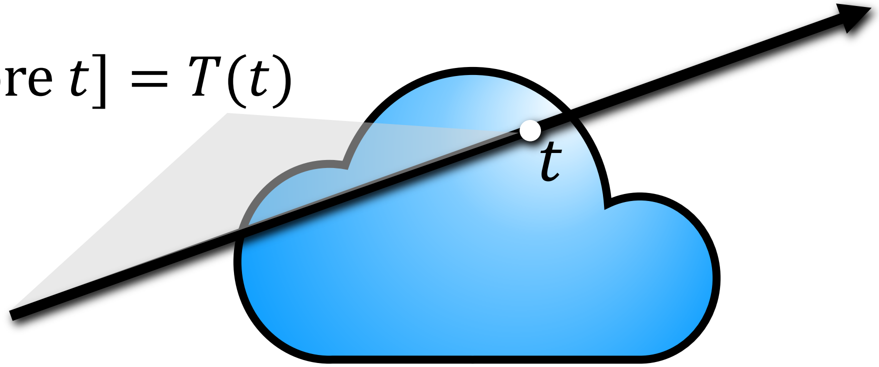
If a ray traveling through the scene hits a particle at distance t along the ray, we return its color $\mathbf{c}(t)$

What does it mean for a ray to “hit” the volume?



This notion is *probabilistic:* chance that ray hits a particle in a small interval around t is $\sigma(t)dt$.
σ is called the “volume density”

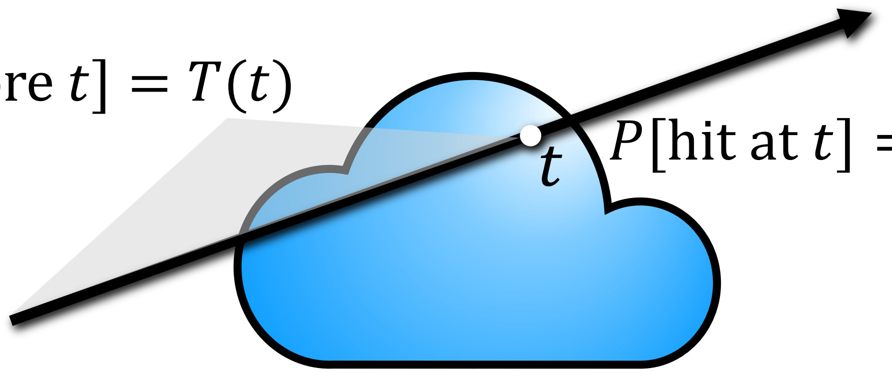
Probabilistic interpretation



To determine if t is the *first* hit along the ray, need to know $T(t)$: the probability that the ray makes it through the volume up to t.

$T(t)$ is called "transmittance"

Probabilistic interpretation

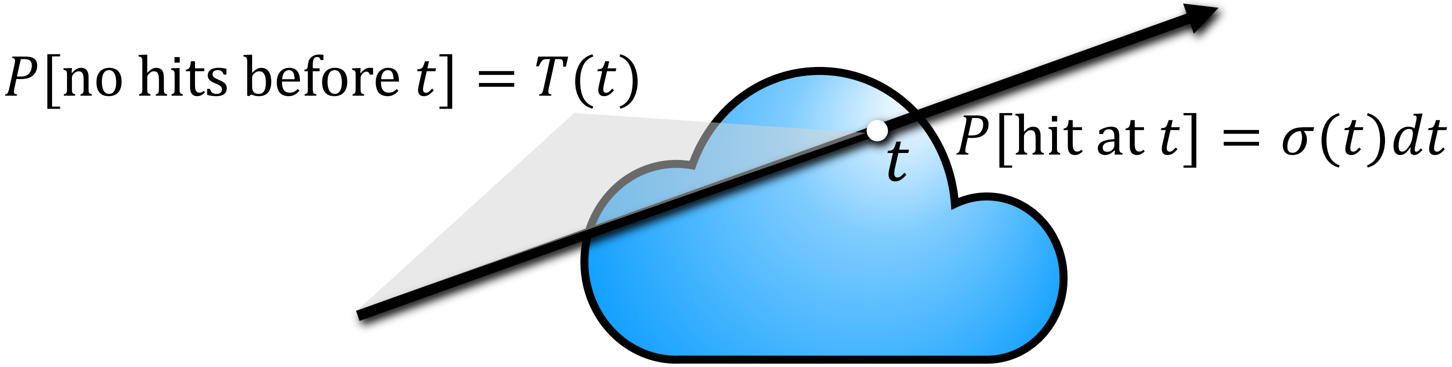


The product of these probabilities tells us how much you see the particles at t:

$P[\text{first hit at } t] = P[\text{no hit before } t] \times P[hit\ at\ t] = T(t)\sigma(t)dt$

Let's write T as a function of σ ! How?

Calculating T given σ



σ and T are related by the probabilistic fact that

$$\underbrace{P[\text{no hit before } t + dt]}_{T(t+dt)} = \underbrace{P[\text{no hit before } t]}_{T(t)} \times \underbrace{P[\text{no hit at } t]}_{(1-\sigma(t)dt)}$$

Calculating transmittance T

$$T(t + dt) = T(t)(1 - \sigma(t)dt)$$

$$T(t + dt) \quad = \quad T(t) \quad (1 - \sigma(t)dt)$$

Now we can solve for T

Solve for T

$$T(t + dt) = T(t)(1 - \sigma(t)dt)$$

Expanded Righthand side

Taylor expansion for T⇒ $T(t) + T'(t)dt = T(t) - T(t)\sigma(t)dt$

Solve for T

$$T(t + dt) = T(t)(1 - \sigma(t)dt)$$

Taylor expansion for T⇒ $\cancel{T(t)} + T'(t)dt = \cancel{T(t)} - T(t)\sigma(t)dt$

Rearrange⇒ $\frac{T'(t)}{T(t)}dt = -\sigma(t)dt$

Solve for T

$$T(t+dt) = T(t)(1-\sigma(t)dt)$$

Taylor expansion for T$\Rightarrow T(t) + T'(t)dt = T(t) - T(t)\sigma(t)dt$

Rearrange$\Rightarrow \frac{T'(t)}{T(t)}dt = -\sigma(t)dt$

Integrate$\Rightarrow \log T(t) = -\int_{t_0}^{t}\sigma(s)ds$

Derivative of :

$$\log f(x) = \frac{f'(x)}{f(x)}$$

Integral of:

$$\int \frac{f'(x)}{f(x)}dx = \log f(x)$$

$-\int \sigma(s)ds$

Solve for T

$$T(t+dt) = T(t)(1-\sigma(t)dt)$$

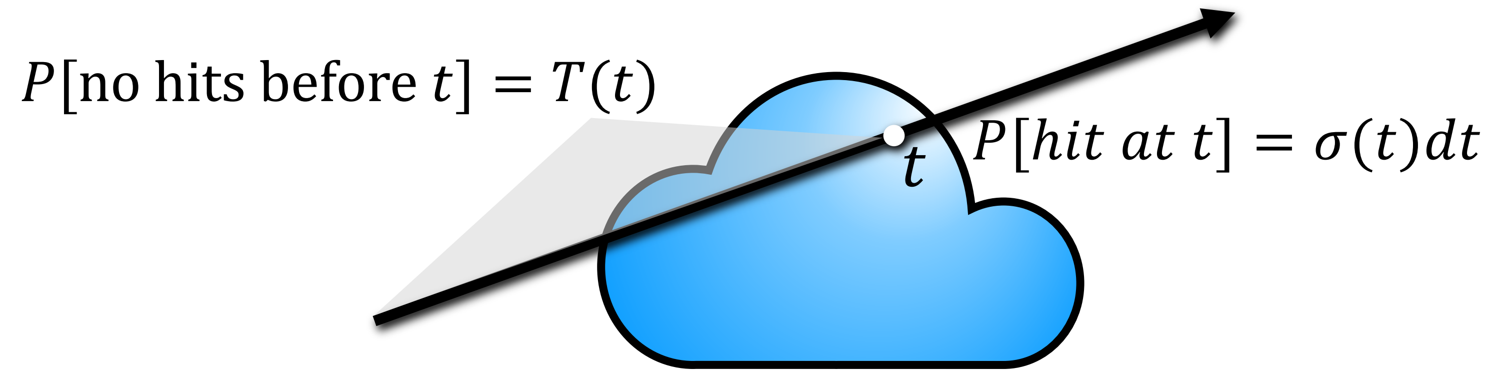
Taylor expansion for T⇒ $T(t) + T'(t)dt = T(t) - T(t)\sigma(t)dt$

Rearrange⇒ $\frac{T'(t)}{T(t)}dt = -\sigma(t)dt$

Integrate⇒ $\log T(t) = -\int_{t_0}^{t}\sigma(s)ds$

Exponentiate⇒ $T(t) = \exp\left(-\int_{t_0}^{t}\sigma(s)ds\right)$

PDF for ray termination



Finally, we can write the probability that a ray terminates at t as a function of only sigma

$$P[\text{first hit at } t] = P[\text{no hit before } t] \times P[\text{hit at } t]$$

$$= T(t)\sigma(t)dt$$

$$= \exp\left(-\int_{t_0}^{t} \sigma(s)ds\right)\sigma(t)dt$$

Expected value of color along ray

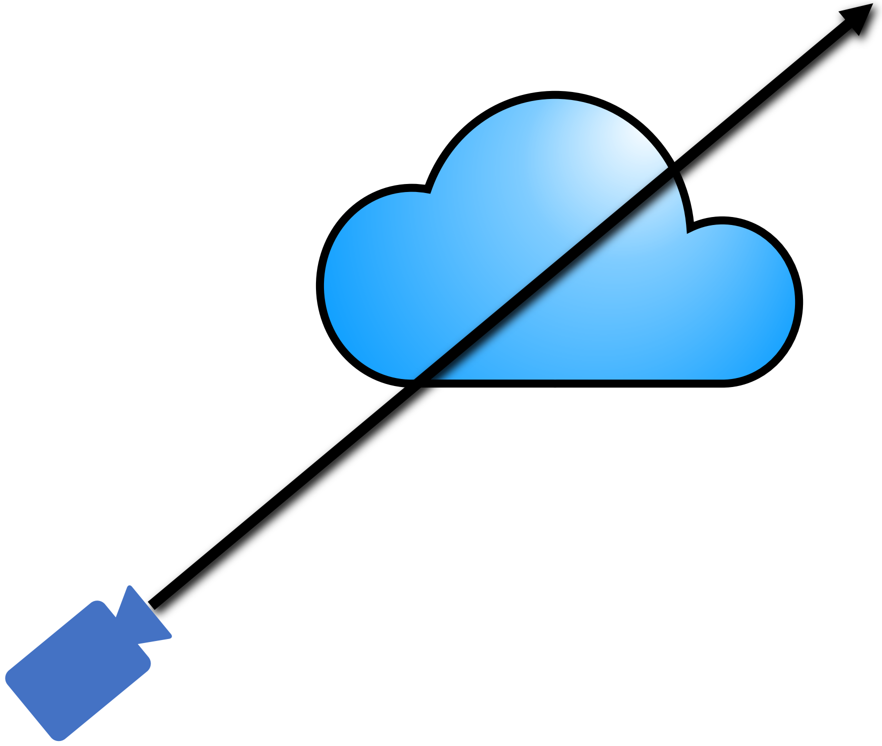
This means the expected color returned by the ray will be

$$\text{expected color of this ray} = \int_{t_0}^{t_1} \underbrace{T(t)\sigma(t)}_{P[first\ hit\ at\ t]} \mathbf{c}(t)dt$$

$$= \int_{t_0}^{t_1} \exp\left(-\int_{t_0}^{t} \sigma(s)ds\right) \sigma(t)\mathbf{c}(t)dt$$

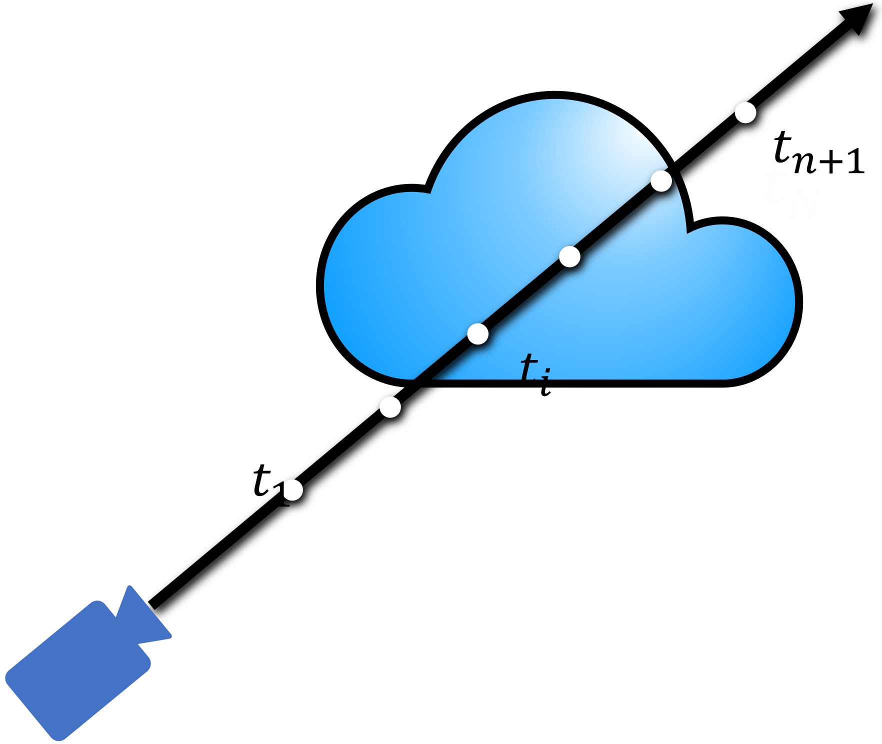
Note the nested integral!

Approximating the nested integral



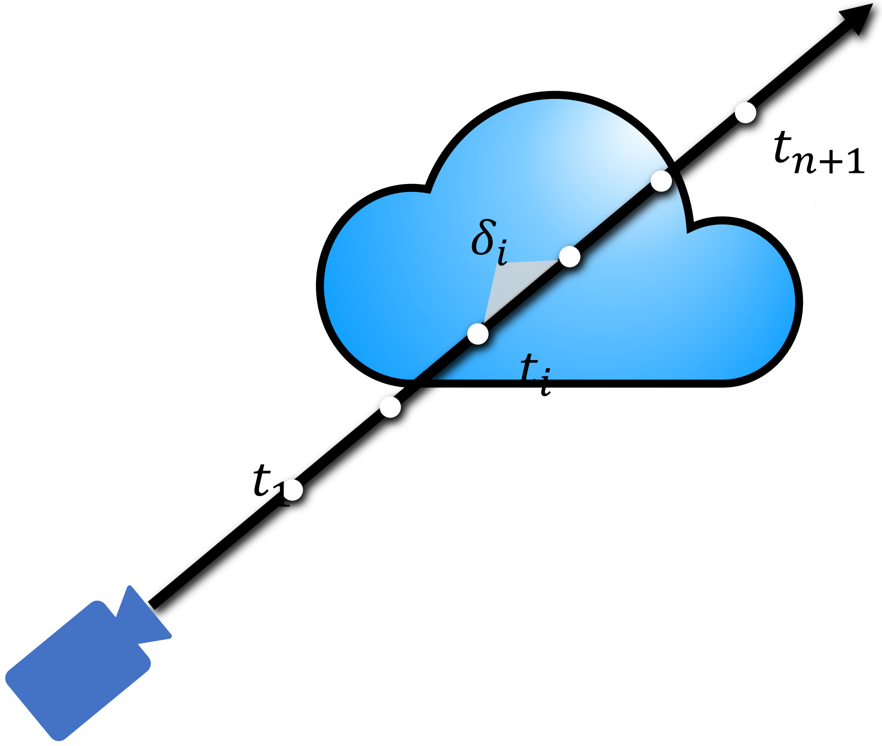
We use quadrature to approximate the nested integral,

Approximating the nested integral



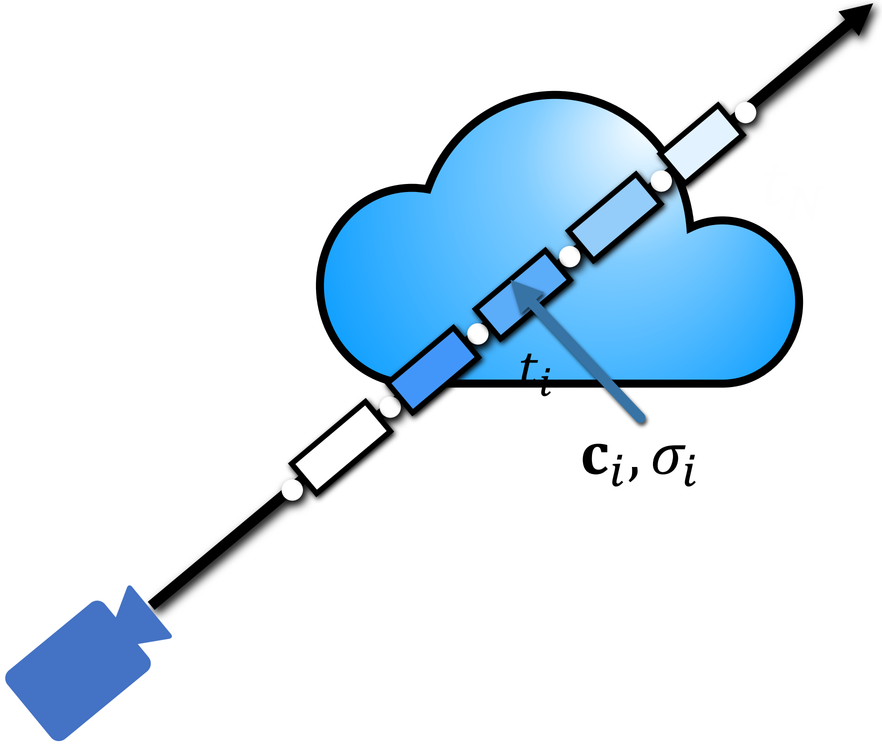
We use quadrature to approximate the nested integral, splitting the ray up into n segments with endpoints $\{t_1, t_2, \dots, t_{n+1}\}$

Approximating the nested integral



We use quadrature to approximate the nested integral, splitting the ray up into n segments with endpoints $\{t_1, t_2, \ldots, t_{n+1}\}$ with lengths $\delta_i = t_{i+1} - t_i$

Approximating the nested integral



We assume volume density and color are roughly constant within each interval

Deriving quadrature estimate

$$\int T(t)\sigma(t)\mathbf{c}(t)dt \approx$$

This allows us to break the outer integral

Deriving quadrature estimate

$$\int T(t)\sigma(t)\mathbf{c}(t)dt \approx \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} T(t)\sigma_i \mathbf{c}_i dt$$

This allows us to break the outer integral into a sum of analytically tractable integrals

Deriving quadrature estimate

$$\int T(t)\sigma(t)\mathbf{c}(t)dt \approx \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} T(t)\sigma_i \mathbf{c}_i dt$$

Caveat: piecewise constant density and color
do not imply constant transmittance!

Deriving quadrature estimate

$$\int T(t)\sigma(t)\mathbf{c}(t)dt \approx \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} T(t)\sigma_i \mathbf{c}_i dt$$

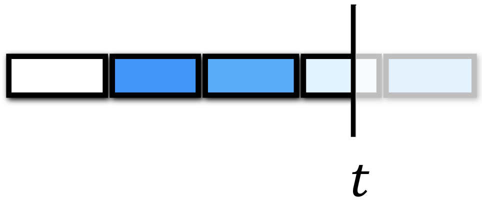
Caveat: piecewise constant density and color **do not** imply constant transmittance!

Important to account for how early part of a segment blocks later part when σ_i is high

Evaluating T for piecewise constant density

For $t \in [t_i, t_{i+1}]$, $T(t) = \exp\left(-\int_{t_1}^{t_i} \sigma_i ds\right) \exp\left(-\int_{t_i}^{t} \sigma_i ds\right)$

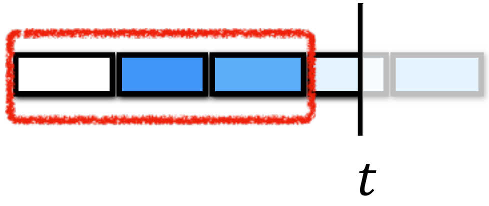
We need to evaluate at continuous t values that can lie *partway through* an interval



Evaluating T for piecewise constant density

For $t \in [t_i, t_{i+1}]$, $T(t) = \exp\left(-\int_{t_1}^{t_i} \sigma_i ds\right) \exp\left(-\int_{t_i}^{t} \sigma_i ds\right)$

$$\exp\left(-\sum_{j=1}^{i-1} \sigma_j \delta_j\right) = T_i$$ "How much light is blocked by all previous segments?"

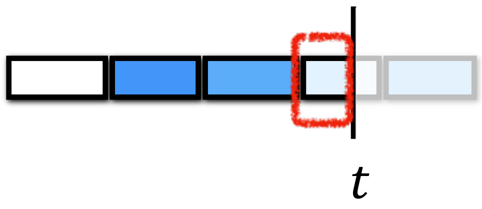


Evaluating T for piecewise constant density

For $t \in [t_i, t_{i+1}]$, $T(t) = \exp\left(-\int_{t_1}^{t_i} \sigma_i ds\right) \exp\left(-\int_{t_i}^{t} \sigma_i ds\right)$

$$\exp(-\sigma_i(t - t_i))$$

"How much light is blocked partway through the current segment?"



Deriving quadrature estimate

$$\int T(t)\sigma(t)\mathbf{c}(t)dt \approx \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} T(t)\sigma_i \mathbf{c}_i dt$$

Deriving quadrature estimate

$$\int T(t)\sigma(t)\mathbf{c}(t)dt \approx \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} T(t)\sigma_i \mathbf{c}_i dt$$

Substitute $= \sum_{i=1}^{n} T_i \sigma_i \mathbf{c}_i \int_{t_i}^{t_{i+1}} \exp(-\sigma_i(t - t_i))dt$

Deriving quadrature estimate

$$\int T(t)\sigma(t)\mathbf{c}(t)dt \approx \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} T(t)\sigma_i \mathbf{c}_i dt$$

$$= \sum_{i=1}^{n} T_i \sigma_i \mathbf{c}_i \int_{t_i}^{t_{i+1}} \exp(-\sigma_i(t - t_i))dt$$

Integrate $= \sum_{i=1}^{n} T_i \sigma_i \mathbf{c}_i \dfrac{\exp(-\sigma_i(t_{i+1} - t_i)) - 1}{-\sigma_i}$

Integral of Exponential:

$$\int \exp(-ax)\, dx = -\frac{1}{a}\exp(-ax)$$

$$\int_{t_i}^{t_{i+1}} \exp(-\sigma(t - t_i))\, dt = -\frac{1}{\sigma}\exp(-\sigma(t - t_i) \mid_{t_i}^{t_{i+1}}$$

$$\frac{\exp(-\sigma_i(t_{i+1} - t_i)) - \exp(-\sigma_i(t_i - t_i))}{-\sigma_i} = \frac{\exp(-\sigma_i(t_{i+1} - t_i)) - 1}{-\sigma_i}$$

Deriving quadrature estimate

$$\int T(t)\sigma(t)\mathbf{c}(t)dt \approx \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} T(t)\sigma_i \mathbf{c}_i dt$$

$$= \sum_{i=1}^{n} T_i \sigma_i \mathbf{c}_i \int_{t_i}^{t_{i+1}} \exp(-\sigma_i(t - t_i))dt$$

$$= \sum_{i=1}^{n} T_i \sigma_i \mathbf{c}_i \frac{\exp(-\sigma_i(t_{i+1} - t_i)) - 1}{-\sigma_i}$$

Cancel σ_i $= \sum_{i=1}^{n} T_i \mathbf{c}_i (1 - \exp(-\sigma_i \delta_i))$

Expected Color $= \sum_{i=1}^{n} T_i \mathbf{c}_i (1 - \exp(-\sigma_i \delta_i))$

Putting it all together

$$\text{Expected Color} = \sum_{i=1}^{n} T_i \mathbf{c}_i (1 - \exp(-\sigma_i \delta_i))$$

where $$T_i = \exp\left(-\sum_{j=1}^{i-1} \sigma_j \delta_j\right)$$

Connection to alpha compositing

$$\text{Expected Color} = \sum_{i=1}^{n} T_i \mathbf{c}_i \underbrace{(1 - \exp(-\sigma_i \delta_i))}_{\text{segment opacity } \alpha_i}$$

segment
opacity α_i

$$\text{Expected Color} = \sum_{i=1}^{n} T_i \mathbf{c}_i \alpha_i$$

where $$T_i = \exp\left(-\sum_{j=1}^{i-1} \sigma_j \delta_j\right)$$

$$= \prod_{j=1}^{i-1} (1 - \alpha_j)$$

$$\prod_i \exp(x_i) = \exp(\sum_i x_i)$$
$$\alpha_i = 1 - \exp(\sigma_i \delta_i)$$
$$1 - \alpha_i = -\exp(\sigma_i \delta_i)$$

Summary

for a ray $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$:

differentiable w.r.t. $\mathbf{c}, \sigma$

$$\mathbf{c} \approx \sum_{i=1}^{n} w_i \mathbf{c}_i = \sum_{i=1}^{n} T_i \alpha_i \mathbf{c}_i$$

weights

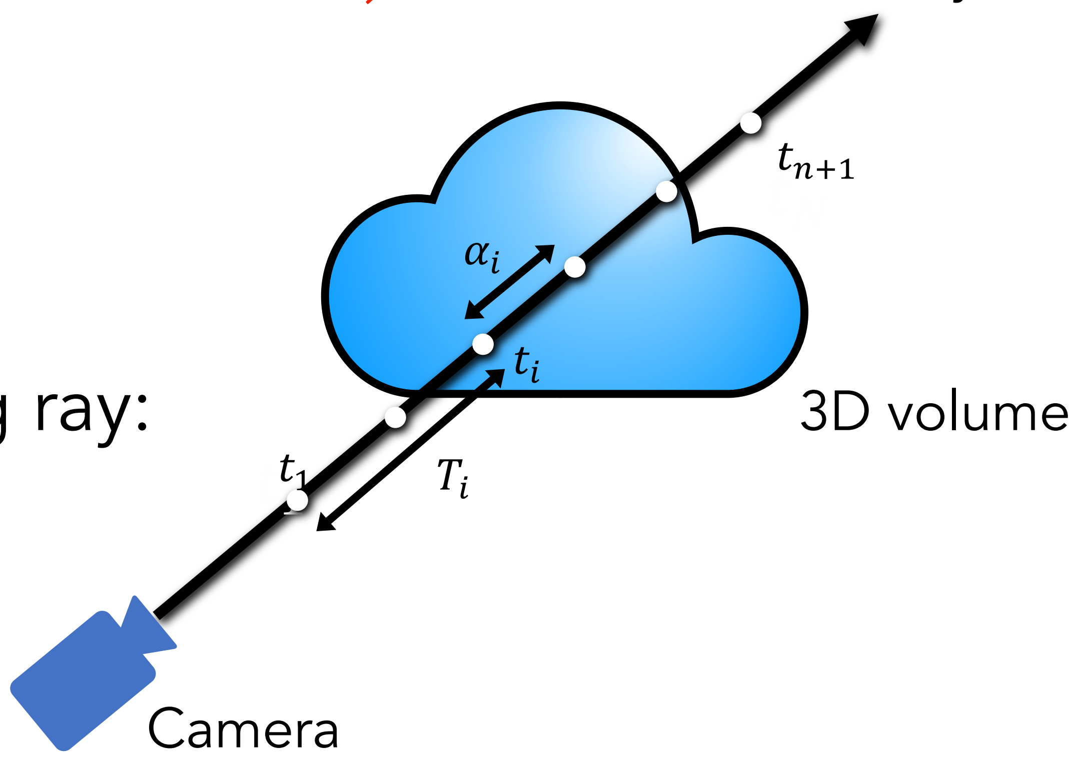
colors

How much light is blocked earlier along ray:

$$T_i = \prod_{j=1}^{i-1} (1 - \alpha_j)$$

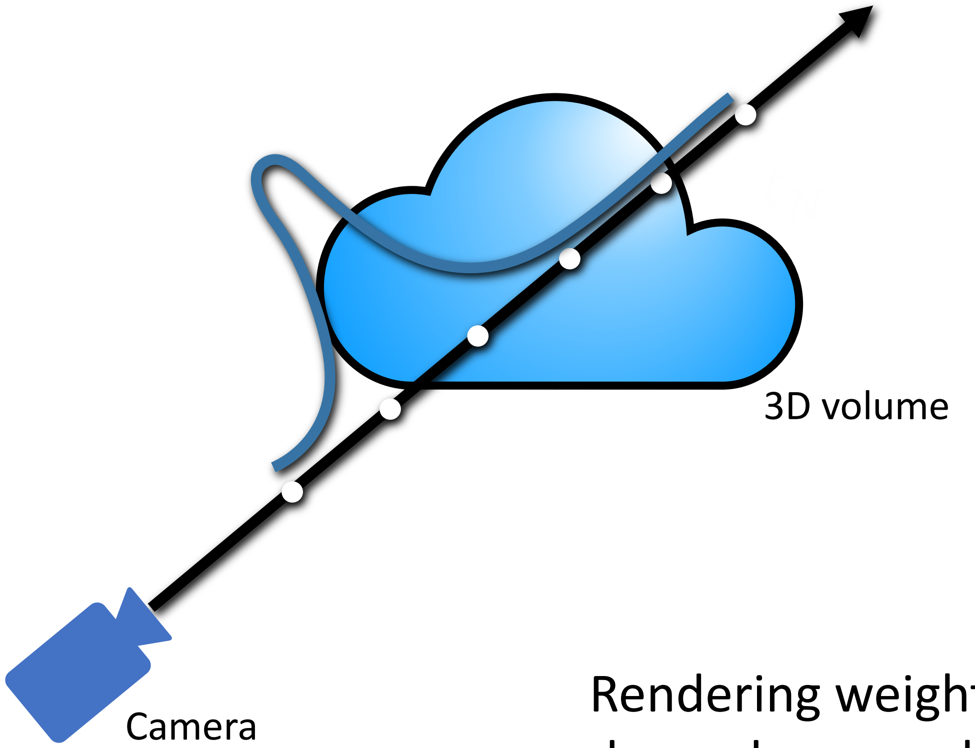
How much light is contributed by ray segment *i*:

$$\alpha_i = 1 - \exp(-\sigma_i \delta_i)$$



Visual intuition: rendering weights is specific to a ray

$$C \approx \sum_{i=1}^{N} T_i \alpha_i c_i$$



Rendering weights are not a 3D function — depends on ray, because of tranmisttance!

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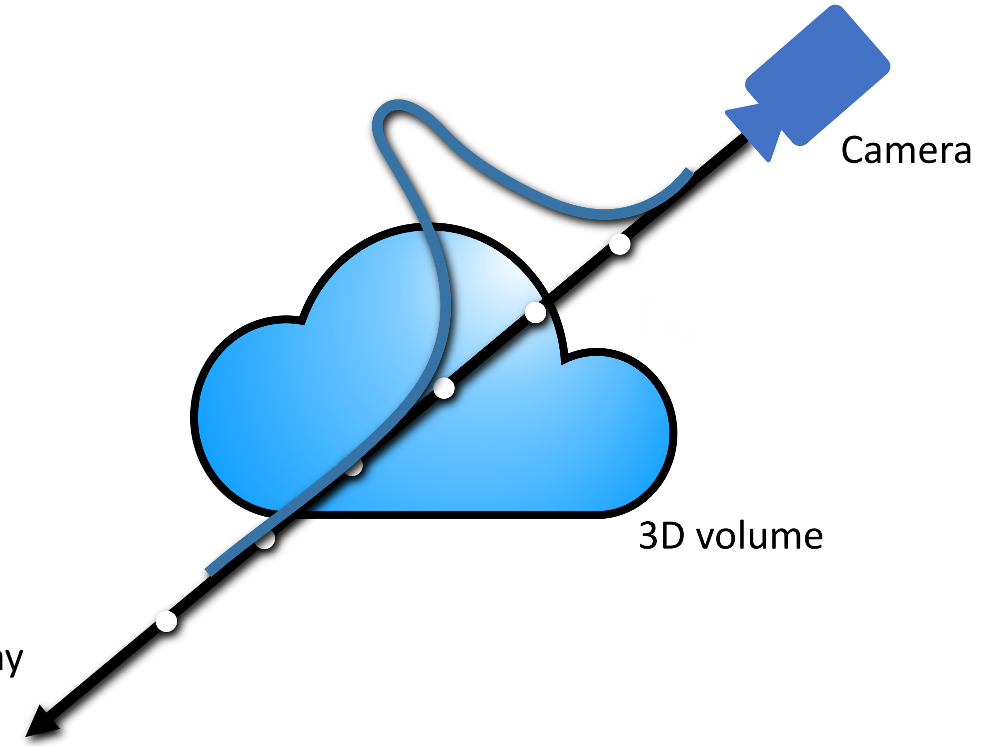
Visual intuition: rendering weights is specific to a ray

$$C \approx \sum_{i=1}^{N} T_i \alpha_i c_i$$



Rendering weights are not a 3D function — depends on ray, because of tranmisttance!

Rendering weight PDF is important

Remember, expected color is equal to

$$\int T(t)\sigma(t)\mathbf{c}(t)dt \approx \sum_i T_i \alpha_i \mathbf{c}_i = \sum_i w_i \mathbf{c}_i$$

$T(t)\sigma(t)$ and $T_i \alpha_i$ are "rendering weights" — probability distribution along the ray (continuous and discrete, respectively)

You can also render entities other than color in 3D, for example it's depth, or any other N-D vector $\boldsymbol{v}_i$

Volume rendered "feature" = $\sum_i w_i \boldsymbol{v}_i$

Rendering weight PDF is important — depth

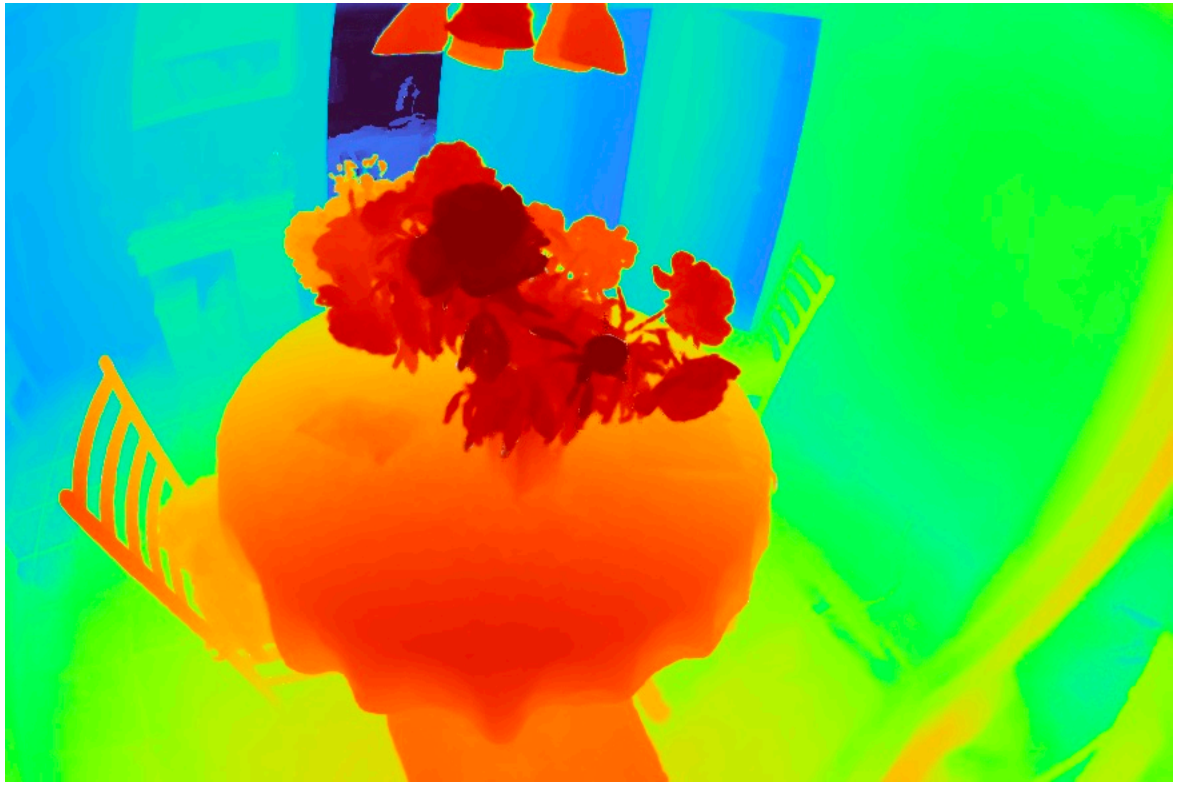
We can use this distribution to compute expectations for other quantities, e.g. “expected depth”:

$$\bar{t} = \sum_i T_i \alpha_i t_i$$

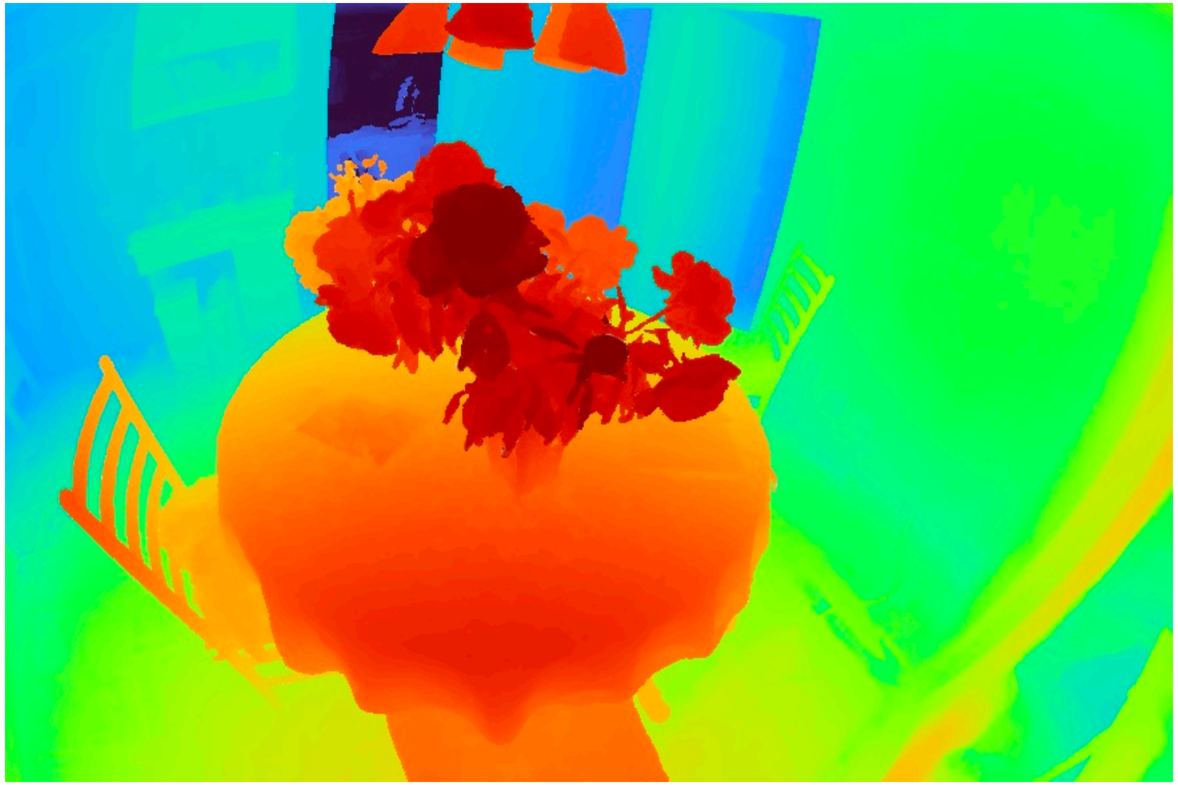
This is often how people visualise NeRF depth maps.

Alternatively, other statistics like mode or median can be used.

Rendering weight PDF is important — depth

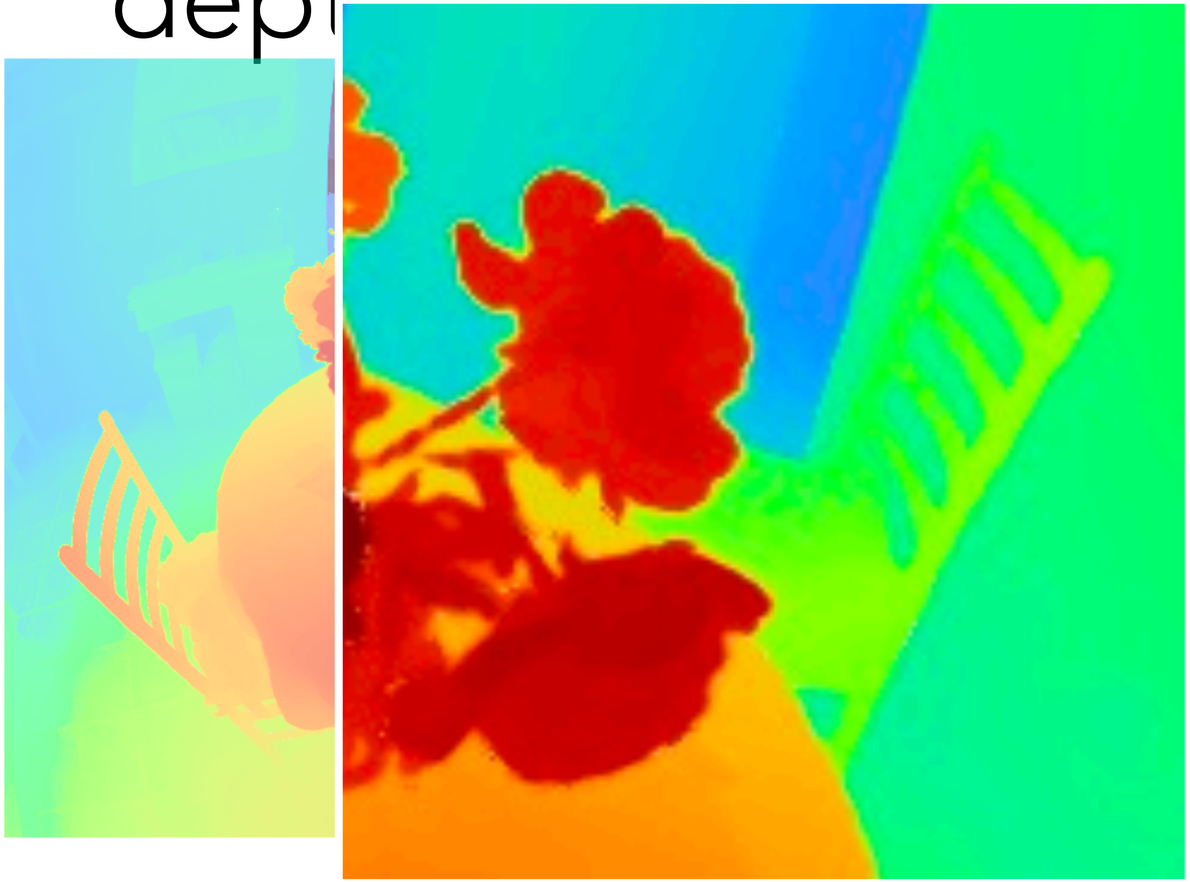


Mean depth

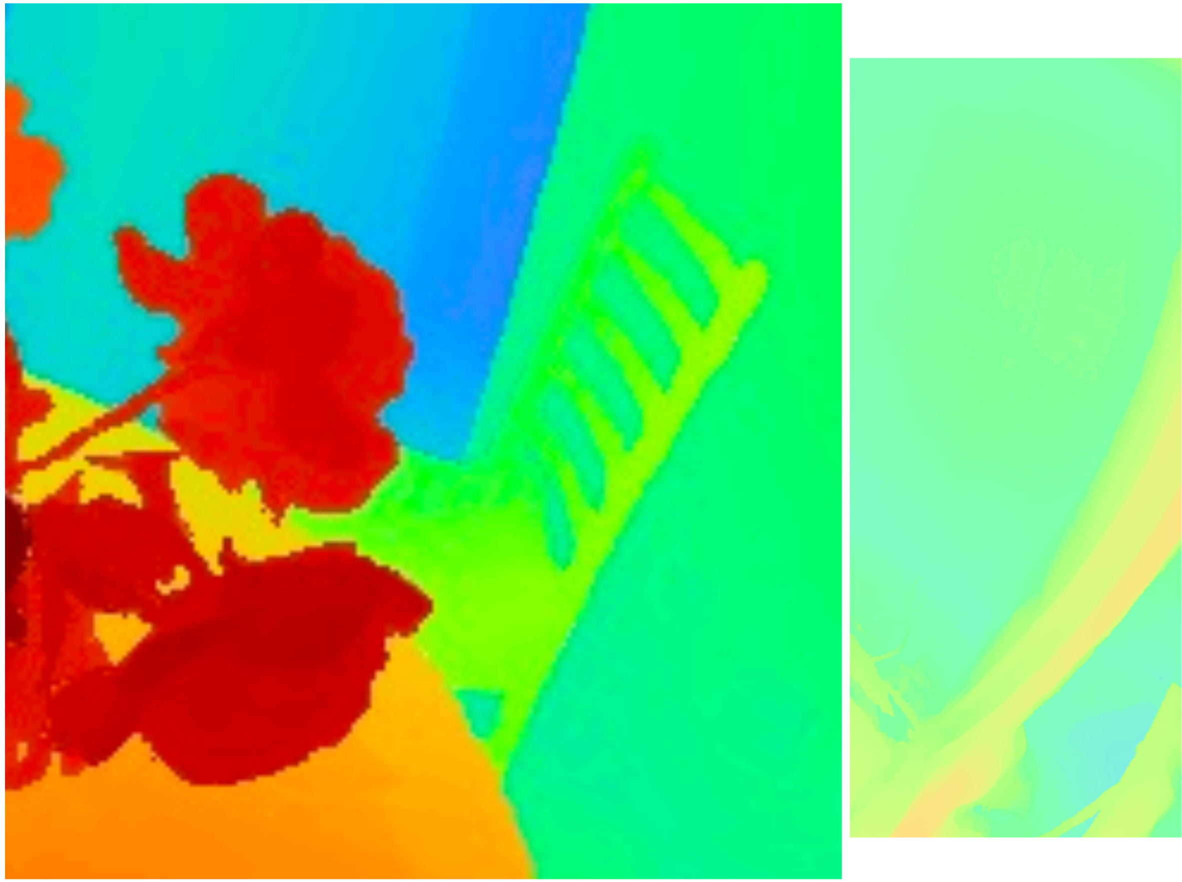


Median depth

Rendering weight PDF is important — depth



Mean depth



Median depth

Volume rendering other quantities

This idea can be used for any quantity we want to “volume render” into a 2D image.
If $\mathbf{v}$ lives in 3D space (semantic features, normal vectors, etc.)

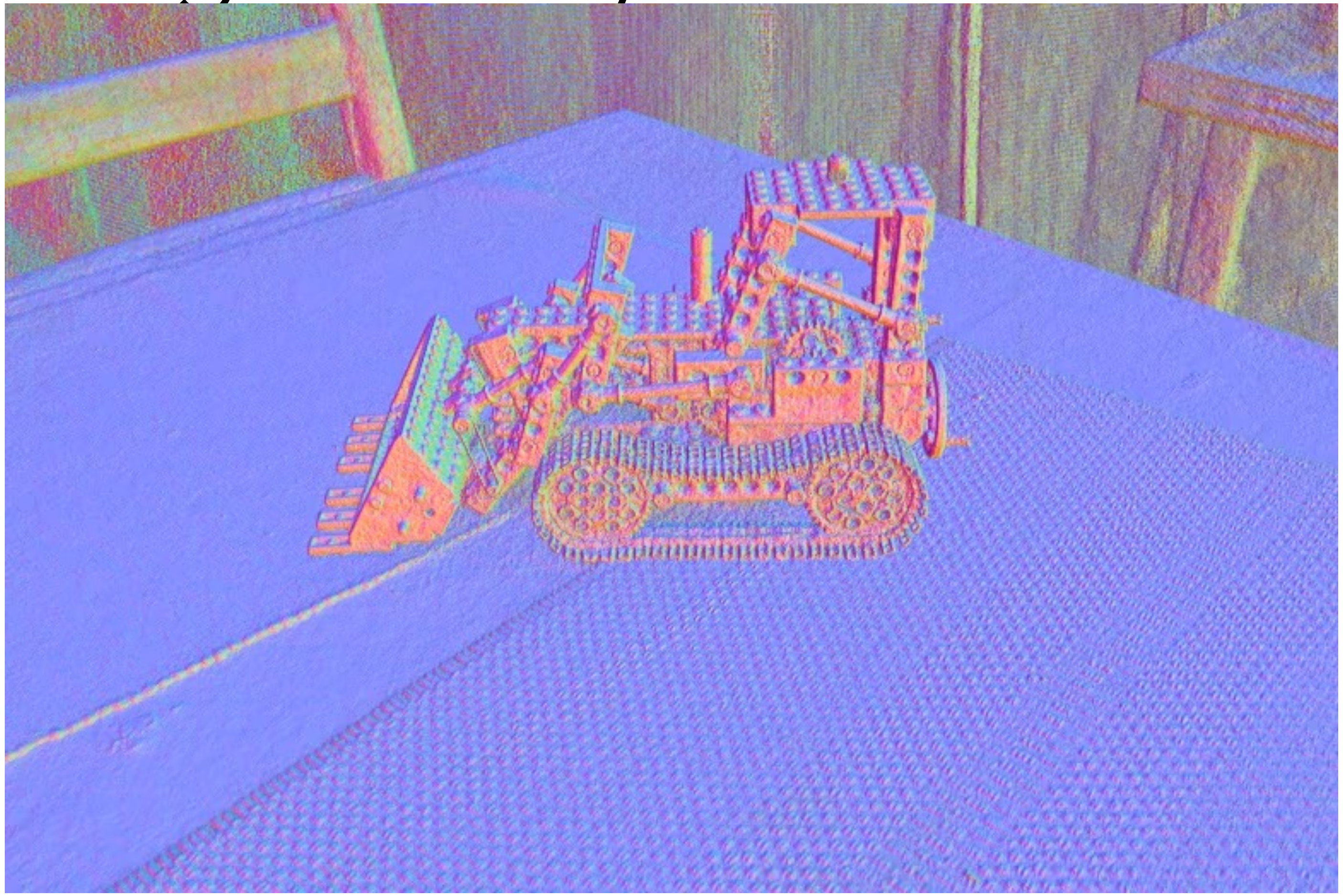
$$\sum_i T_i \alpha_i \mathbf{v}_i$$

can be taken per-ray to produce 2D output images.

Volume Rendering CLIP features

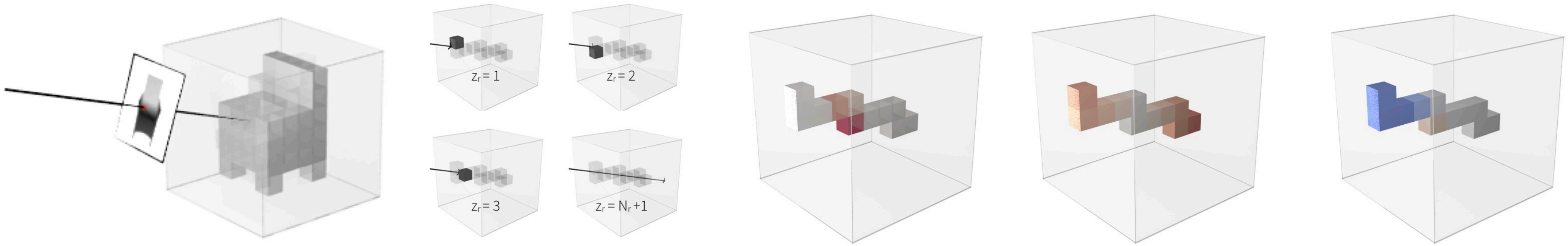
LERF: Language Embedded Radiance Fields, Kerr* and Kim* et al. ICCV 2023

Density as geometry



Normal vectors (from analytic gradient of density)

Previous Papers



Differentiable ray consistency work used a forward model with "probabilistic occupancy" to supervise 3D-from-single-image prediction. Same rendering model as alpha compositing!

$$p(z_r = i) = \begin{cases} (1 - x_i^r) \prod_{j=1}^{i-1} x_j^r, & \text{if } i \leq N_r \\ \prod_{j=1}^{N_r} x_j^r, & \text{if } i = N_r + 1 \end{cases}$$

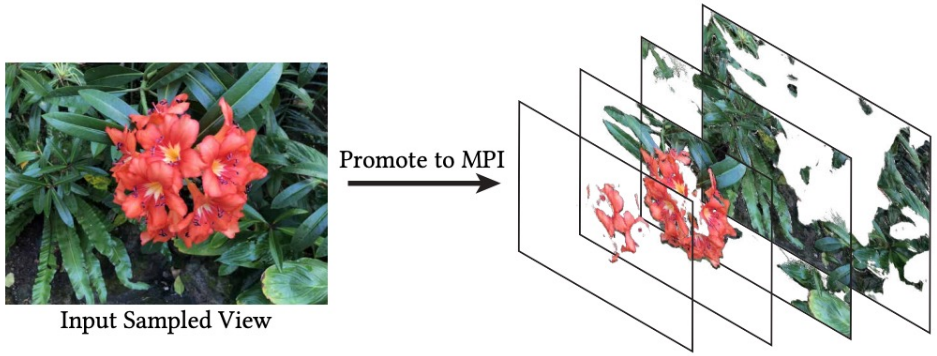
Tulsiani et al 2017, *Multi-view Supervision for Single-view Reconstruction via Differentiable Ray Consistency*

Similar Ideas before NeRF

Multiplane image methods

Stereo Magnification (Zhou et al. 2018)
Pushing the Boundaries… (Srinivasan et al. 2019)
Local Light Field Fusion (Mildenhall et al. 2019)
DeepView (Flynn et al. 2019)
Single-View… (Tucker & Snavely 2020)

Typical deep learning pipelines - images go into a 3D CNN, big RGBA 3D volume comes out



Neural Volumes

(Lombardi et al. 2019)

Direct gradient descent to optimize an RGBA volume, regularized by a 3D CNN