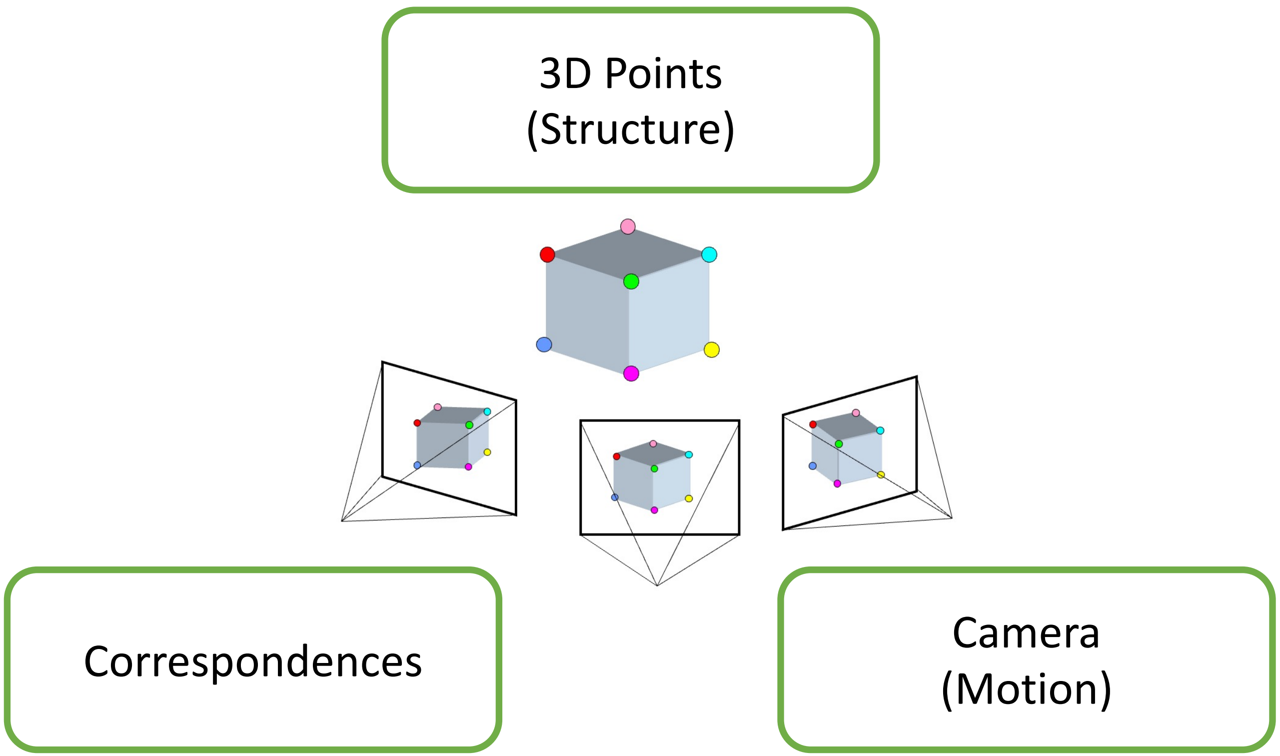
Stereo (Binocular)

credit: clavivs

Shree Nayar’s YT series: First principals of Computer Vision

CS180: Intro to Computer Vision and Comp. Photo
Angjoo Kanazawa & Alexei Efros, UC Berkeley, Fall 2023

Last week: 3 key components in 3D



Coordinate frames + Transforms

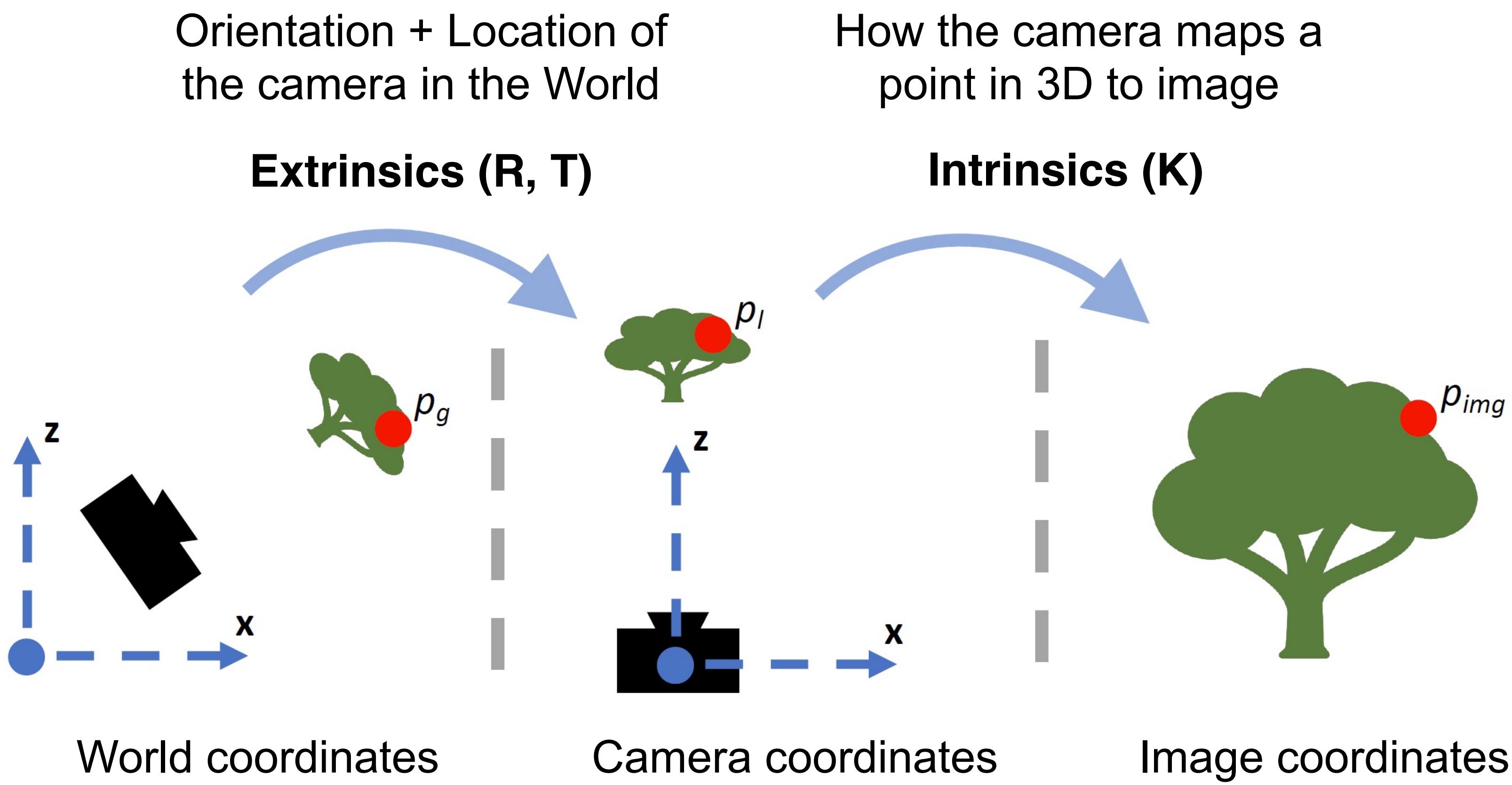


Figure credit: Peter Hedman

Camera: Specifics

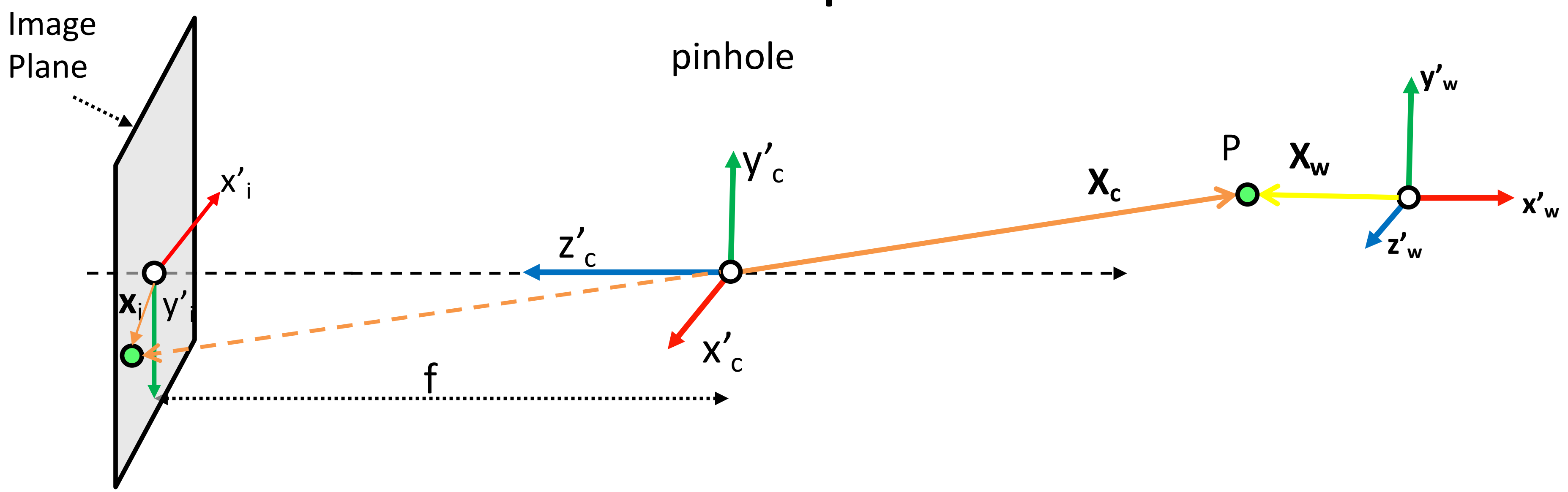


Image Coordinates

$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Perspective Projection (3D to 2D)

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Coordinate Transformation (3D to 3D)

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Slide inspired by Shree Nayar

Camera: Specifics

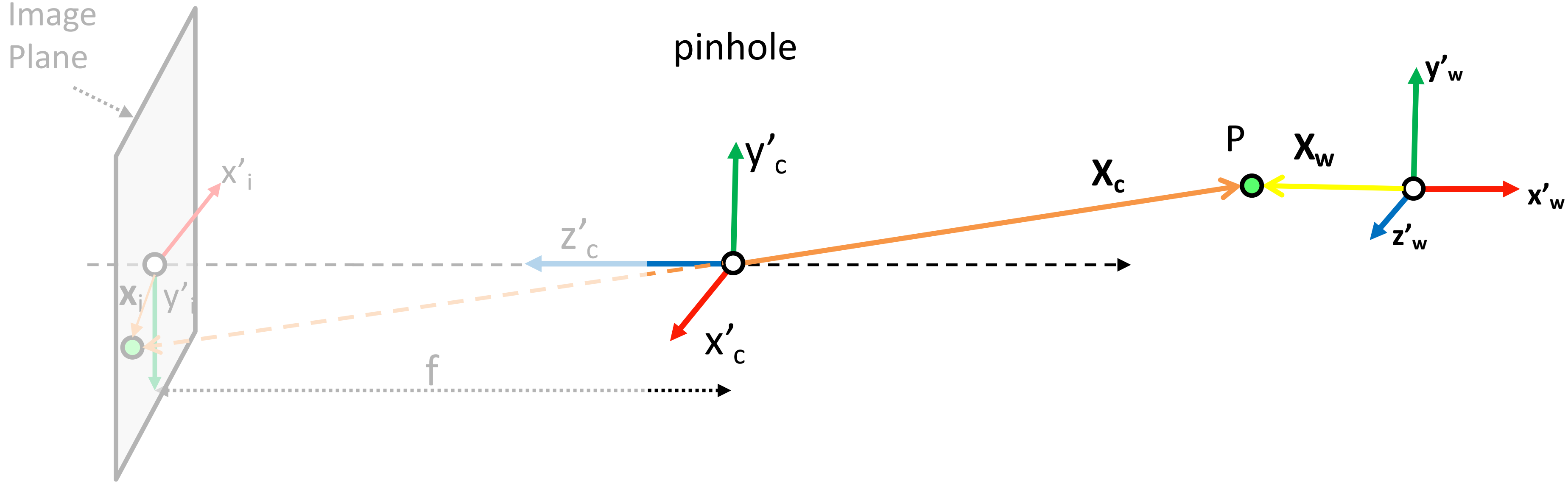


Image Coordinates

Camera Coordinates

World Coordinates

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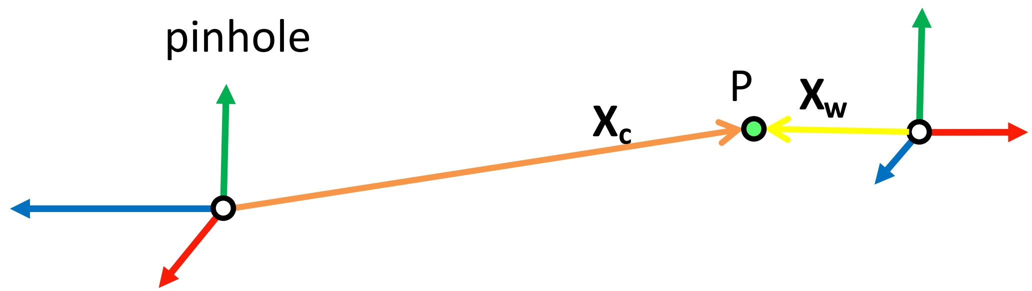
Perspective Projection (3D to 2D)

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Coordinate Transformation (3D to 3D)

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Camera Transformation (3D-to-3D)



Camera Coordinates

World Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Coordinate Transformation

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R_{3\times 3} & \mathbf{t} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Extrinsic

Matrix

Camera: Specifics

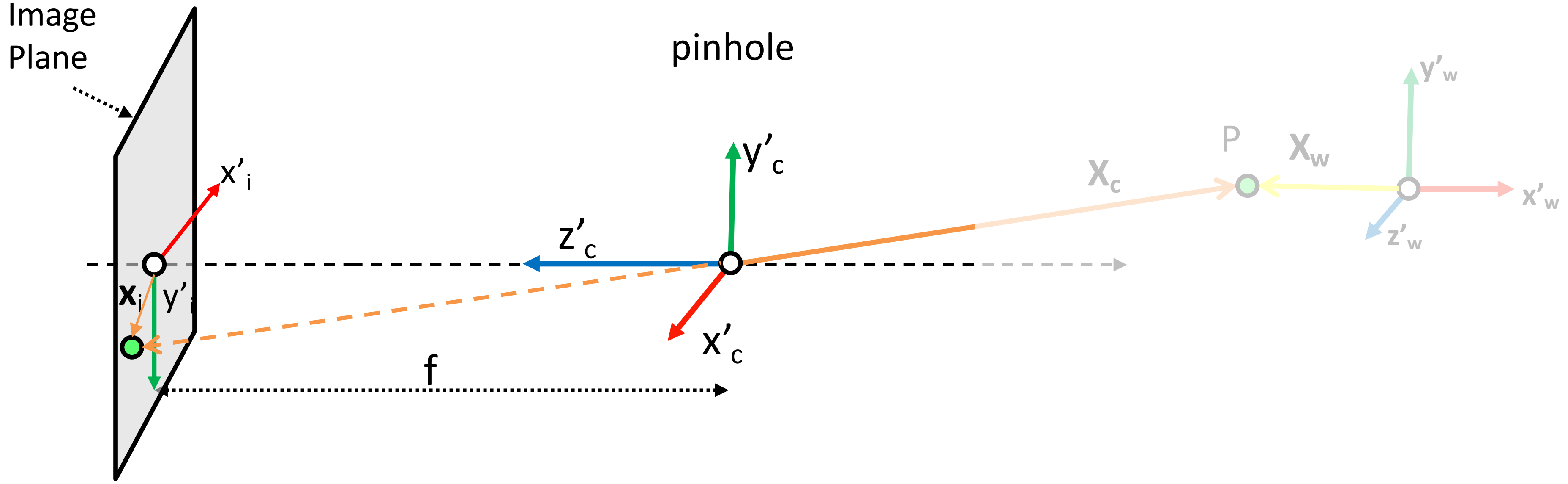


Image Coordinates

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Perspective Projection (3D to 2D)

Camera Coordinates

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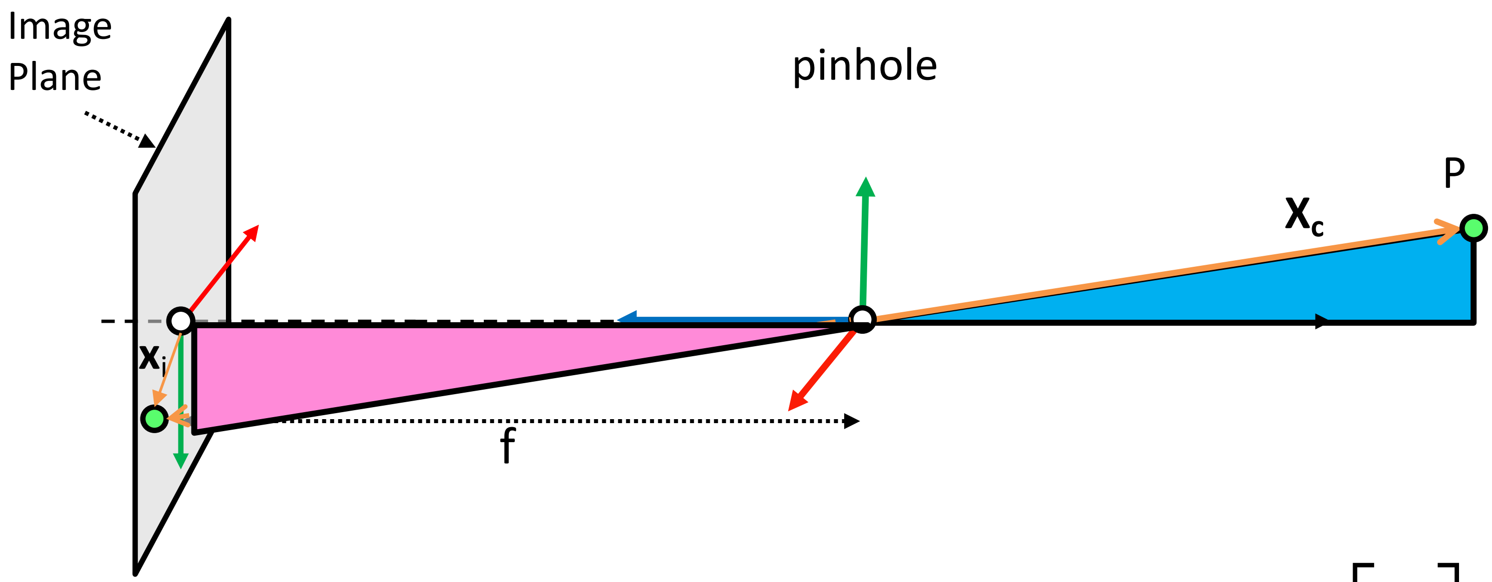
Coordinate Transformation (3D to 3D)

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Perspective Projection



$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Image Coordinates

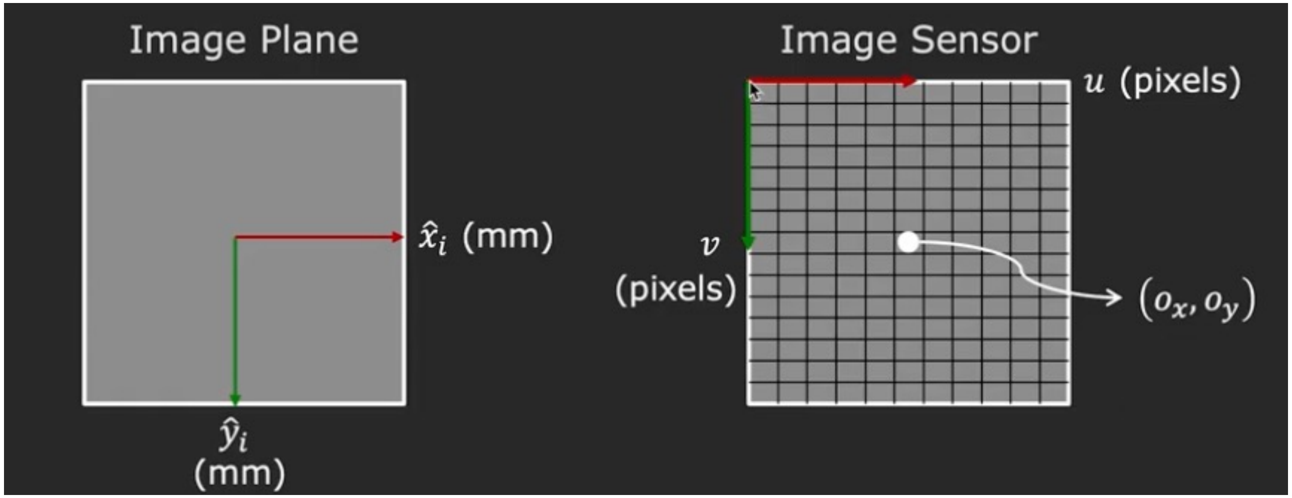
$$\frac{x_i}{f} = \frac{x_c}{z_c}$$

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Camera Coordinates

$$x_i = f\frac{x_c}{z_c}$$

Image Plane to Image Sensor Mapping



1. Scale: For pixel density (pixel/mm) & aspect ratio: $[m_x, m_y]$

$$m_x \hat{x}_i, m_y \hat{y}_i$$

2. Shift: In an image, top left corner is the origin. But in the image plane, the origin is where the optical axis pierces the plane! Need to shift by: (o_x, o_y)

$$u_i = m_x \hat{x}_i + o_x = m_x f \frac{x_c}{z_c} + o_x$$

Putting it all together, pixel coordinates: where $[f_x, f_y] = [m_x f, m_y f]$

$$u_i = \boxed{f_x} \frac{x_c}{z_c} + \boxed{o_x} \quad v_i = \boxed{f_y} \frac{y_c}{z_c} + \boxed{o_y}$$

With homogeneous coordinates

Perspective projection + Transformation to Pixel Coordinates:

$$u_i = f_x \frac{x_c}{z_c} + o_x \quad v_i = f_y \frac{y_c}{z_c} + o_y$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Intrinsic Matrix

Putting it all together

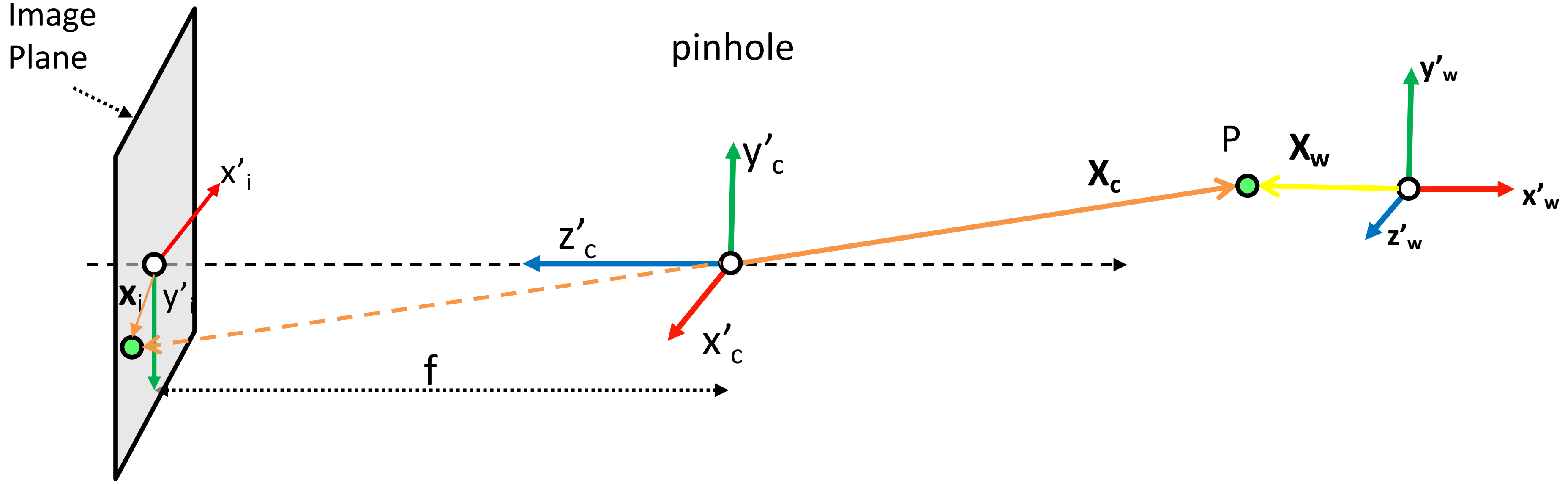


Image Coordinates

$$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Intrinsics: Perspective Projection & pixel conversion

$$\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Extrinsics: Coordinate Transformation

$$\begin{bmatrix} R_{3\times 3} & \mathbf{t} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}$$

Slide inspired by Shree Nayar

Projection Matrix

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{3\times 3} & \mathbf{t} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

3 x 4 Projection matrix

What's the Degrees of Freedom?

For completeness, we need to add **skew** (this is 0 unless pixels are shaped like rhombi/parallelograms)

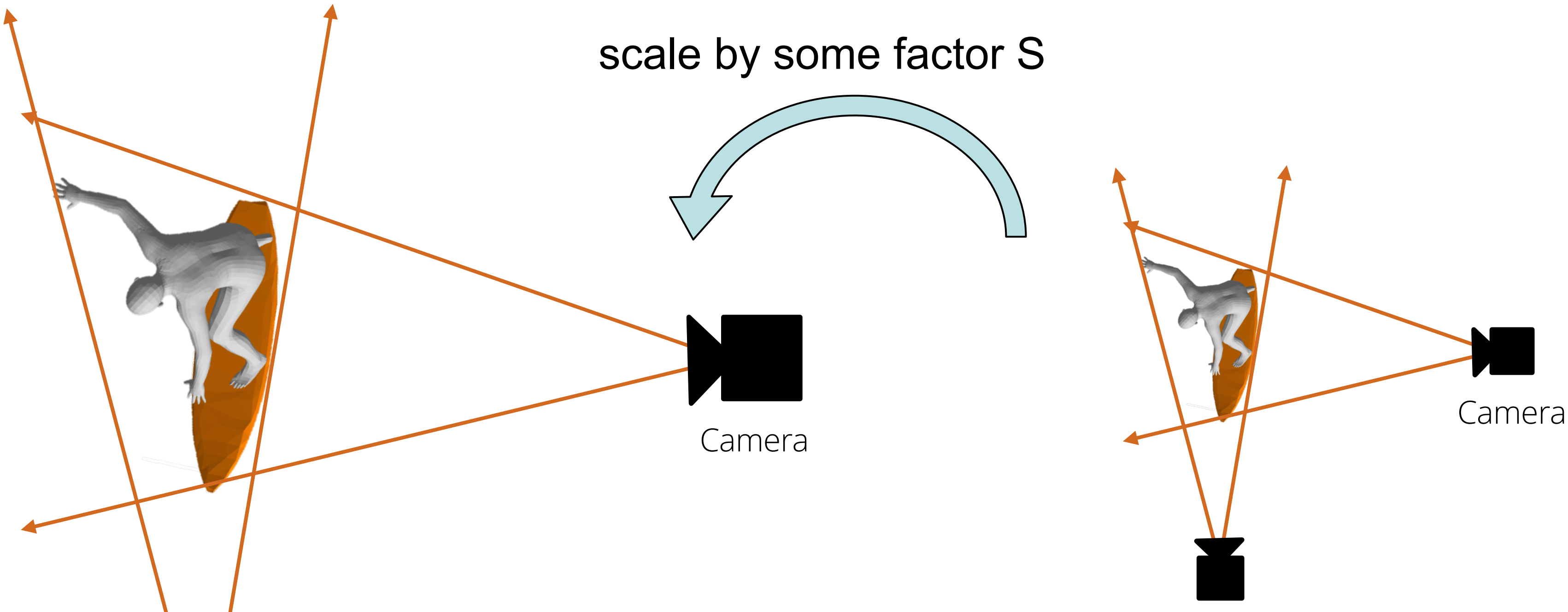
$$K = \begin{bmatrix} f_x & s & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

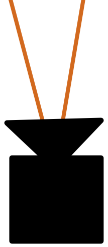
Intrinsics: 4 + 1 (skew)

Extrinsic: 3 + 3 = 6

11 unknowns (up to scale)

Fundamental Scale Ambiguity





Reconstruction is only possible *up to* global scale

Scaling the world & camera doesn't change the projection

Unless you know something metric about the scene

e.g. surfboard is 2.1m

Exercises

Going from World to Camera

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Extrinsic Matrix:

$$T_{w2c} = \begin{bmatrix} R_{3\times 3} & \mathbf{t} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}$$

$$\mathbf{X}_c = T_{w2c}\mathbf{X}_w$$

Going from Camera to World

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

World Coordinates

$$\mathbf{X}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Extrinsic Matrix:

$$T_{w2c} = \begin{bmatrix} R_{3\times 3} & \mathbf{t} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}$$

$$T_{w2c}^{-1}\mathbf{X}_c = \mathbf{X}_w$$

Where is the camera center in the world?

$$\mathbf{X}_c = T_{w2c}\mathbf{X}_w \longleftrightarrow X_c = RX_w + T$$

$$T_{w2c}^{-1}\mathbf{X}_c = \mathbf{X}_w \longleftrightarrow R^T(X_c - T) = X_w$$

Set X_c to zero (origin in camera = camera center)

$$R^T(\vec{0} - T) = C_w$$

$$C_w = -R^T T$$

Now you can derive Camera to World transform as well

Camera to Image

Image Coordinates

$$\mathbf{x}_i = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix}$$

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Intrinsic Matrix:

$$K = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{x}_i = K\mathbf{X}_c$$

Image to Camera?

Image Coordinates

$$\mathbf{x}_i = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix}$$

Camera Coordinates

$$\mathbf{X}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

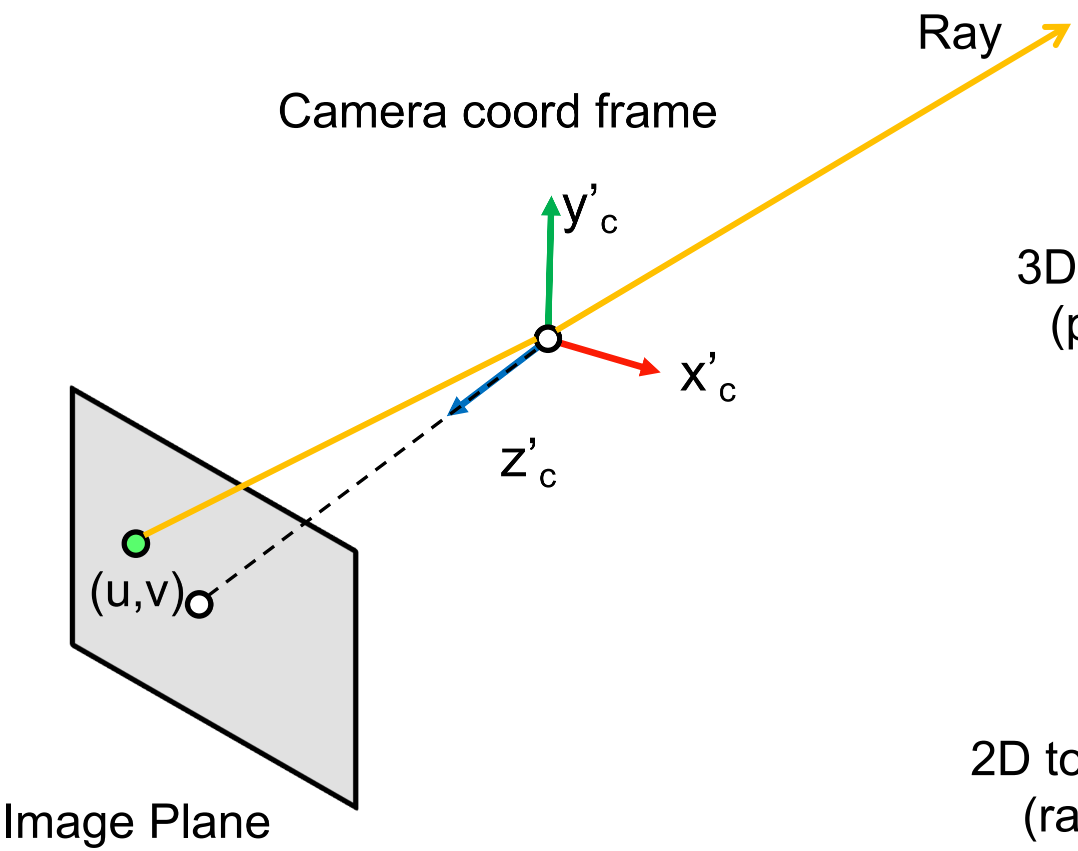
Intrinsic Matrix:

$$K = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$u = f_x \frac{x_c}{z_c} + o_x \longrightarrow x = \frac{z}{f_x}(u - o_x)$$

What’s the problem?

We don't know the depth! but we know it will be: on the ray!



3D to 2D: (point)

$$u = f_x \frac{x_c}{z_c} + o_x$$

$$v = f_y \frac{y_c}{z_c} + o_y$$

2D to 3D: (ray) Back projection

$$x = \frac{z}{f_x}(u - o_x)$$

$$y = \frac{z}{f_y}(v - o_y)$$

$$z > 0$$

Slide credit: Shree Nayar

What is your coordinate space?

- In Project 5 (and in life) always make sure you're in the right coordinate space.
- eg. Which space is the ray defined in?

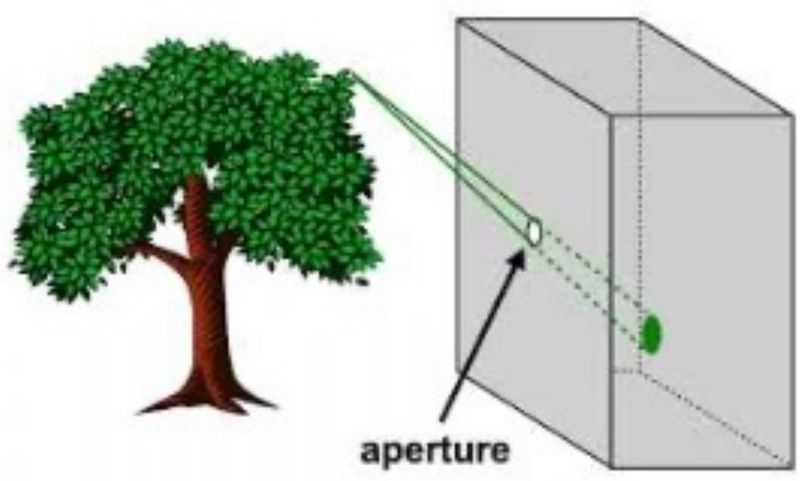
2D to 3D:
(ray)
Back projection

$$x = \frac{z}{f_x}(u - o_x)$$

$$y = \frac{z}{f_y}(v - o_y)$$

$$z > 0$$

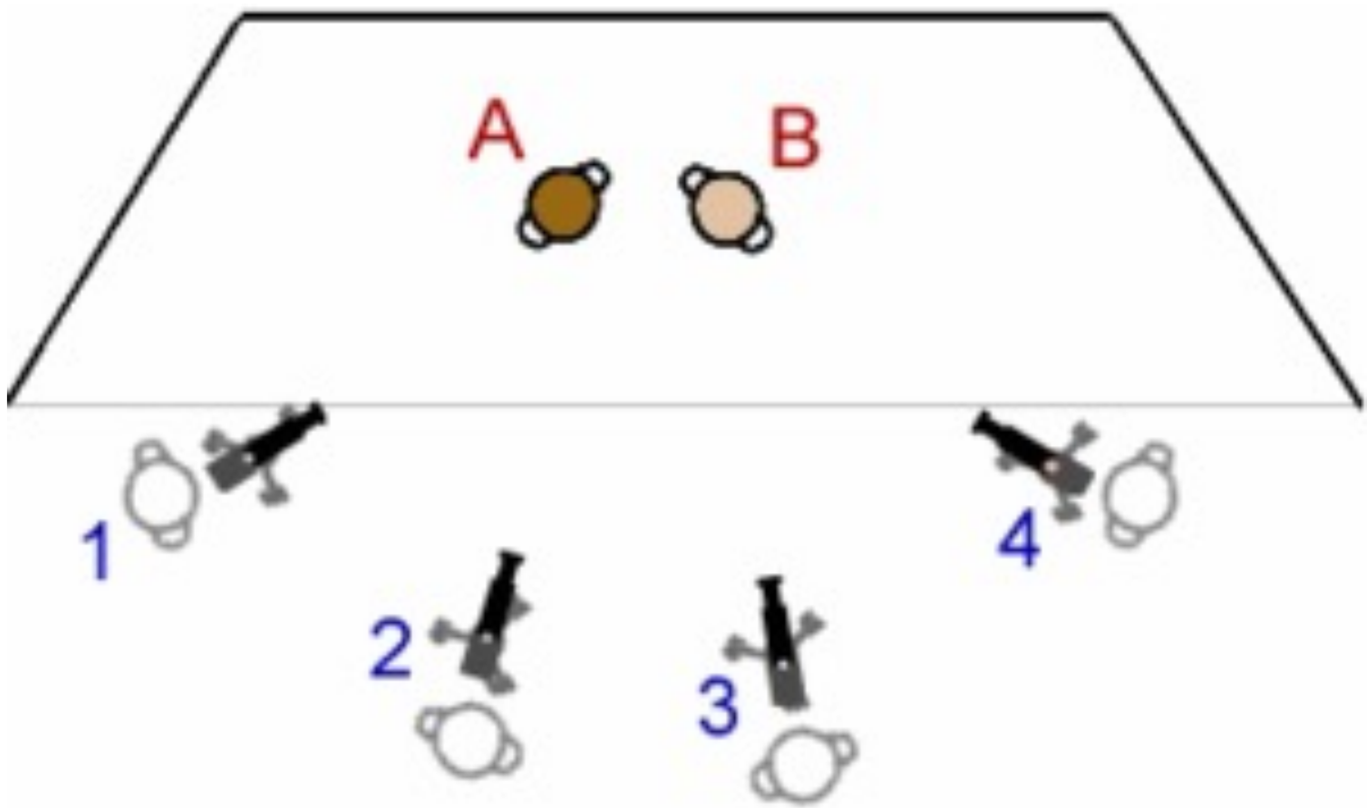
Watch these 5 min videos



https://www.youtube.com/watch?v=F5WA26W4JaM
https://www.youtube.com/watch?v=g7Pb8mrwcJ0

Calibration: What are my cameras?

Problem Setup



What are the Extrinsic and Intrinsic matrices for each camera?

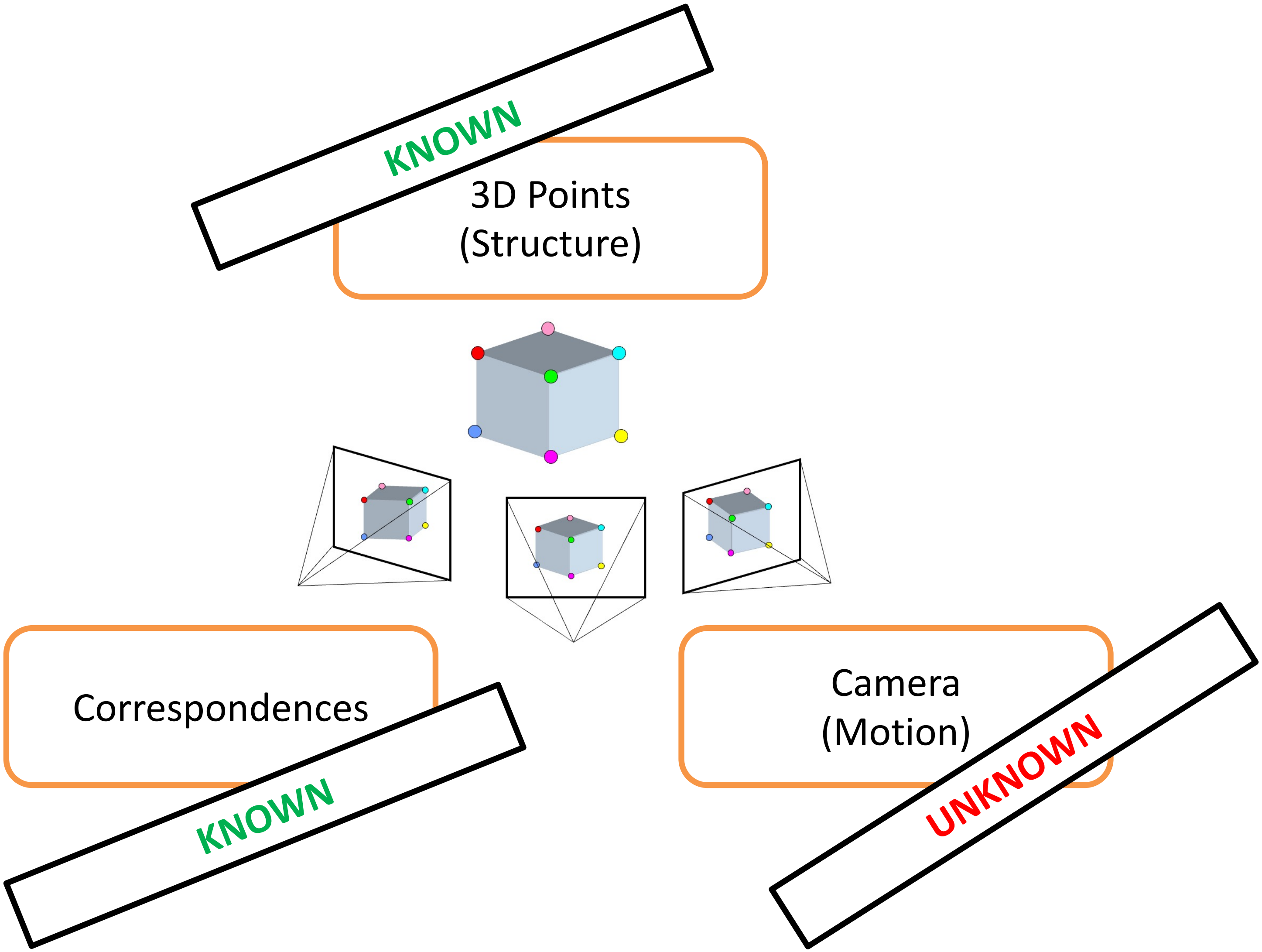
How to calibrate the camera?

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

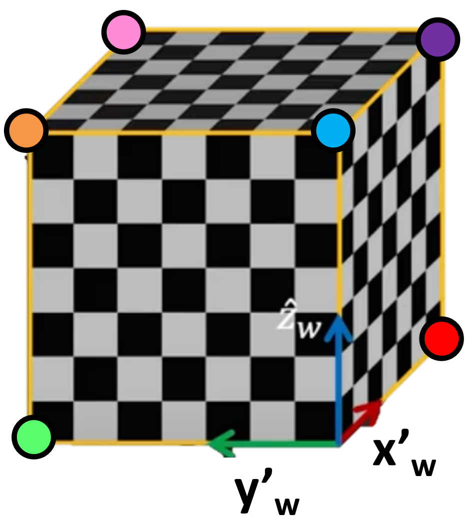
If we know the points in 3D we can estimate the camera!!

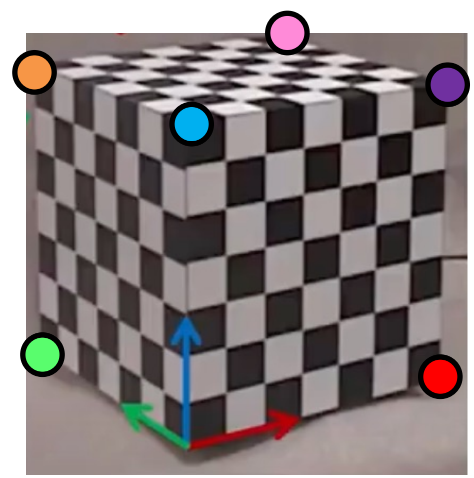
Problem setup: Camera Calibration



Step 1: With a known 3D object

1. Take a picture of an object with known 3D geometry

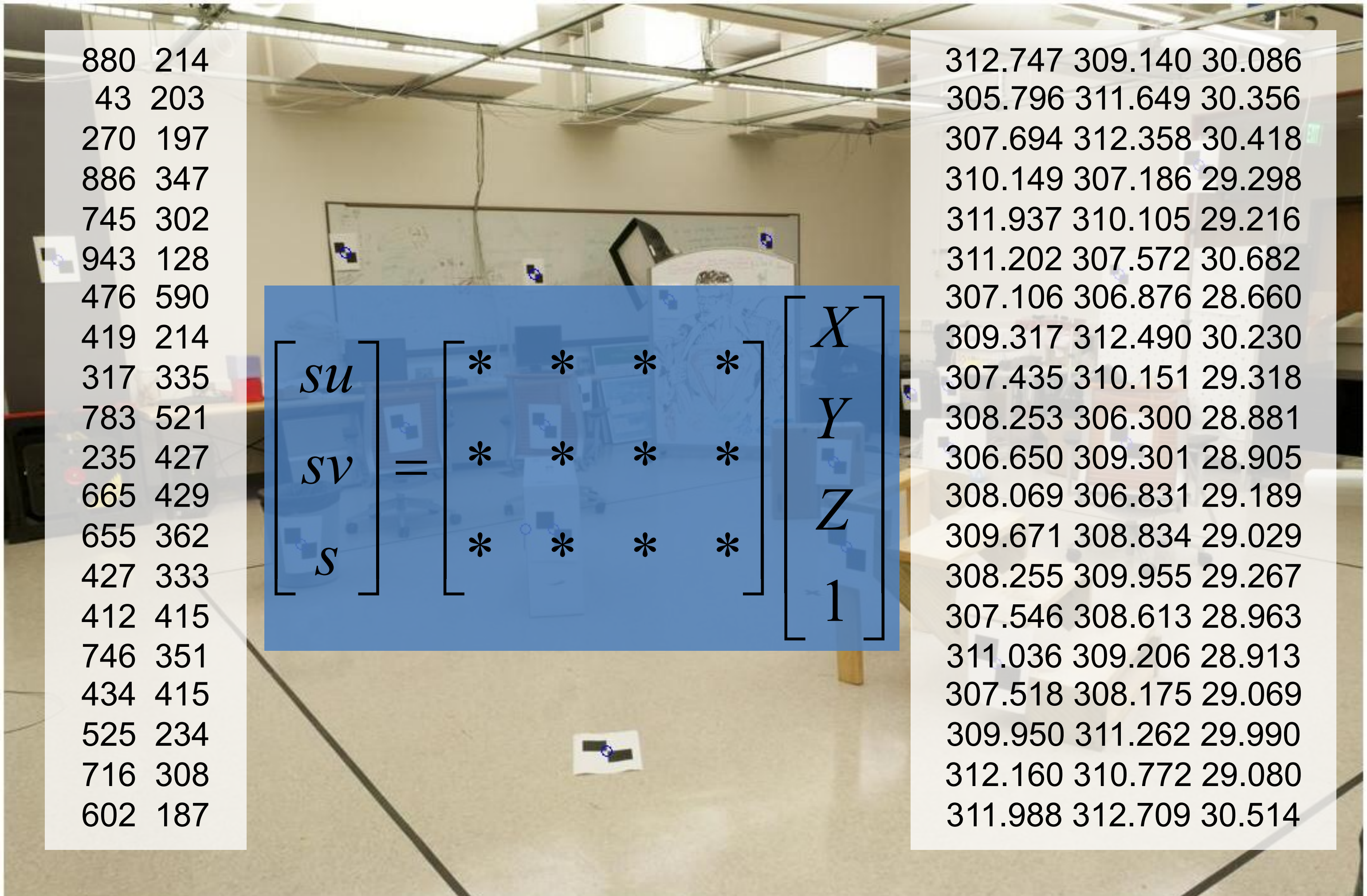




2. Identify correspondences

Slide from Shree Nayar

How do we calibrate a camera?



880	214
43	203
270	197
886	347
745	302
943	128
476	590
419	214
317	335
783	521
235	427
665	429
655	362
427	333
412	415
746	351
434	415
525	234
716	308
602	187

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

312.747	309.140	30.086
305.796	311.649	30.356
307.694	312.358	30.418
310.149	307.186	29.298
311.937	310.105	29.216
311.202	307.572	30.682
307.106	306.876	28.660
309.317	312.490	30.230
307.435	310.151	29.318
308.253	306.300	28.881
306.650	309.301	28.905
308.069	306.831	29.189
309.671	308.834	29.029
308.255	309.955	29.267
307.546	308.613	28.963
311.036	309.206	28.913
307.518	308.175	29.069
309.950	311.262	29.990
312.160	310.772	29.080
311.988	312.709	30.514

Method: Set up a linear system

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- Solve for m's entries using linear least squares

Ax=0 form

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Just like how you solved for homography!

Can we factorize M back to K [R | T]?

- Yes.
- Why? because K and R have a very special form:

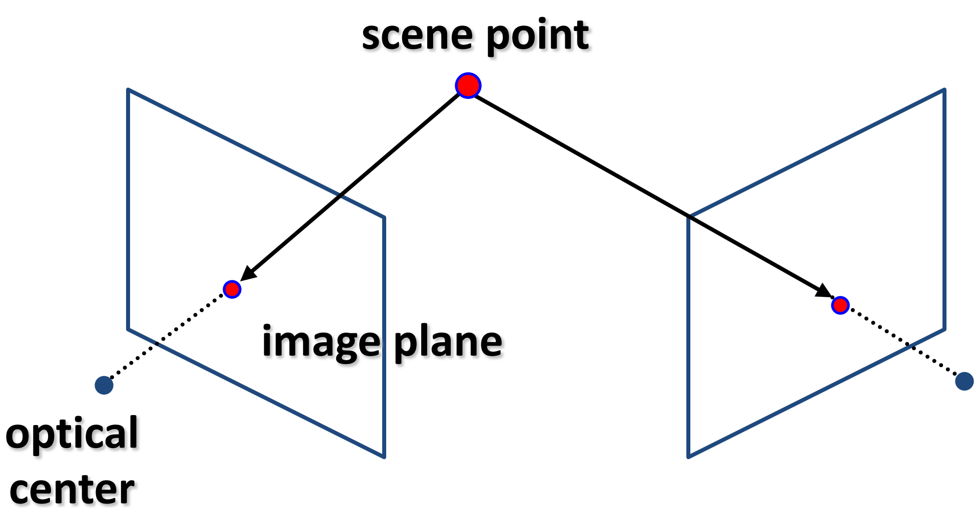
$$\begin{bmatrix} f_x & s & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

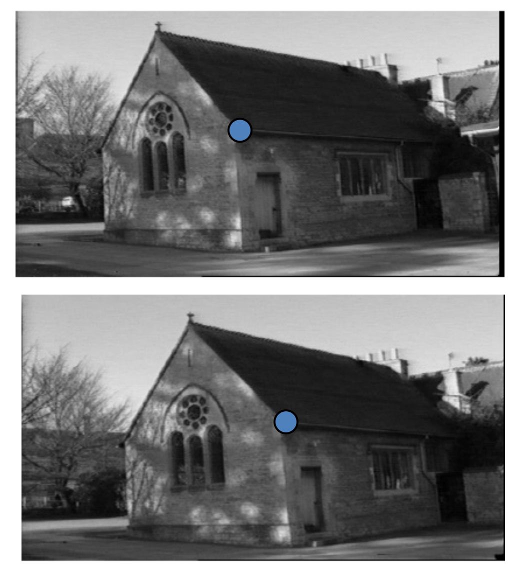
- QR decomposition
- Practically, use camera calibration packages (there is a good one in OpenCV)

Now that our cameras are calibrated, can we find the 3D scene point of a pixel?

Estimating depth with stereo

- **Stereo**: shape from "motion" between **two** views
- We'll need to consider:
  - 1. Camera pose ("calibration")
  - 2. Image point correspondences





Stereo vision

Two cameras, simultaneous views

Single moving camera and static scene

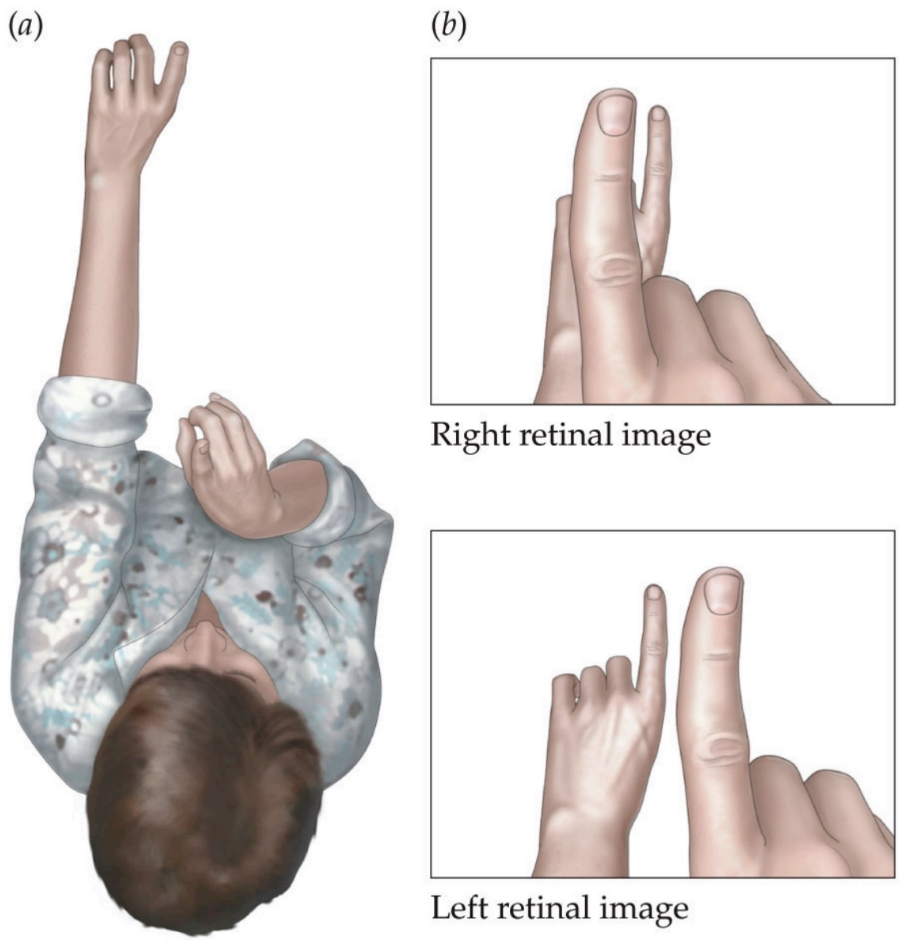
Simple Stereo Setup

- Assume **parallel** optical axes
- Two cameras are calibrated
- Find relative depth

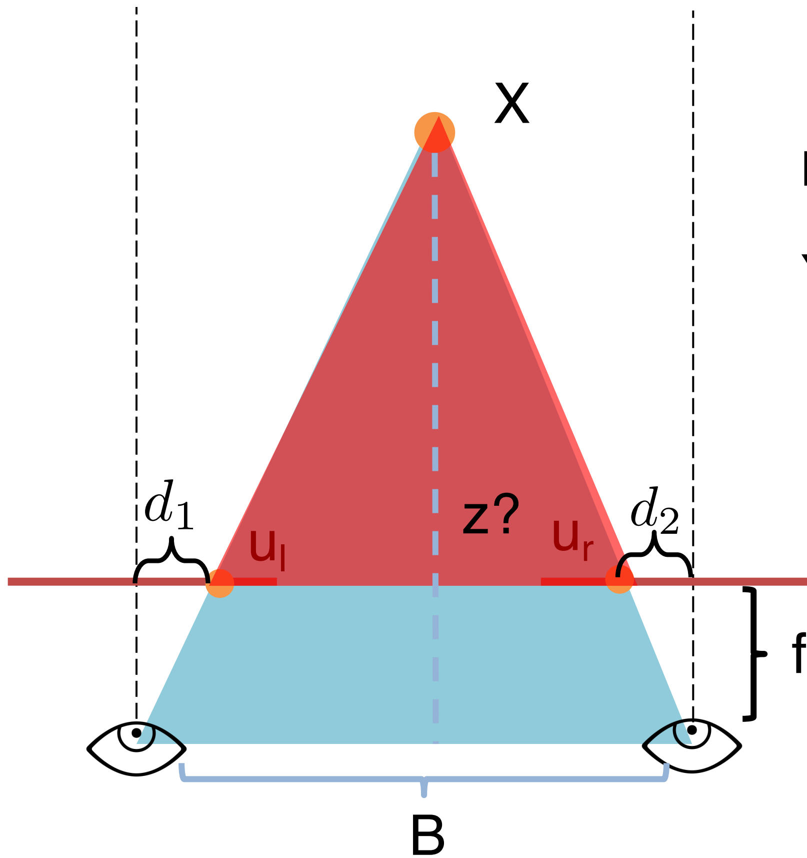
Key Idea: difference in corresponding points to understand shape

Slide credit: Noah Snavely

We are equipped with binocular vision. Let's try!



Solving for Depth in Simple Stereo



Do we have enough to know what is Z?

Yes, similar triangles!

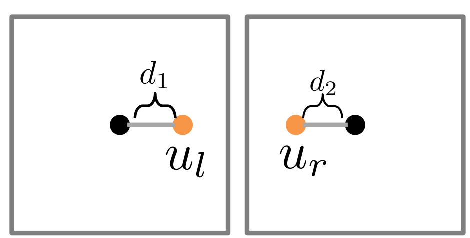
$$\frac{B - (u_l - u_r)}{z - f} = \frac{B}{z}$$

$$z = \frac{fB}{u_l - u_r}$$

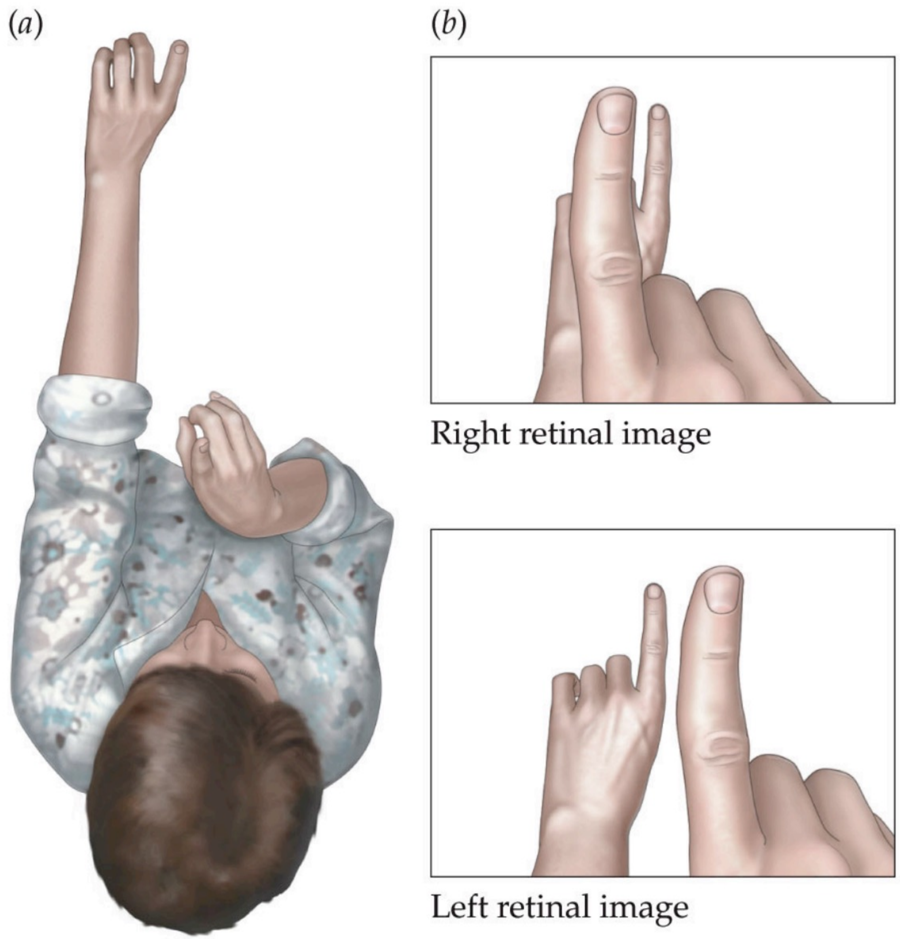
disparity (how much corrsp. pixels move)

Base of : $B - (d1 + d_2)$

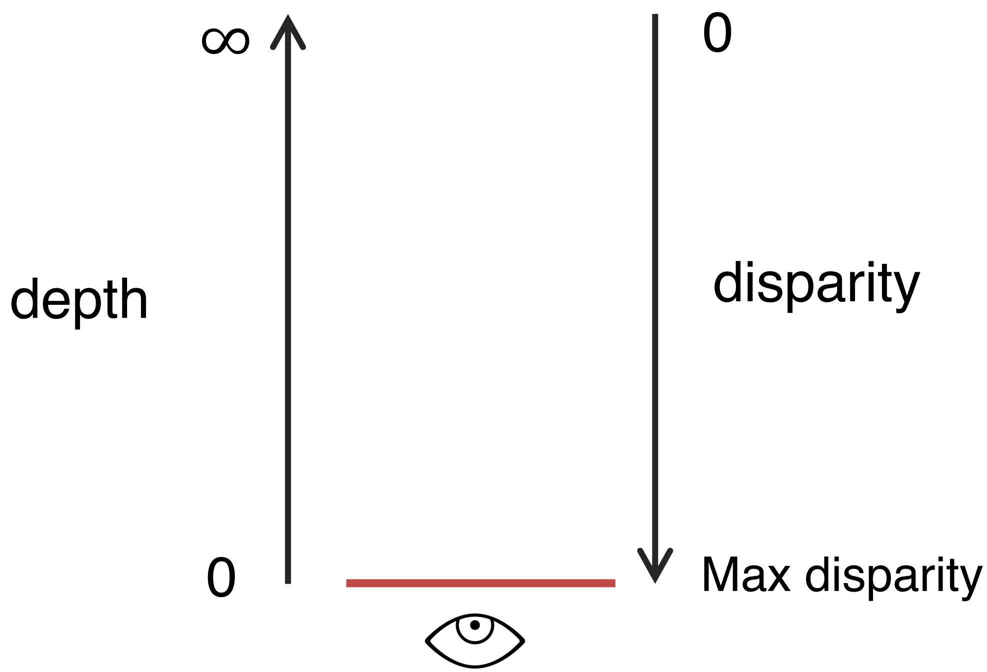
in image coordinates: $= B - (u_l - u_r)$



Try with your hands!



Depth is **inversely** proportional to disparity



$$z = \frac{fB}{u_l - u_r}$$

disparity

$$z \propto \frac{1}{u_l - u_r} = \frac{1}{d}$$

what is the disparity of the closer point?
what is the disparity of the far away point?

Disparity gives you the depth information!

Try again

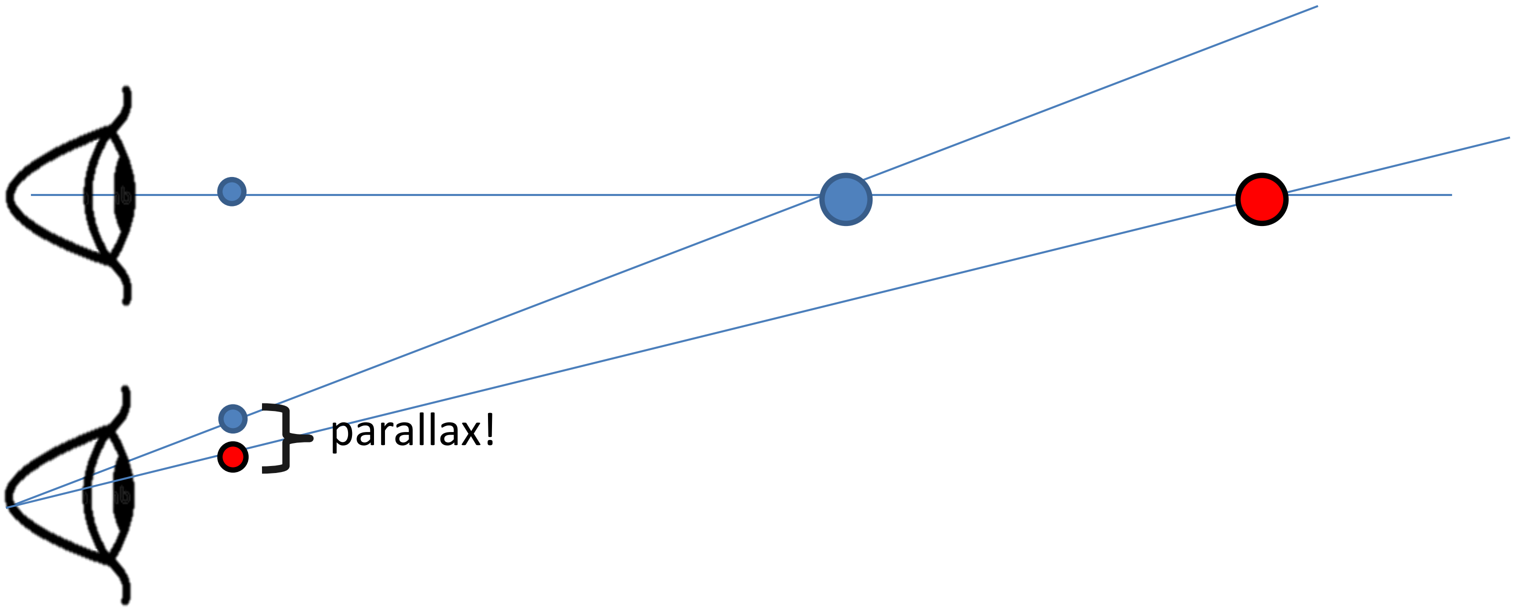
1. Setup so your fingers are on the same line of sight from one eye
2. Now look in the other eye

They move!

Relative displacement is higher as the relative distance grows

== Parallax

Parallax



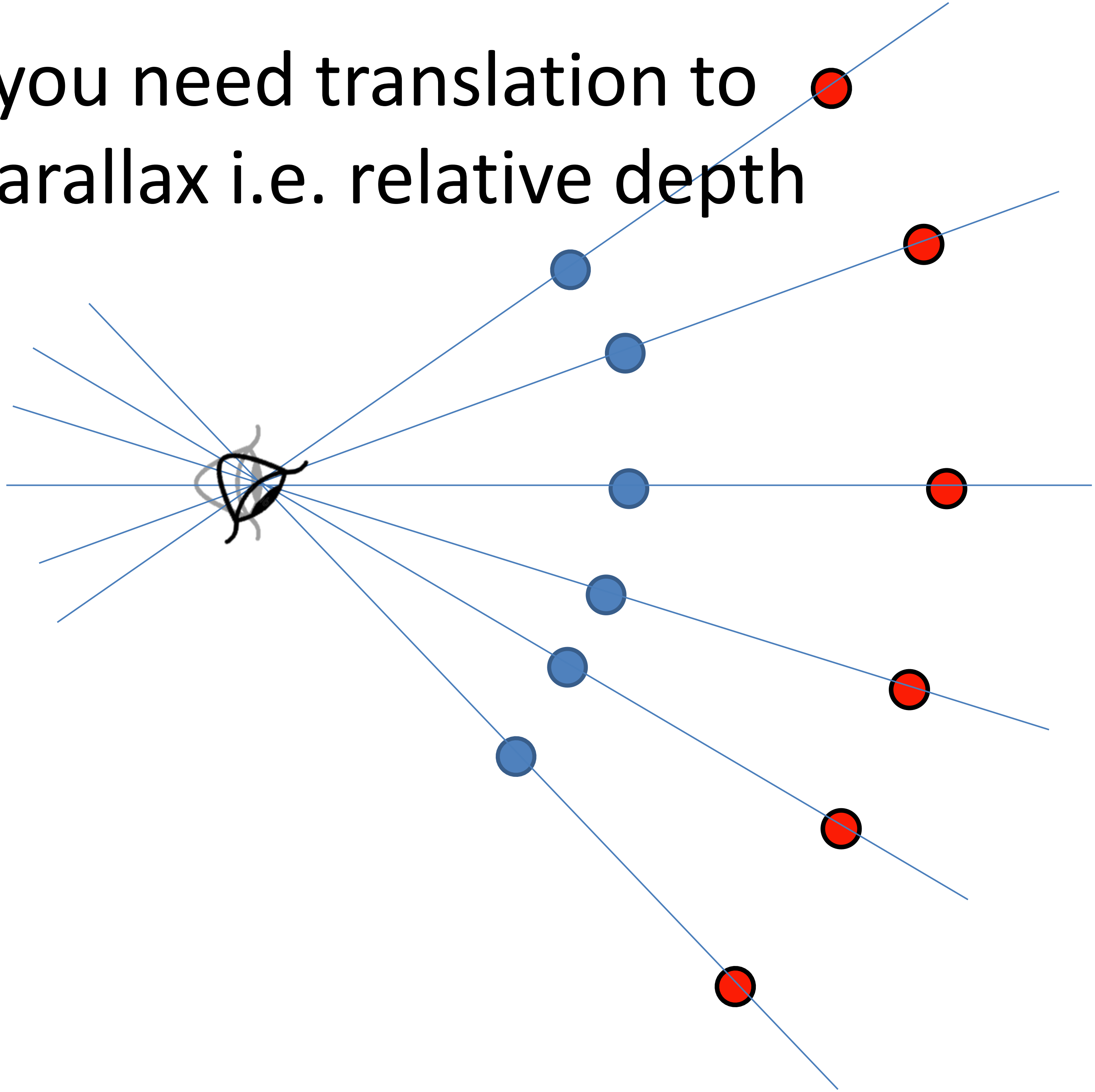
Parallax = *from ancient Greek parállaxis*

= *Para (*side by side) + *allássō*, (to alter)

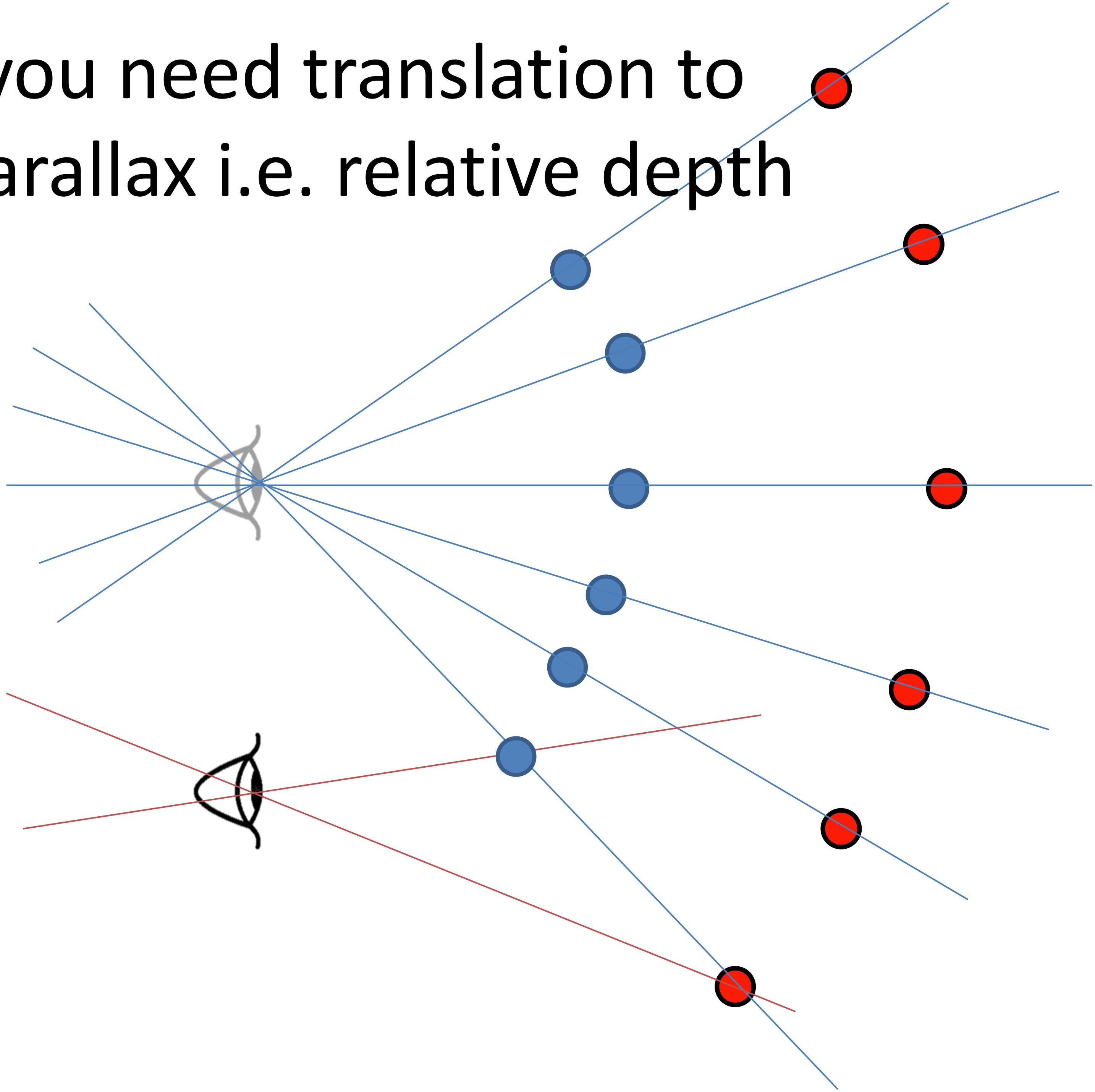
= *Change in position from different view point*

Two eyes give you parallax, you can also move to see more parallax = "Motion Parallax"

Why you need translation to see parallax i.e. relative depth

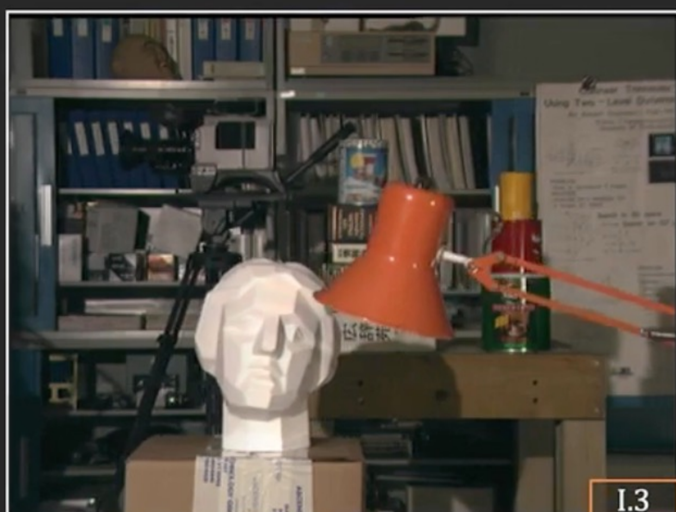


Why you need translation to see parallax i.e. relative depth



So how do we get depth?

- Find the disparity! of corresponding points!
- Called: Stereo Matching

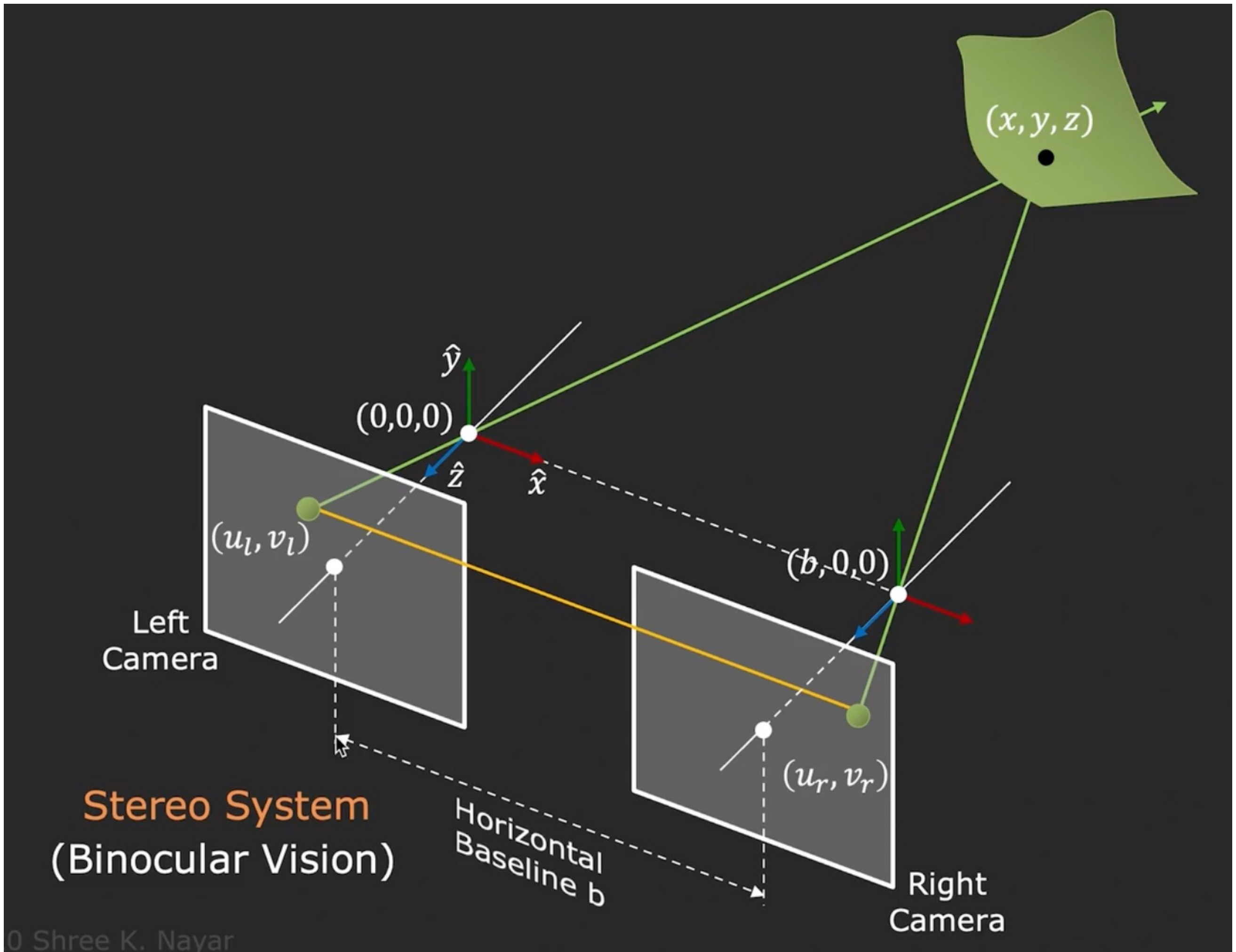


Left/Right Camera Images

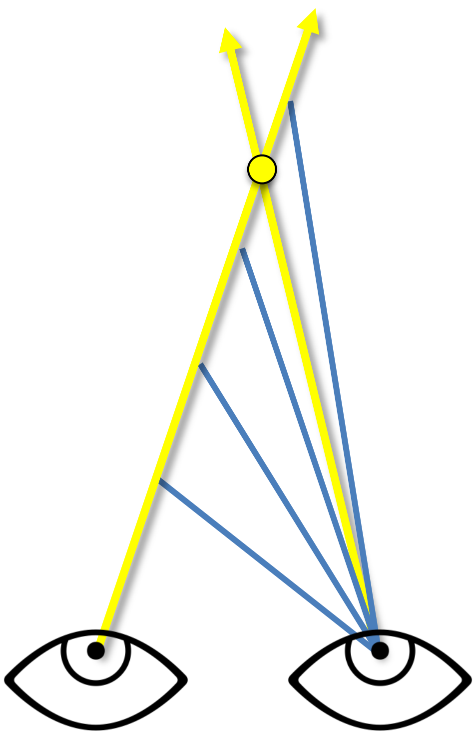
Disparity Map (Ground Truth)

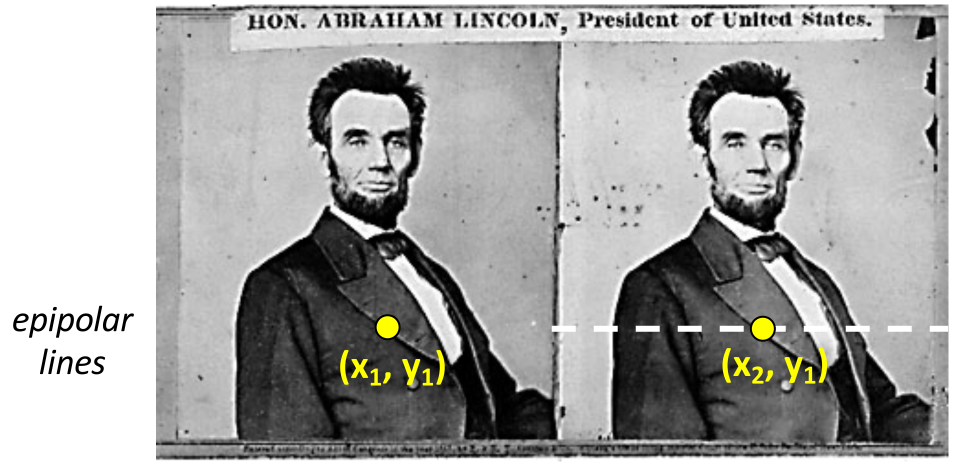
Where is the corresponding point going to be?

Hint



Epipolar Line

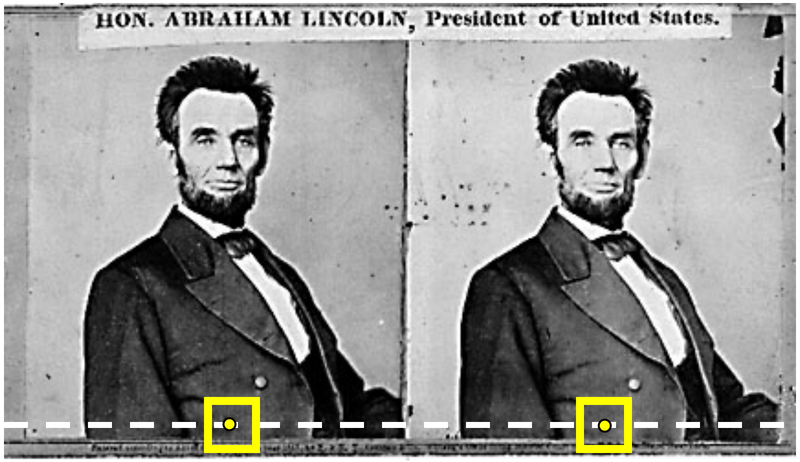




Two images captured by a purely horizontal translating camera (*rectified* stereo pair)

$x_1 - x_2$ = the *disparity* of pixel (x_1, y_1)

Your basic stereo algorithm



For every epipolar line:

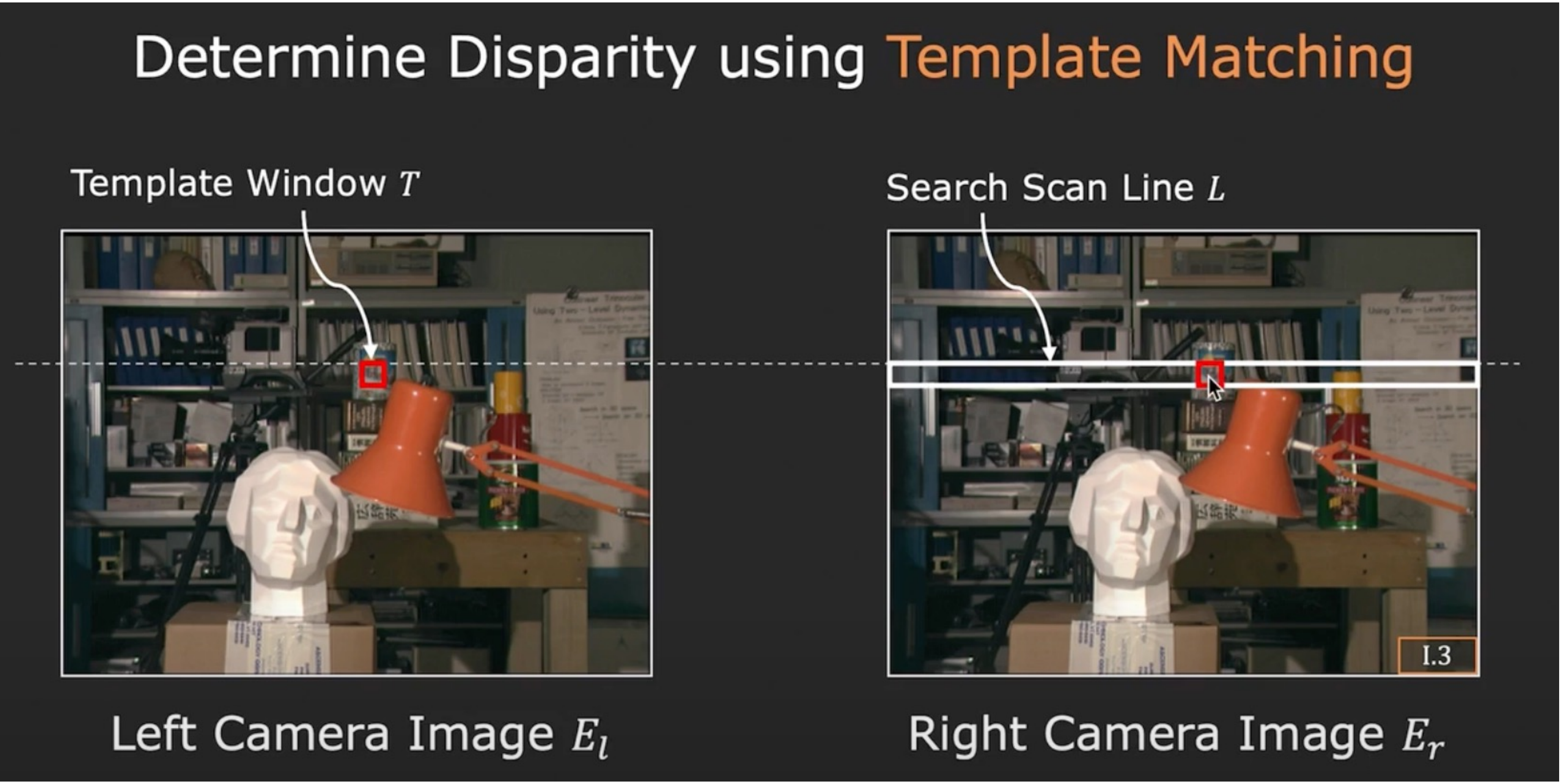
For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

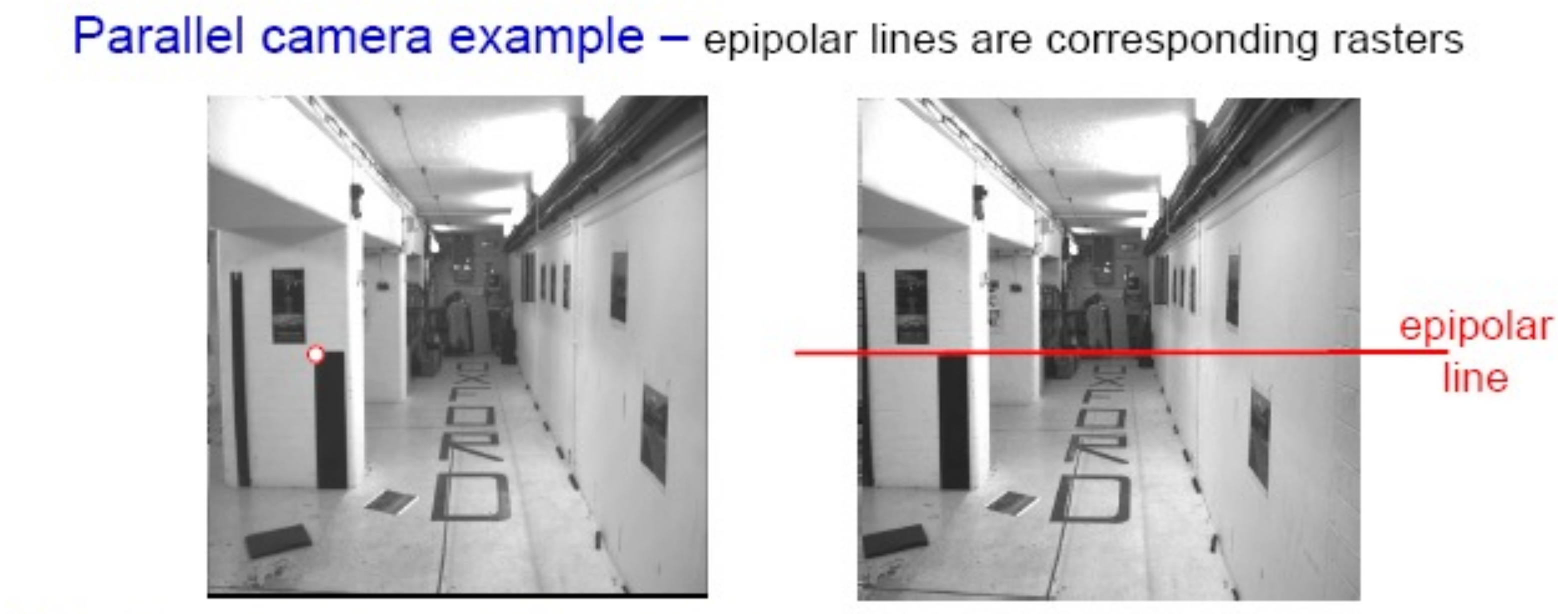
Improvement: match ***windows,*** *+ clearly lots of matching strategies*

Slide credit: Noah Snavely

Your basic stereo algorithm

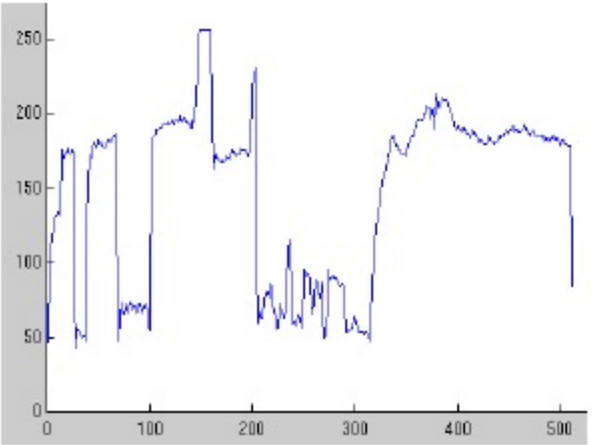


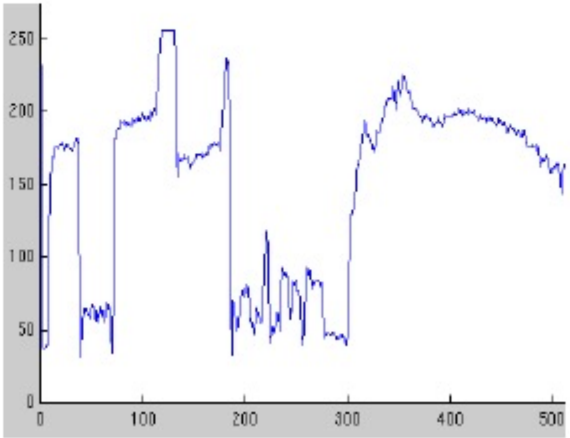
Correspondence problem



Source: Andrew Zisserman

Intensity profiles

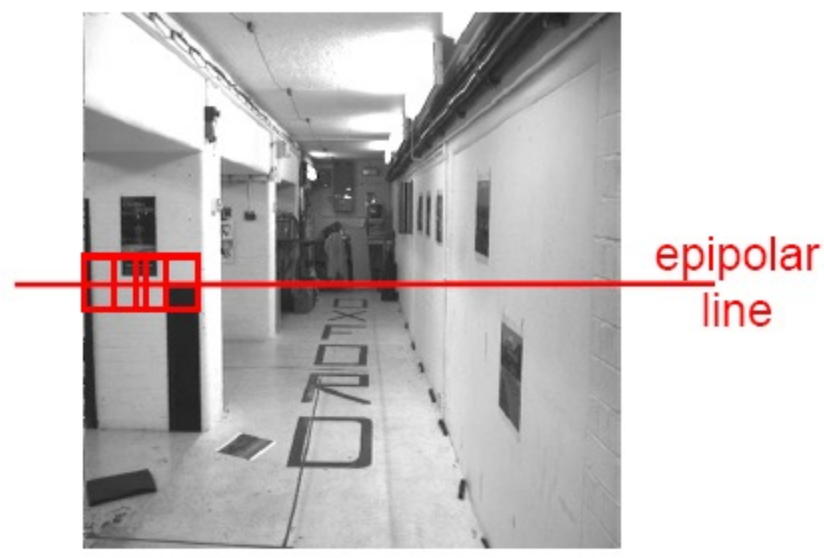




- Clear correspondence between intensities, but also noise and ambiguity

Source: Andrew Zisserman

Correspondence problem



Neighborhood of corresponding points are similar in intensity patterns.

Source: Andrew Zisserman

Normalized cross correlation

subtract mean: $A \leftarrow A - \langle A \rangle, B \leftarrow B - \langle B \rangle$

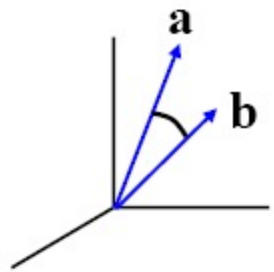
$$\mathrm{NCC} = \frac{\sum_i \sum_j A(i,j)B(i,j)}{\sqrt{\sum_i \sum_j A(i,j)^2}\sqrt{\sum_i \sum_j B(i,j)^2}}$$

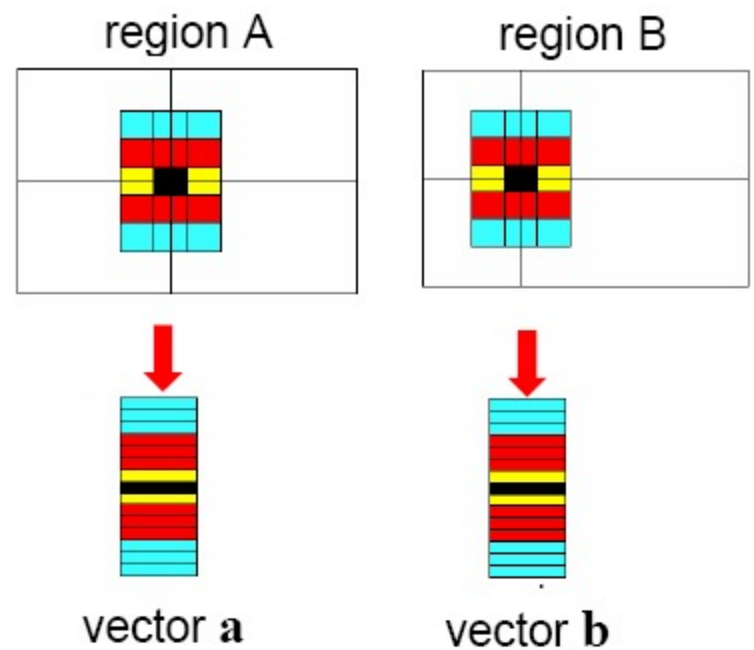
Write regions as vectors

$$\mathtt{A} \rightarrow \mathbf{a}, \ \ \mathtt{B} \rightarrow \mathbf{b}$$

$$\mathrm{NCC} = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

$$-1 \leq \mathrm{NCC} \leq 1$$

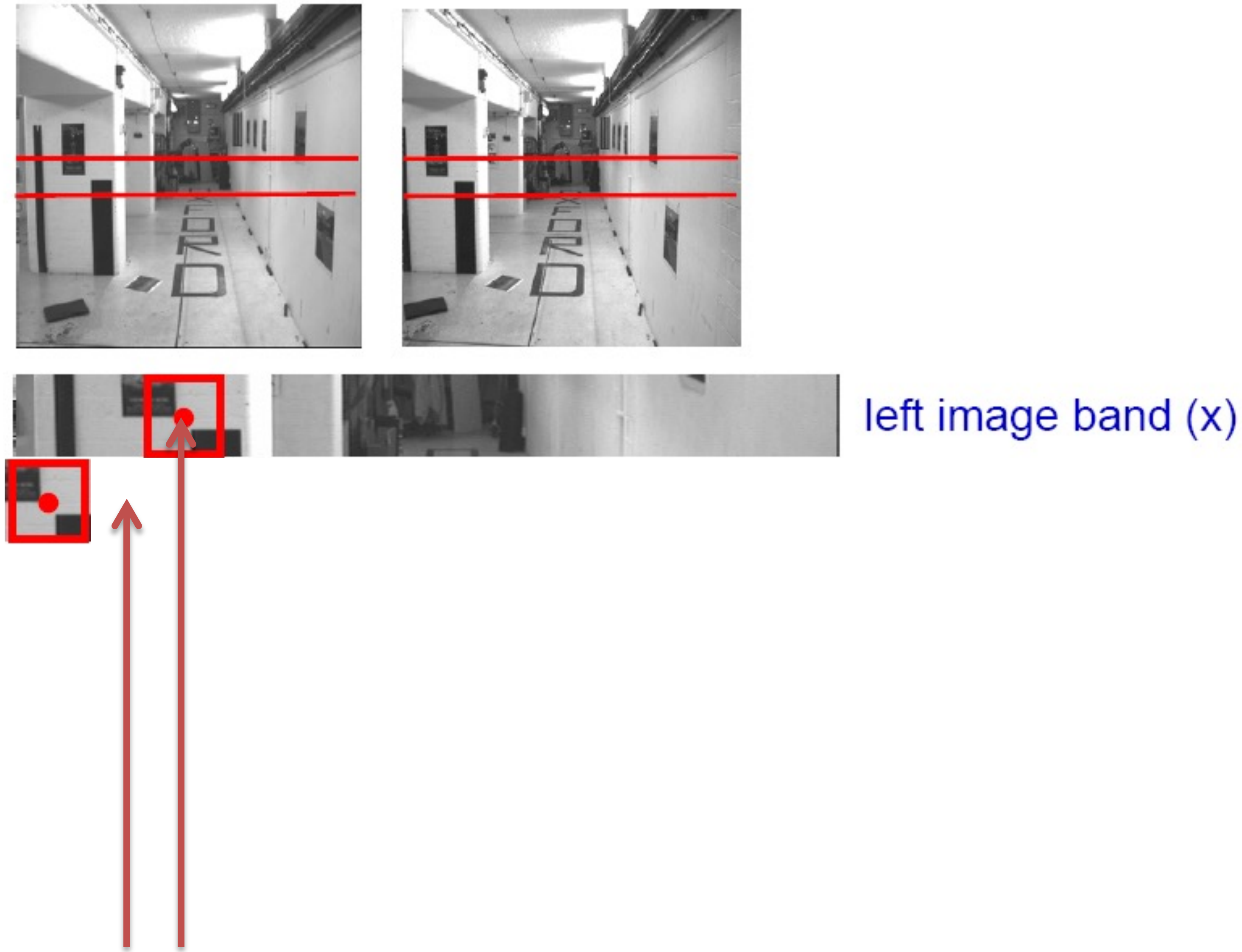




Similar to MOPS descriptor computation

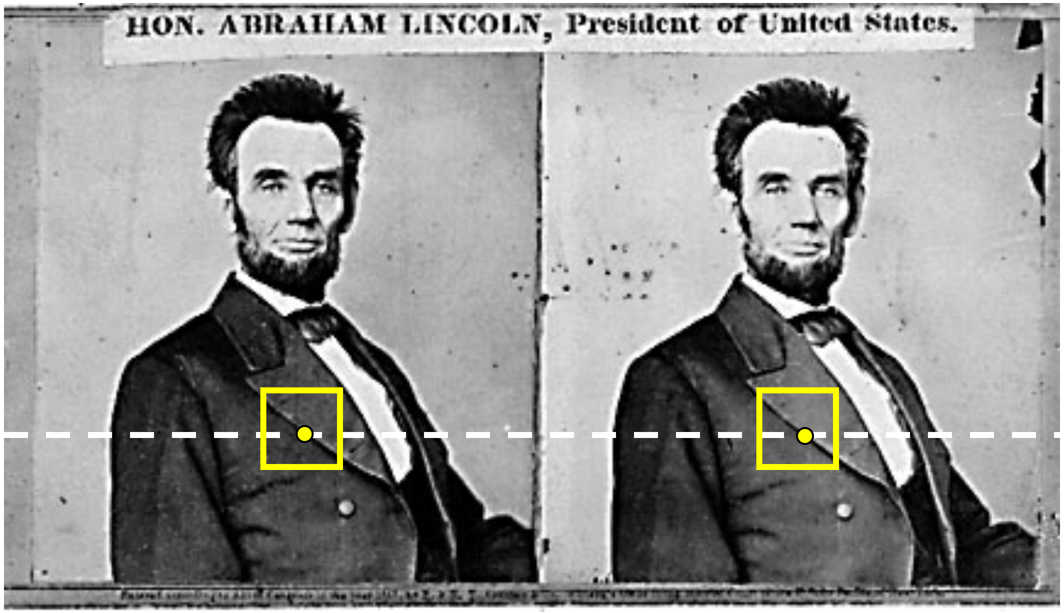
Source: Andrew Zisserman

Correlation-based window matching



Source: Andrew Zisserman

Dense correspondence search



For each epipolar line

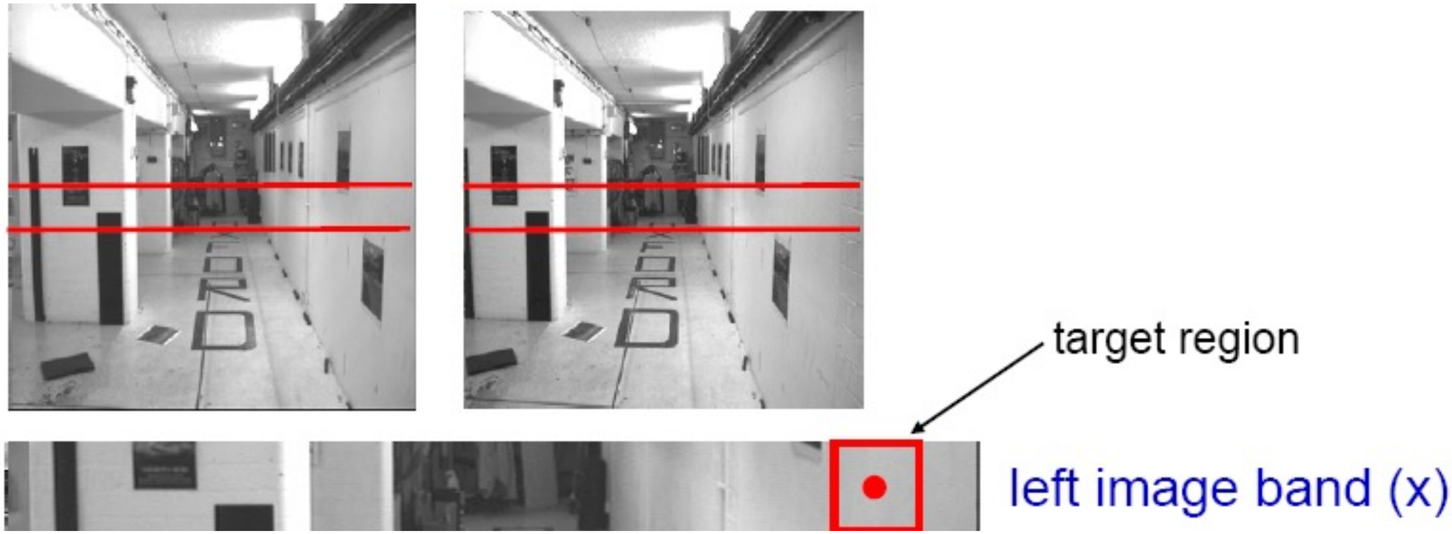
For each pixel / window in the left image

- compare with every pixel / window on same epipolar line in right image
- pick position with minimum match cost (e.g., SSD, correlation)

Adapted from Li Zhang

Grauman

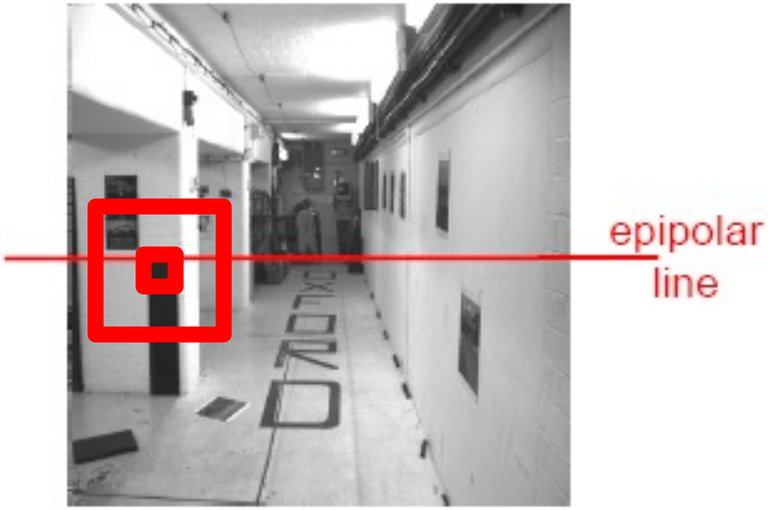
Textureless regions



Source: Andrew Zisserman

Grauman

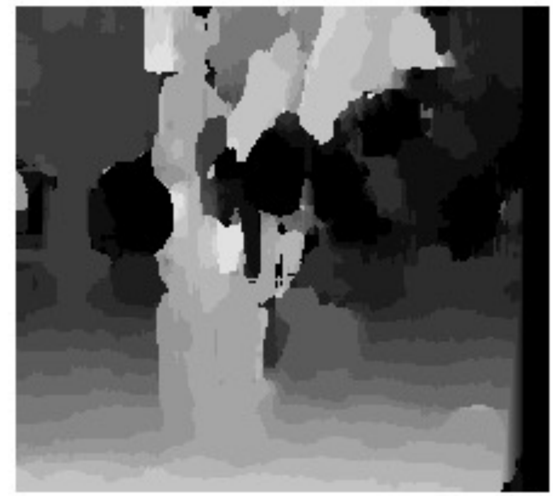
Effect of window size



Source: Andrew Zisserman

Effect of window size

W = 3



W = 20

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

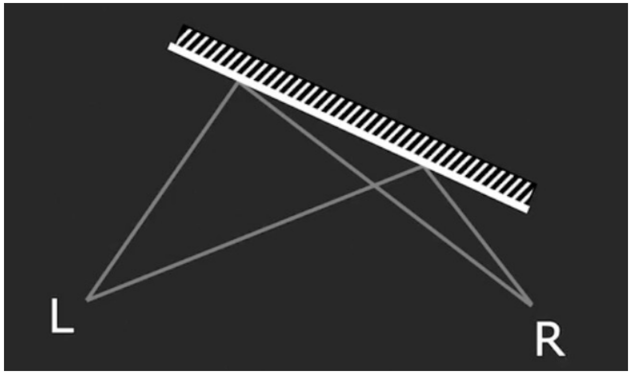
Figures from Li Zhang

Grauman

Issues with Stereo

- Surface must have non-repetitive texture

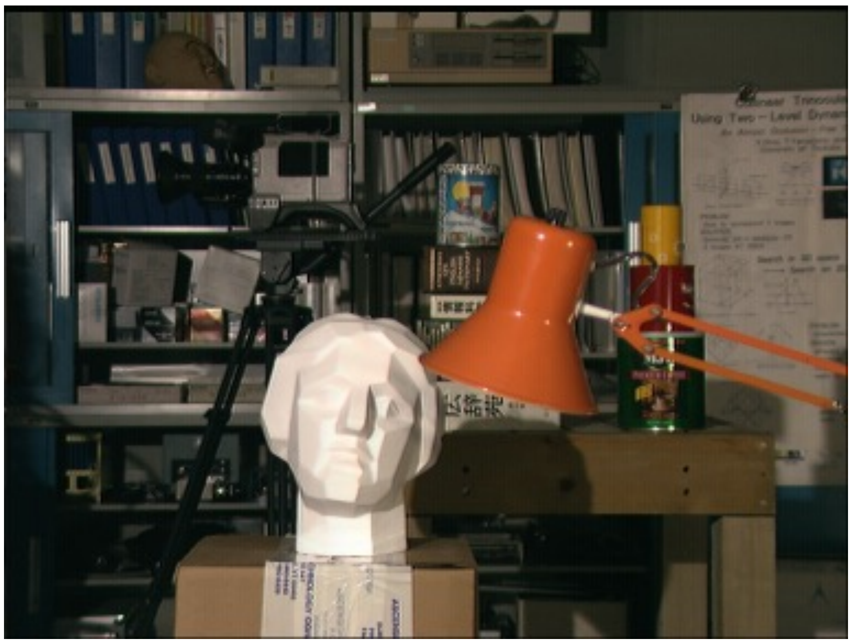
- Foreshortening effect makes matching a challenge



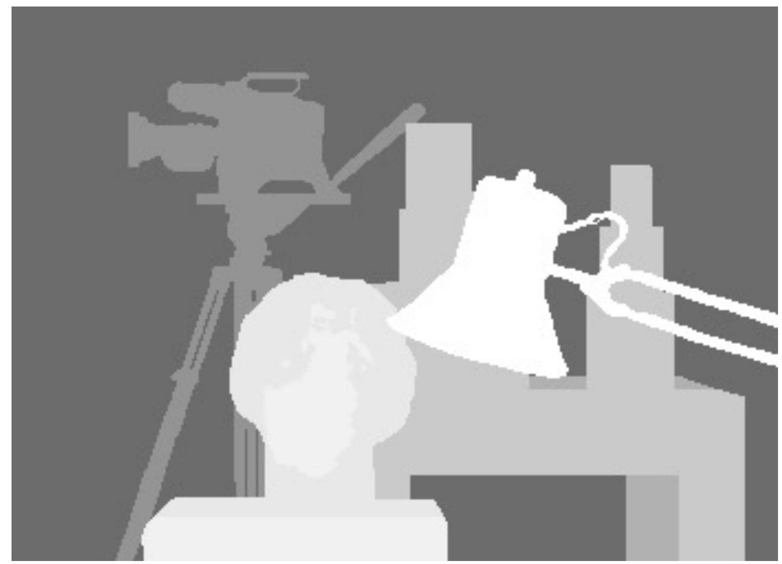
Slide Credit: Shree Nayar

Stereo Results

– Data from University of Tsukuba

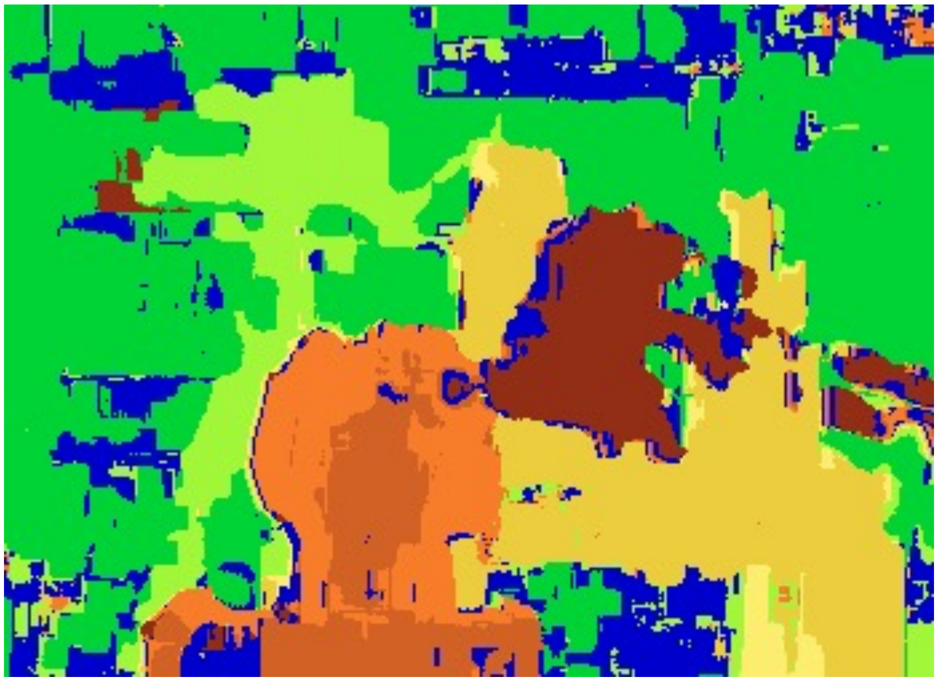


Scene



Ground truth

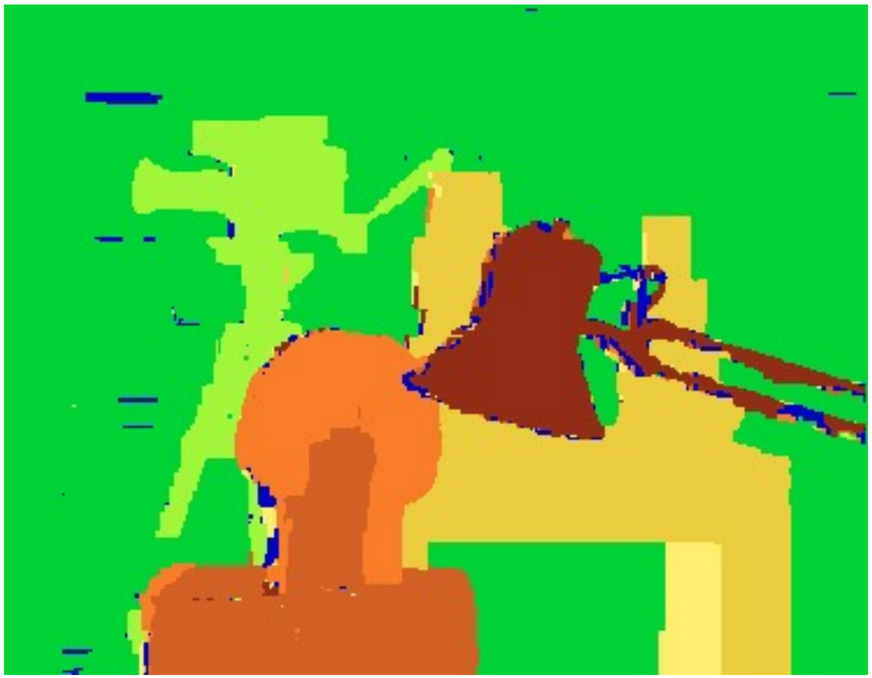
Results with Window Search



Window-based matching
(best window size)

Ground truth

Better methods exist...



Energy Minimization

Boykov et al., [Fast Approximate Energy Minimization via Graph Cuts](), International Conference on Computer Vision, September 1999.

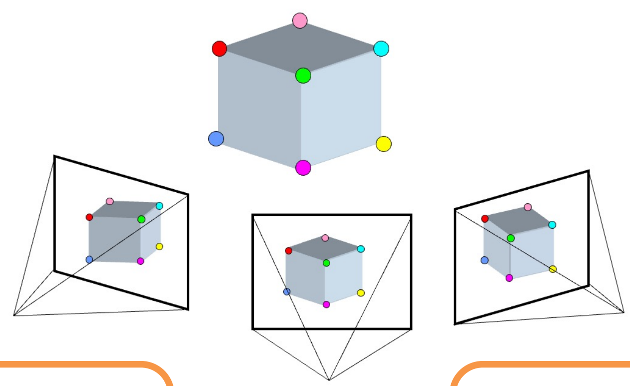
Ground truth

Summary

- With a simple stereo system, **how much pixels move, or “disparity”** give information about the depth
- Correspondences to measure the pixel disparity

Many problems in 3D

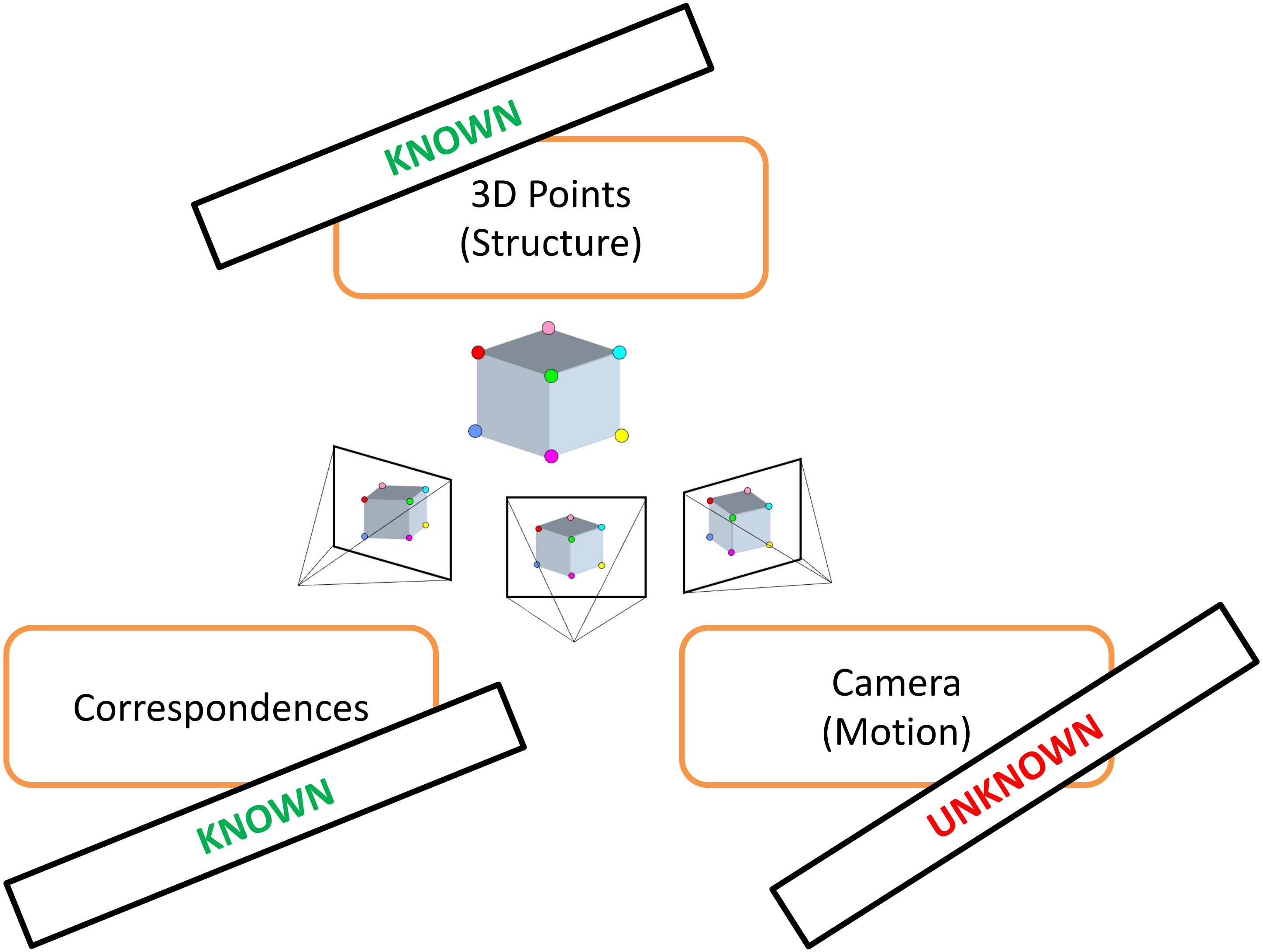
3D Points
(Structure)



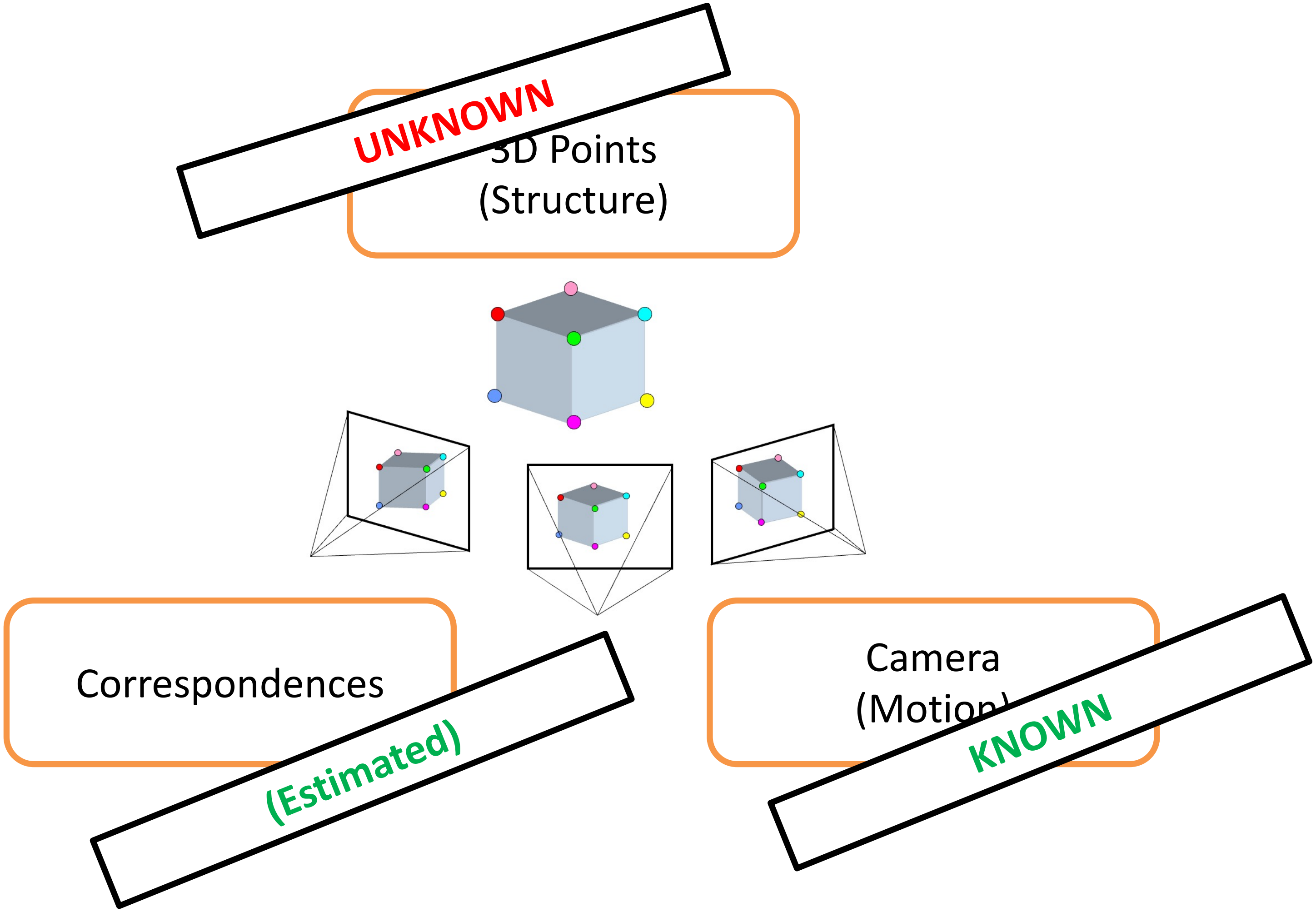
Correspondences

Camera
(Motion)

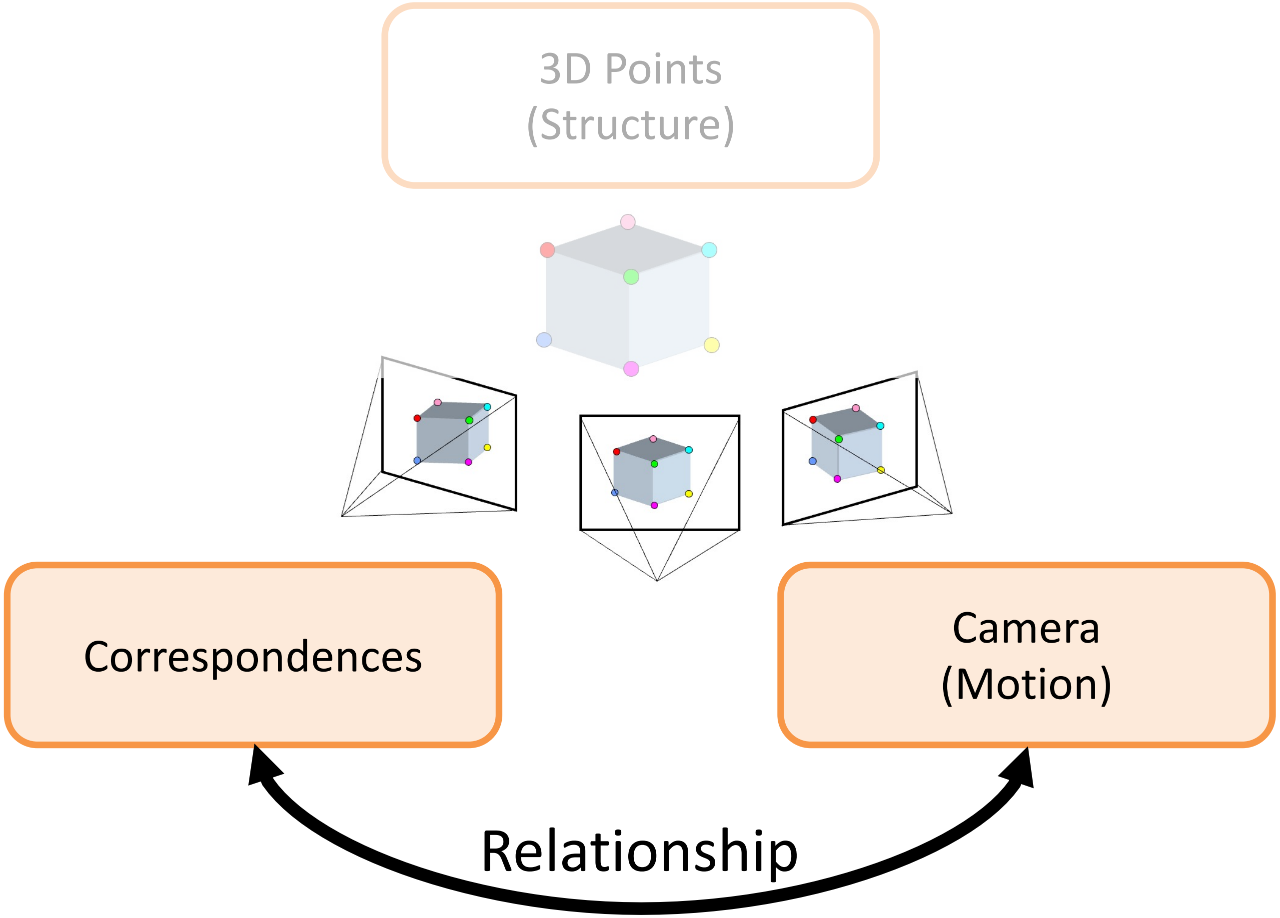
Camera Calibration



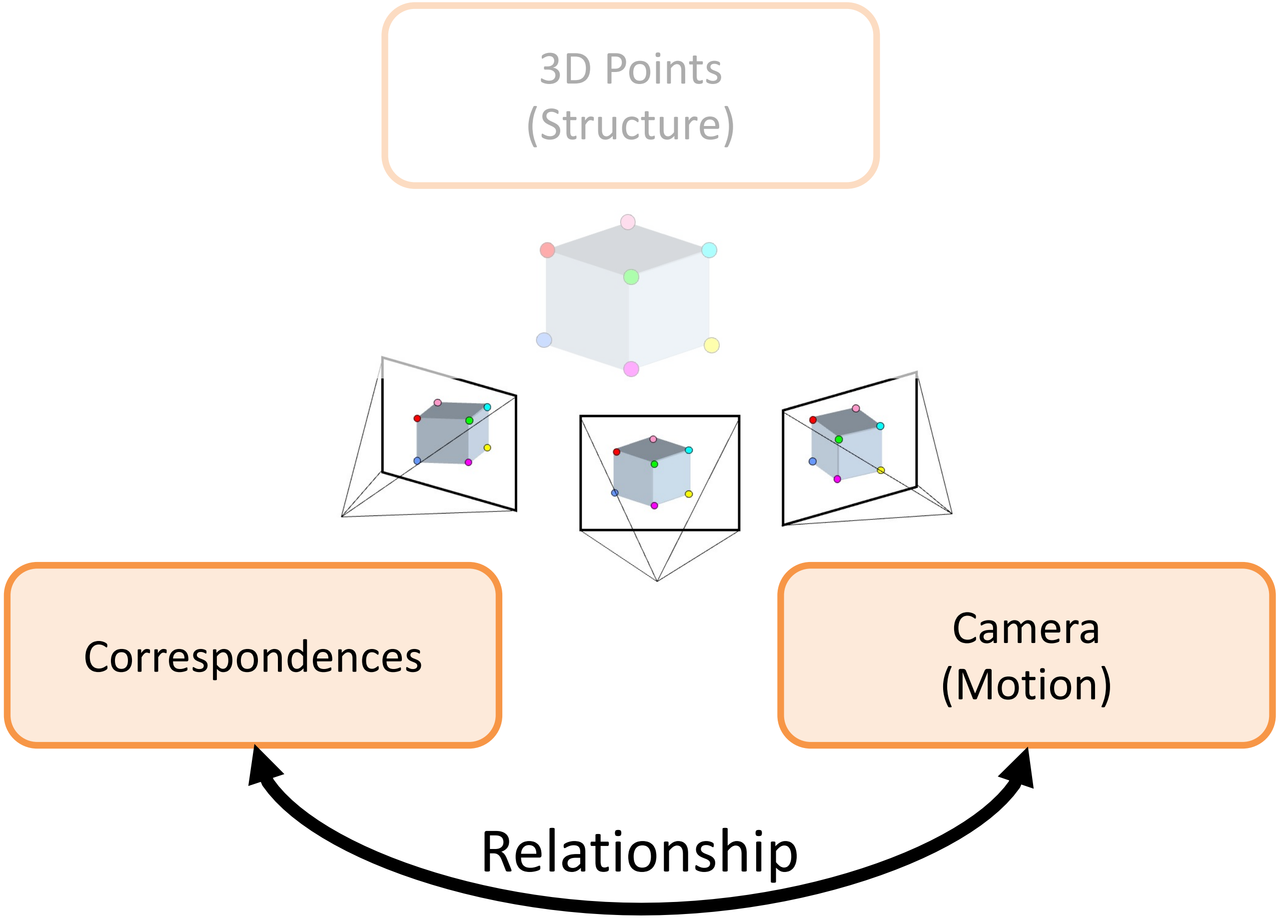
Stereo (w/2 cameras); Multi-view Stereo



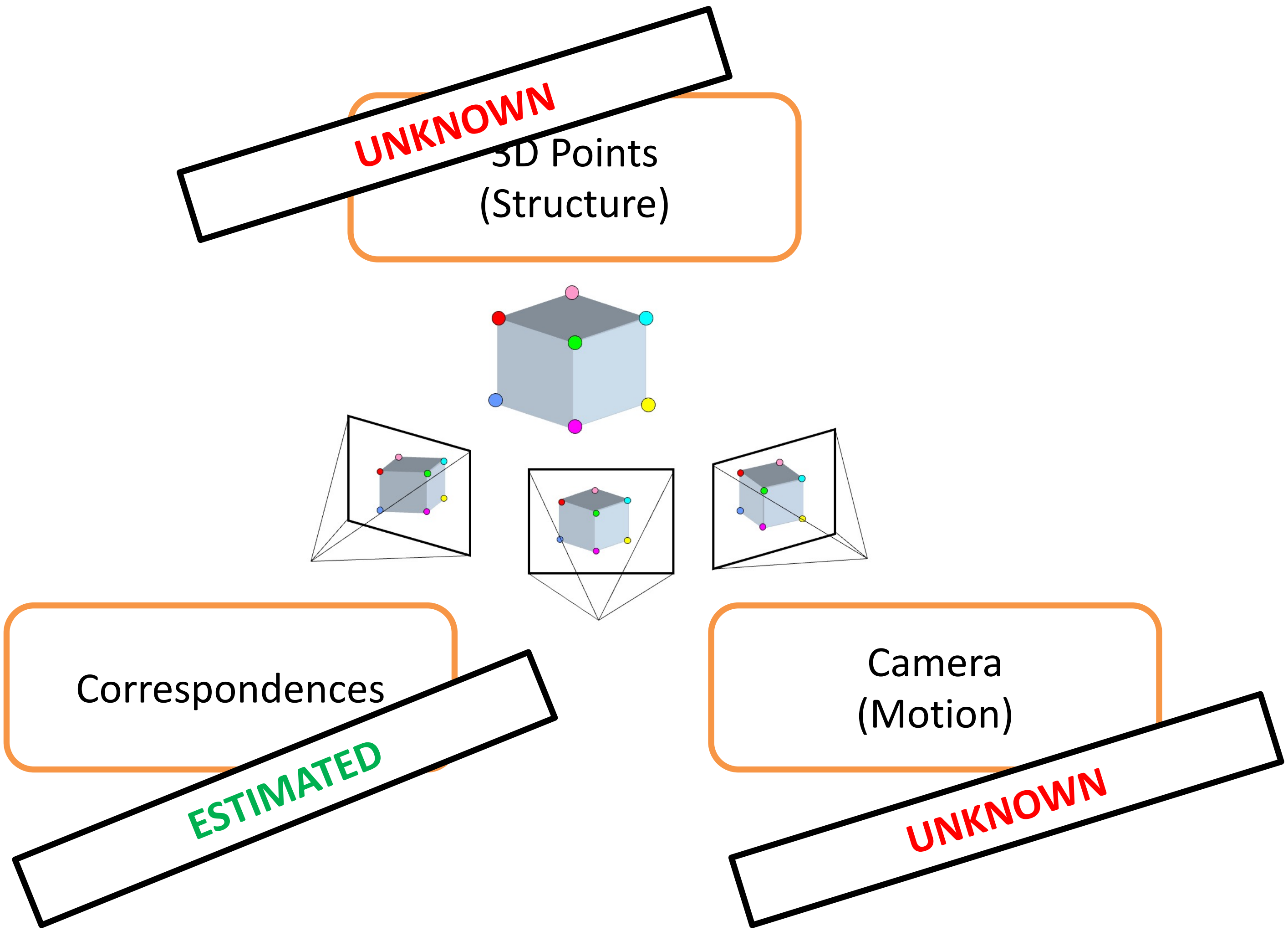
Camera helps Correspondence:
Epipolar Geometry



Correspondence gives camera: **Epipolar Geometry**

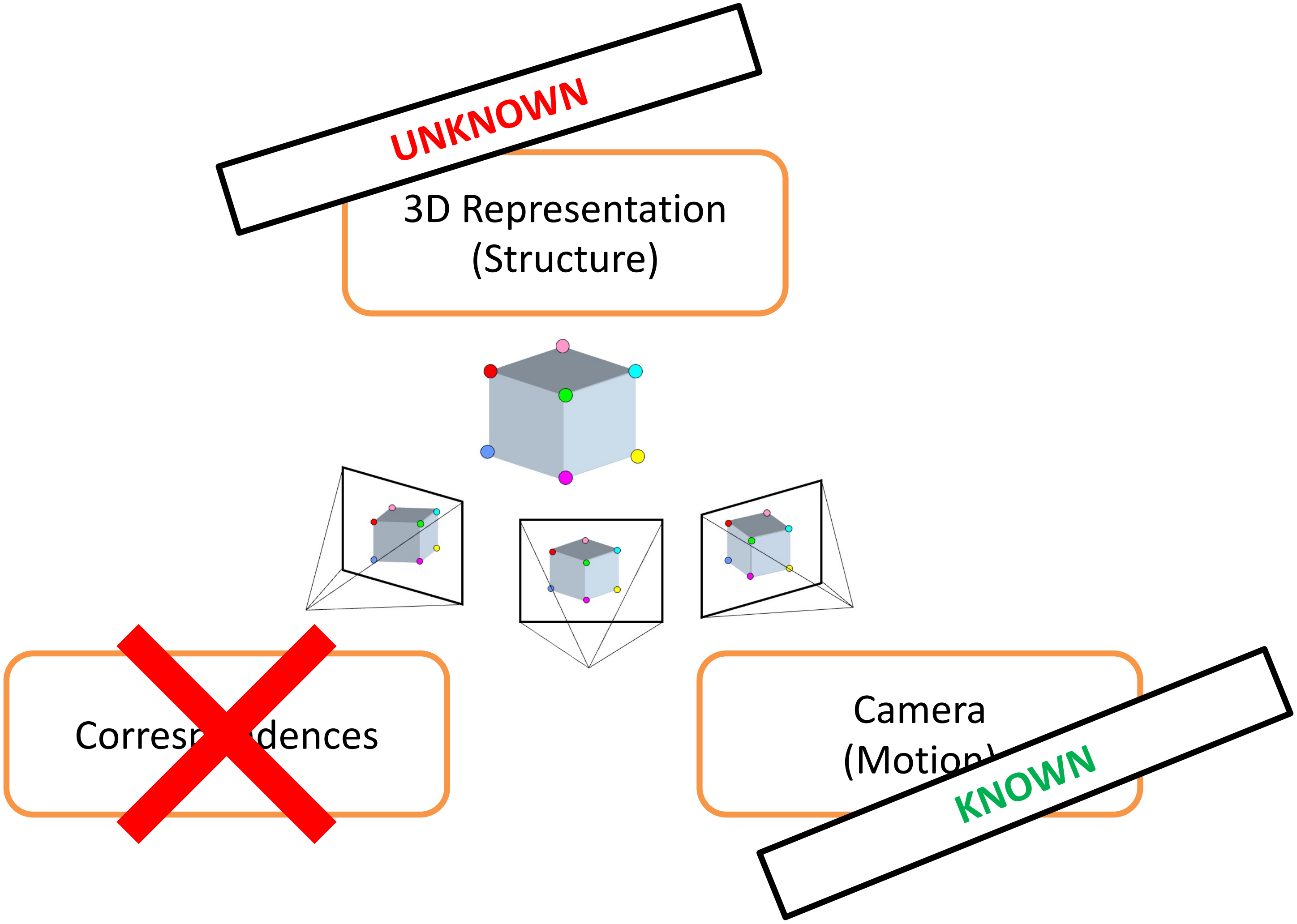


(Next lecture) Ultimate: Structure-from-Motion/SLAM



The starting point for all problems where you can’t calibrate actively

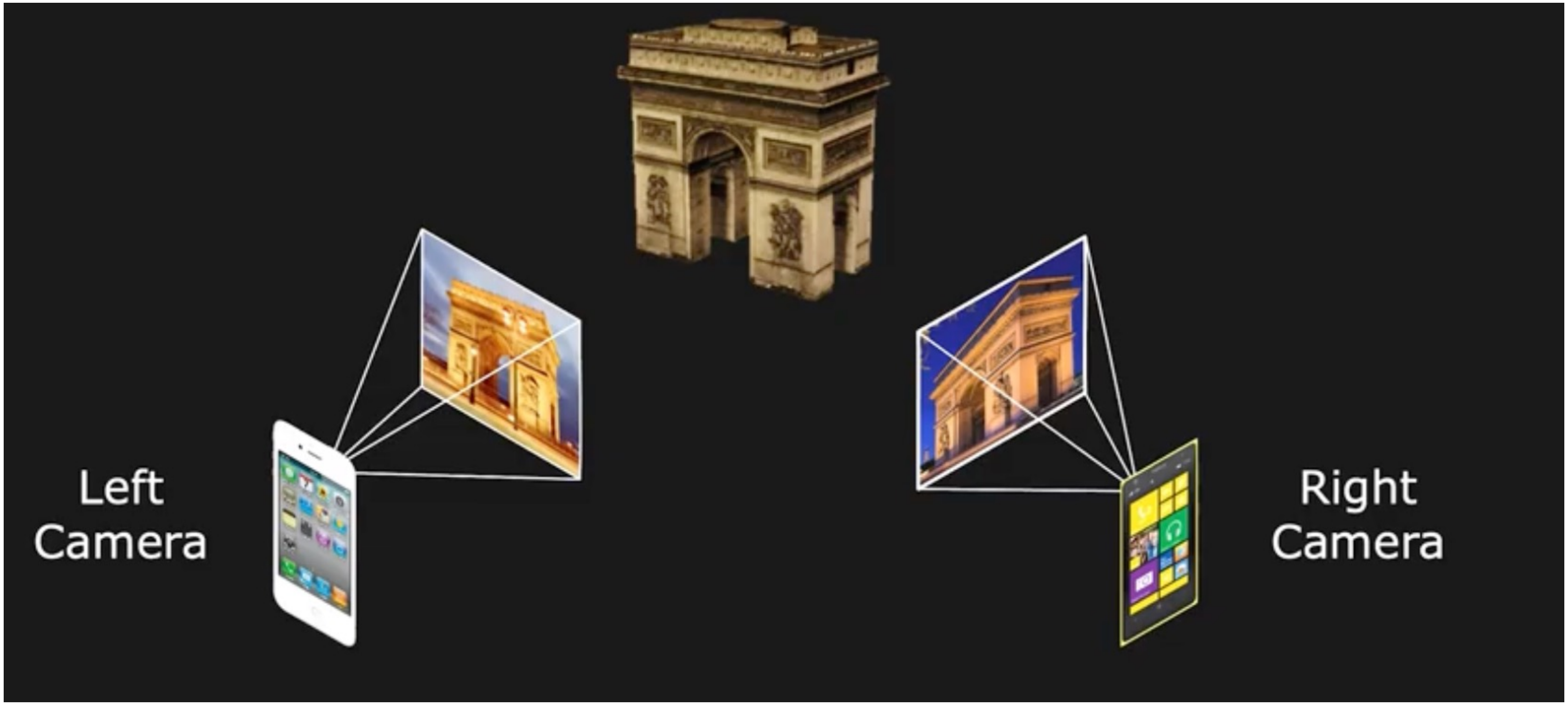
New: Neural Rendering



A form of multi-view stereo, more on this in the NeRF lecture.

Next: Uncalibrated Stereo

- From two arbitrary views



- Assume intrinsics are known (fx, fy, ox, oy)

Slide Credit: Shree Nayar