Homographies and Panoramas

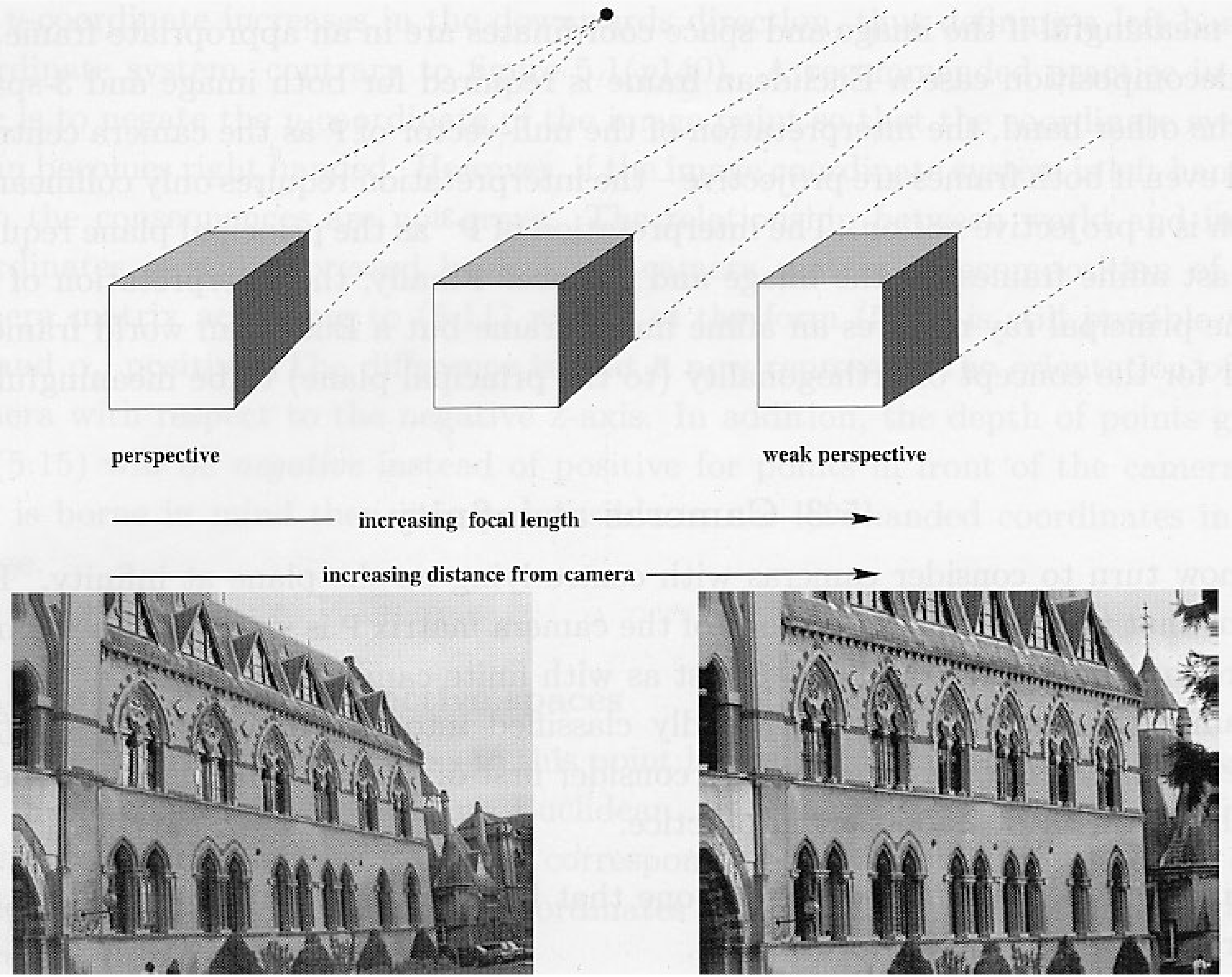
© Andrew Campbell

with a lot of slides stolen from Steve Seitz and Rick Szeliski

CS180: Intro to Computer Vision and Comp. Photo
Alexei Efros, UC Berkeley, Fall 2024

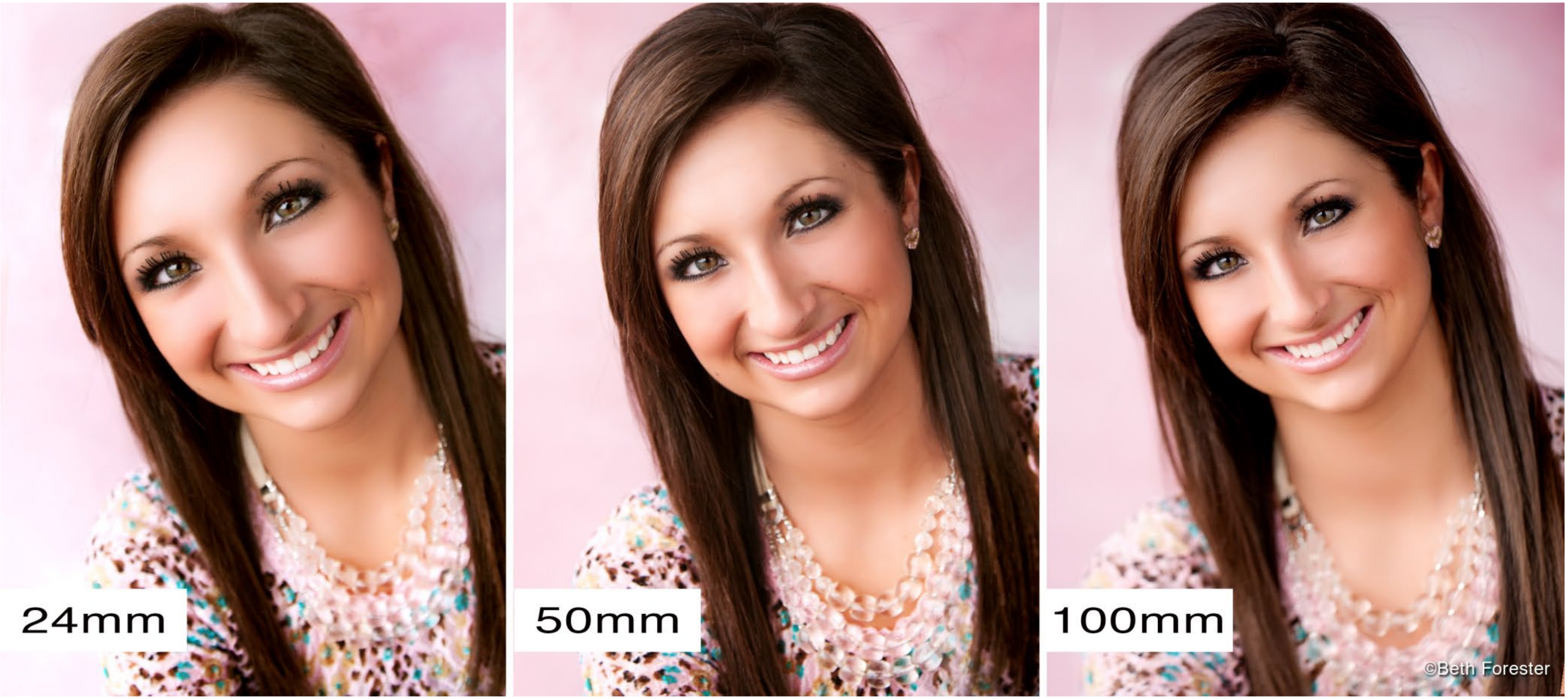
Project 4 out today!

Recap



From Zisserman & Hartley

Focal length / distance in portraiture



©Beth Forester

Perspective Distortion

Not due to lens flaws

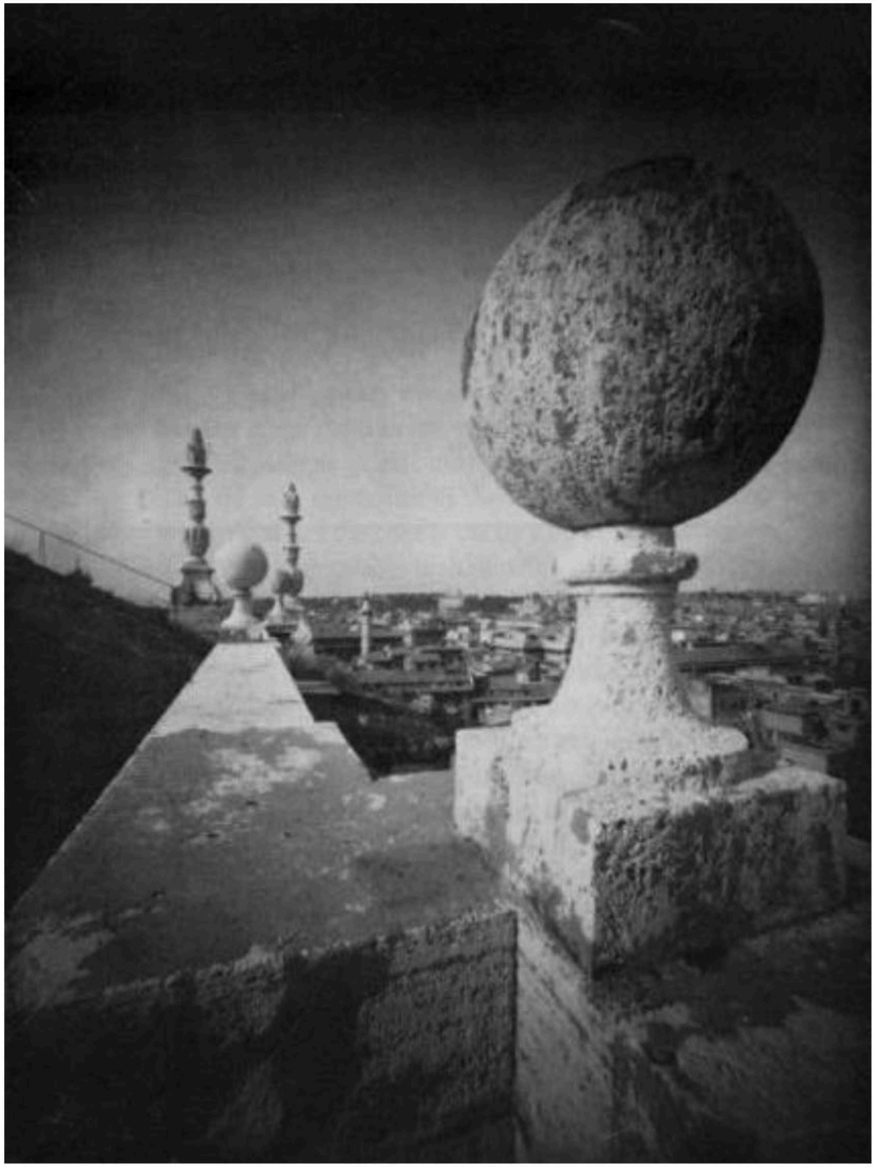
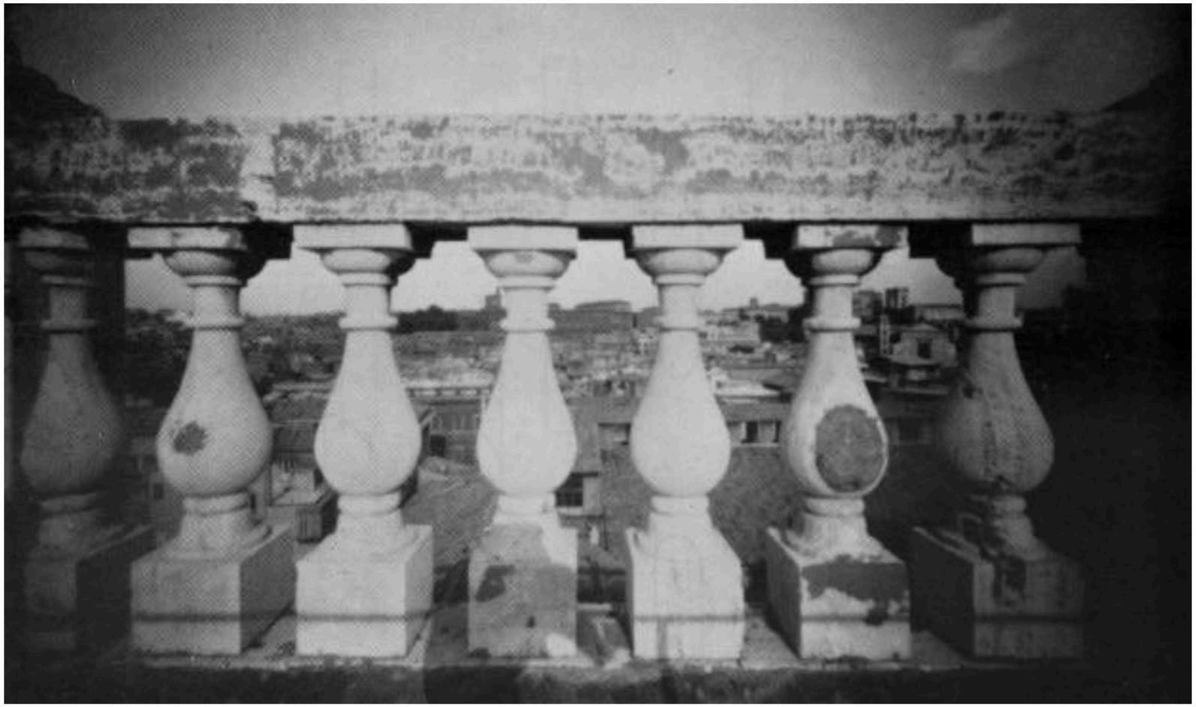
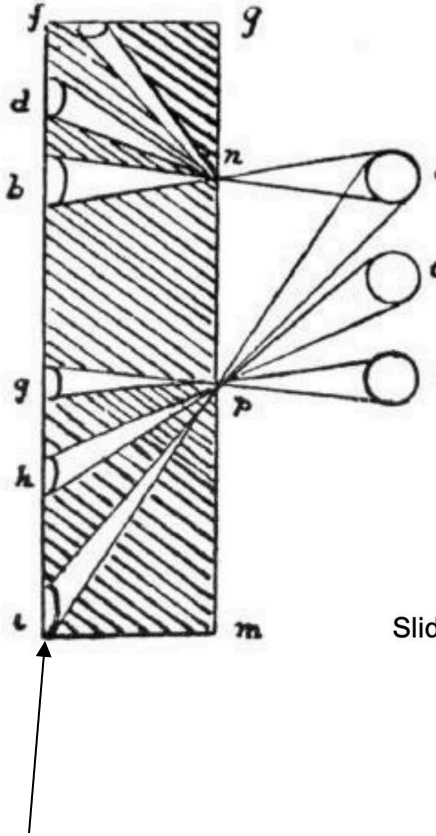


Image source: F. Durand

Problem pointed out by Da Vinci

The exterior columns appear bigger



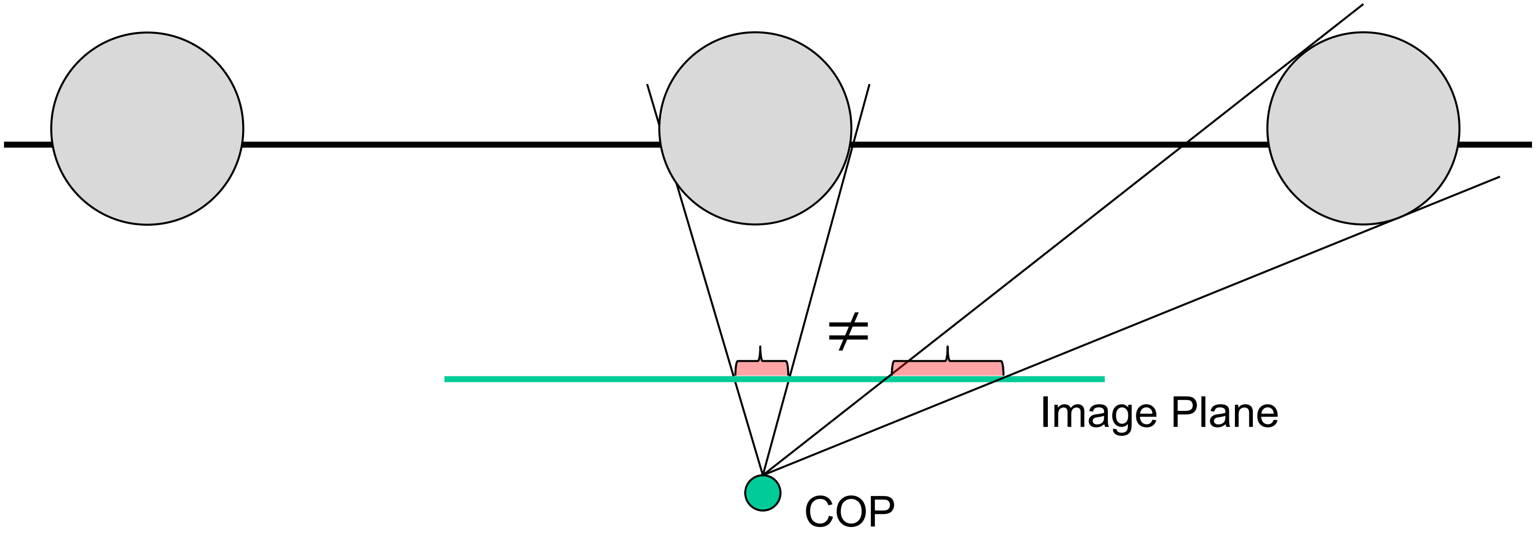


Slide by F. Durand

Image Plane

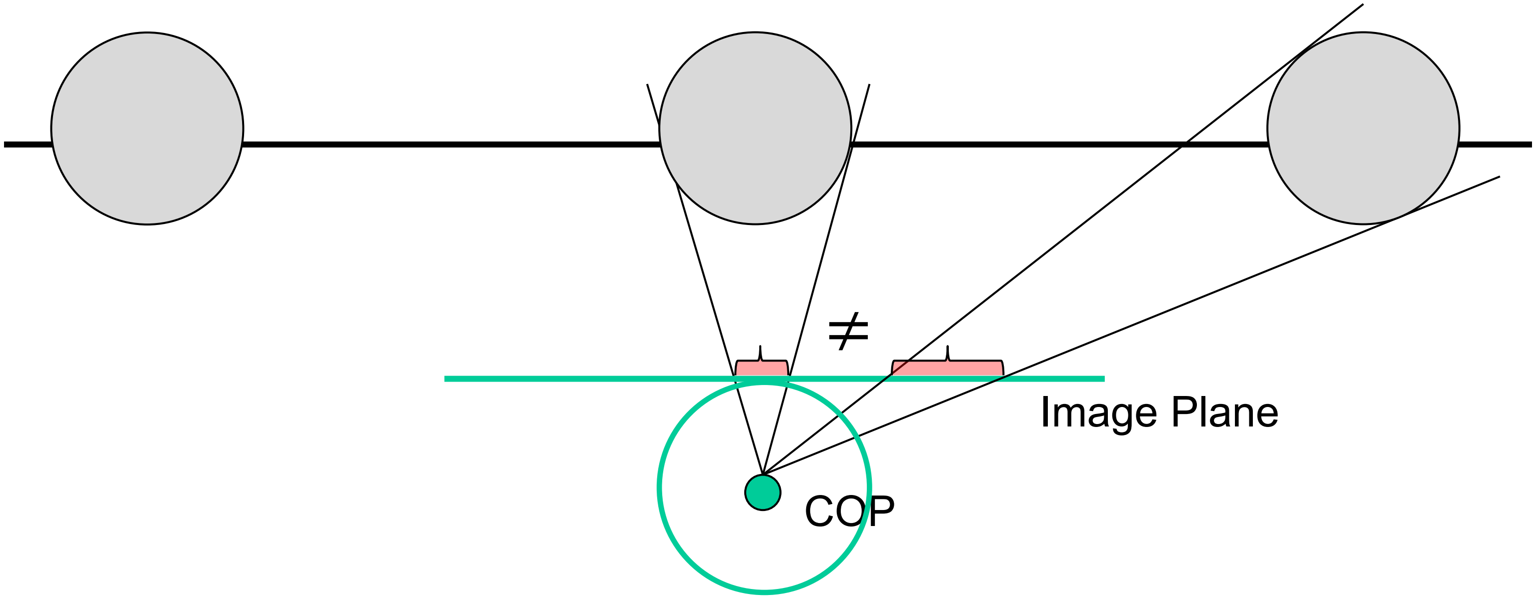
Perspective Distortion

Recall Perspective Projection: $x' = f\frac{X}{Z}$ $y' = f\frac{Y}{Z}$



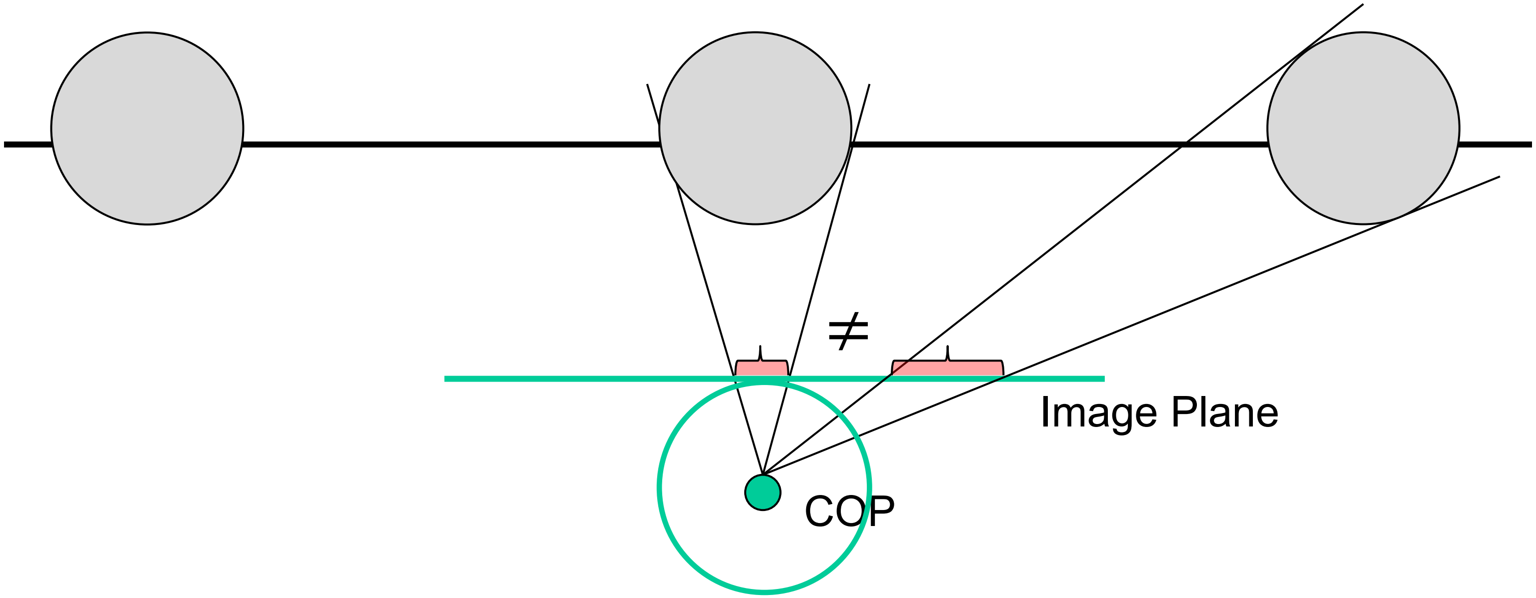
Perspective Distortion

With a spherical projection plane



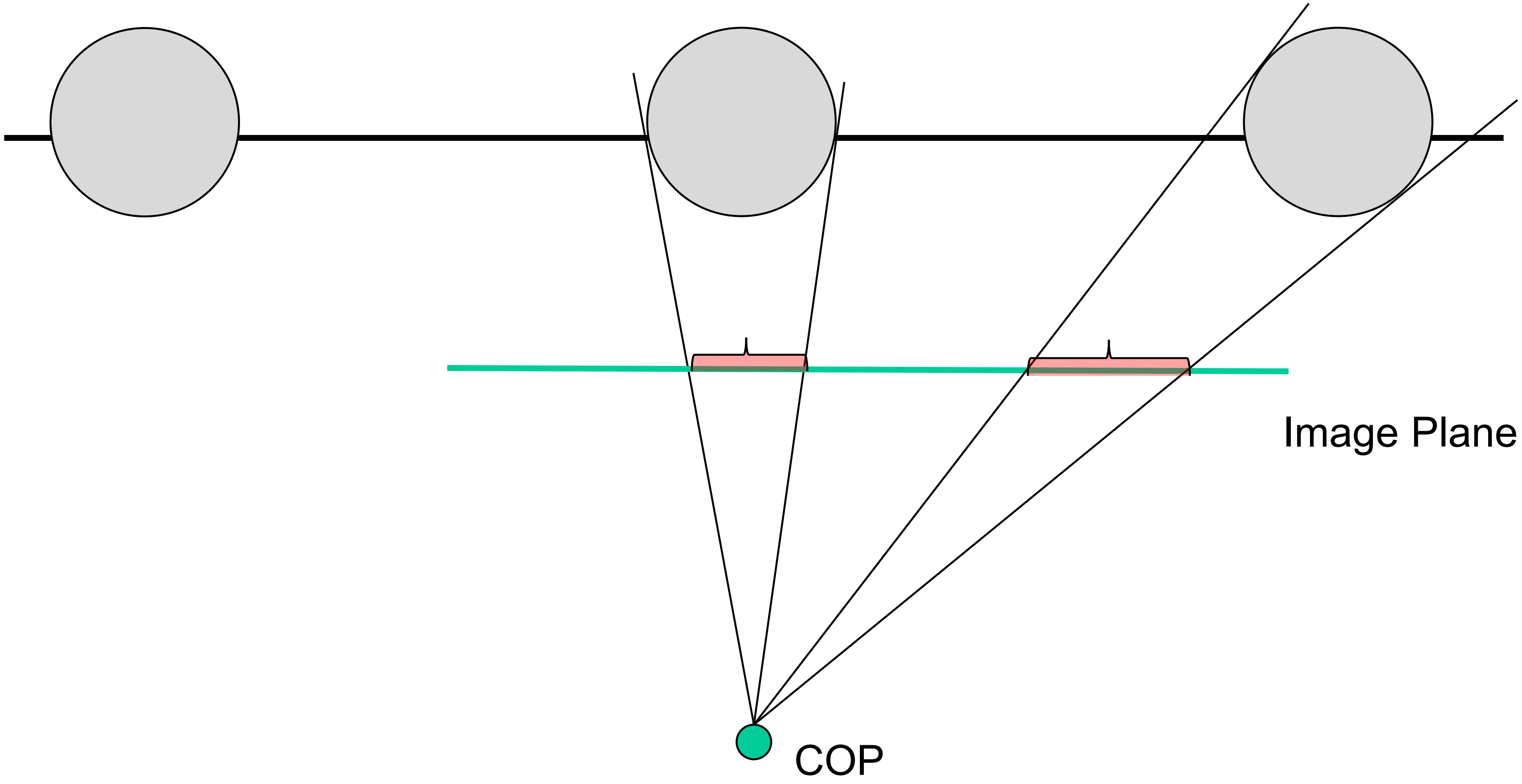
Perspective Distortion

With a spherical projection plane



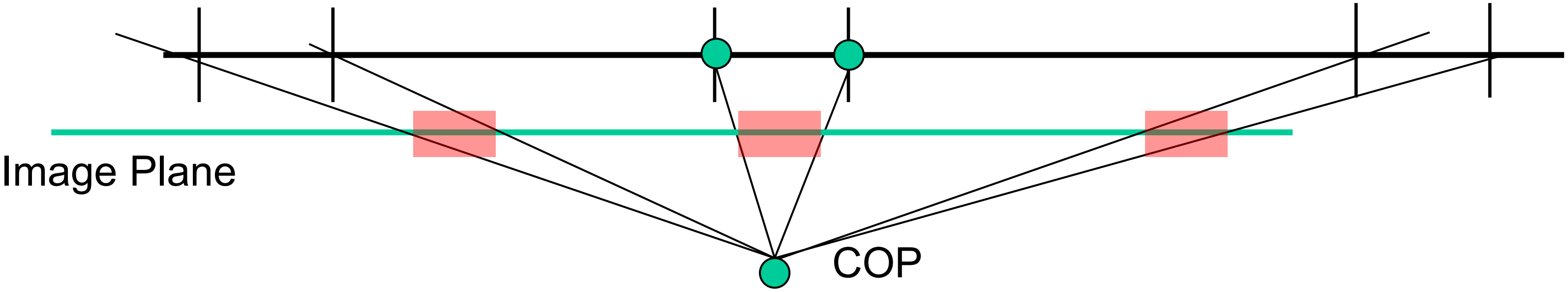
Perspective Distortion

Less noticeable with long focal length (i.e. you see distortion more with wide-angle camera)



Perspective Distortion

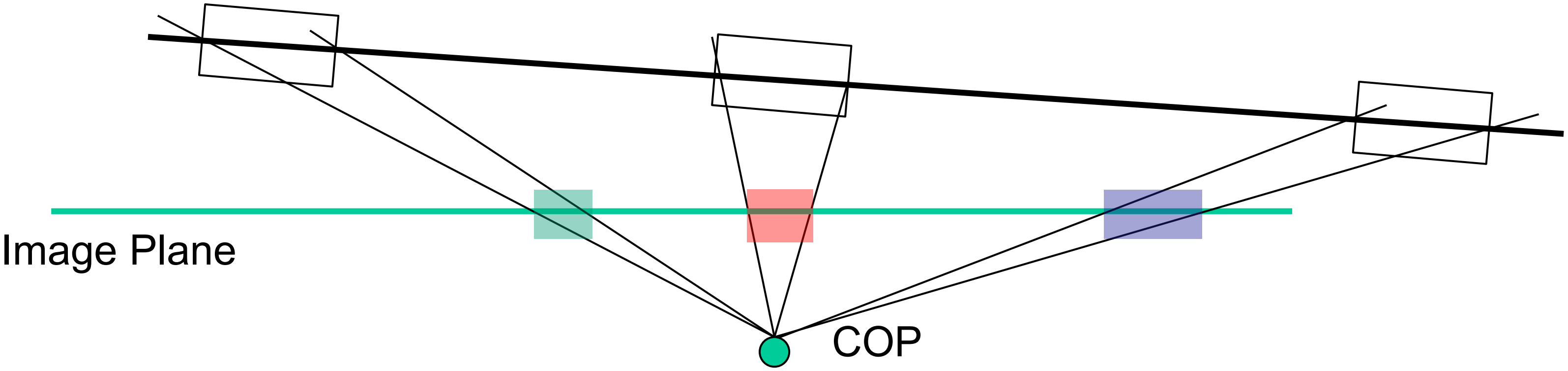
It's about the change in depths



But this is a very special case..

Perspective Distortion

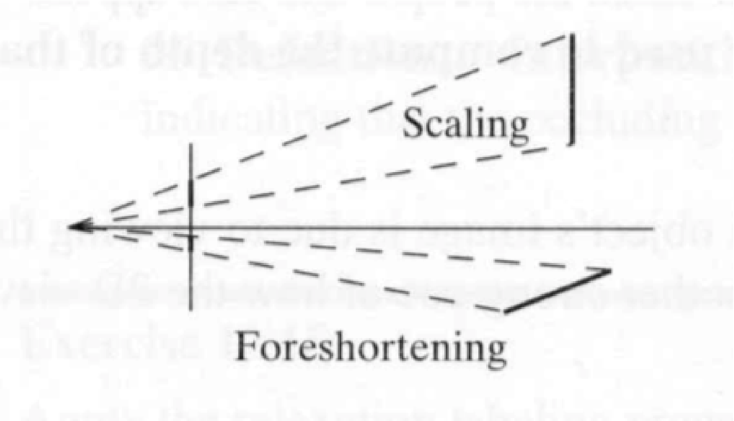
More likely..



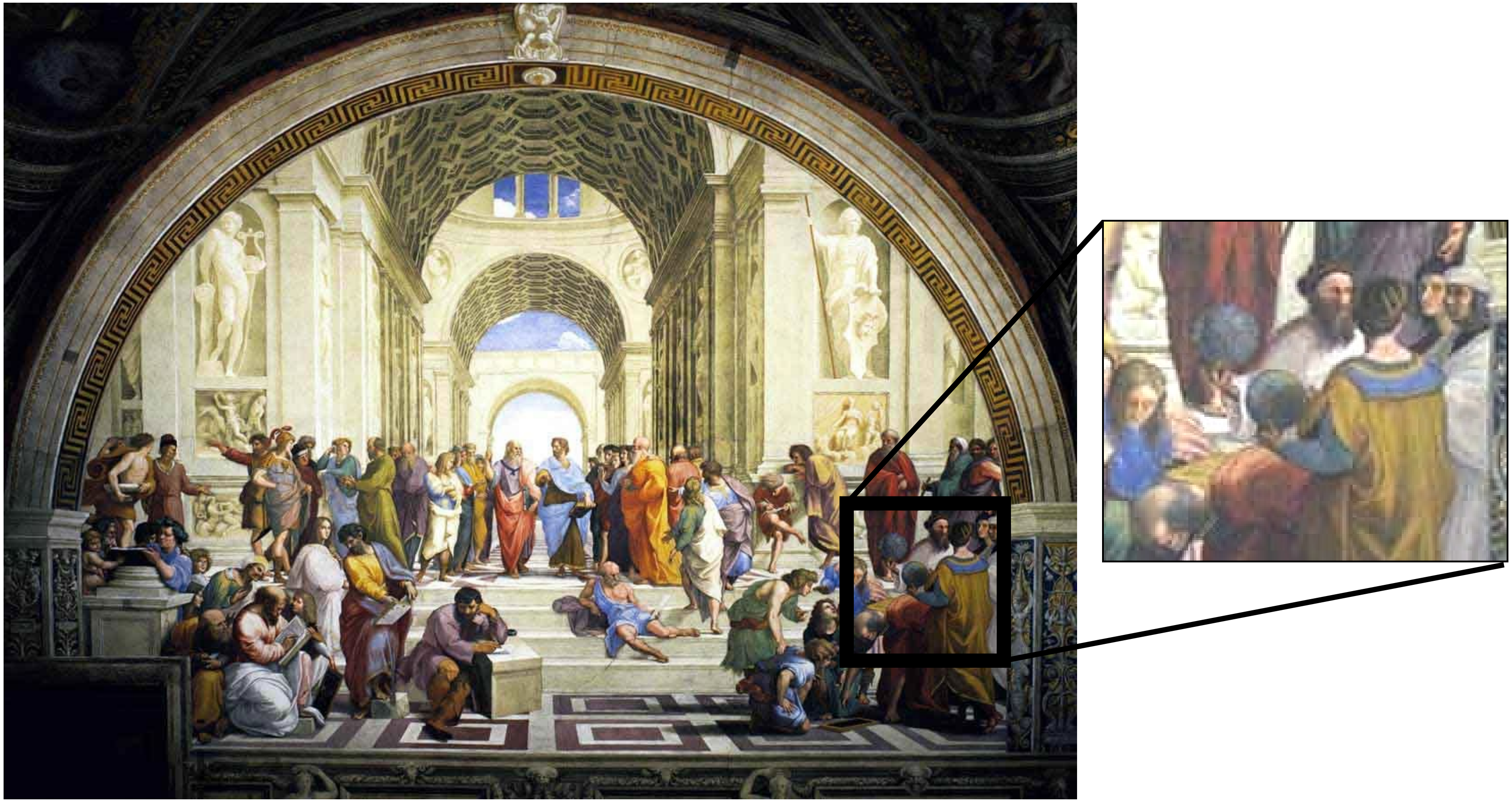
Foreshortening

- When a line (or surface) is parallel to the image plane, the effect of perspective projection is *scaling*.

- When an line (or surface) is not parallel to the image plane, we use the term *foreshortening* to describe the projective distortion (i.e., the dimension parallel to the optical axis is compressed relative to the frontal dimension).



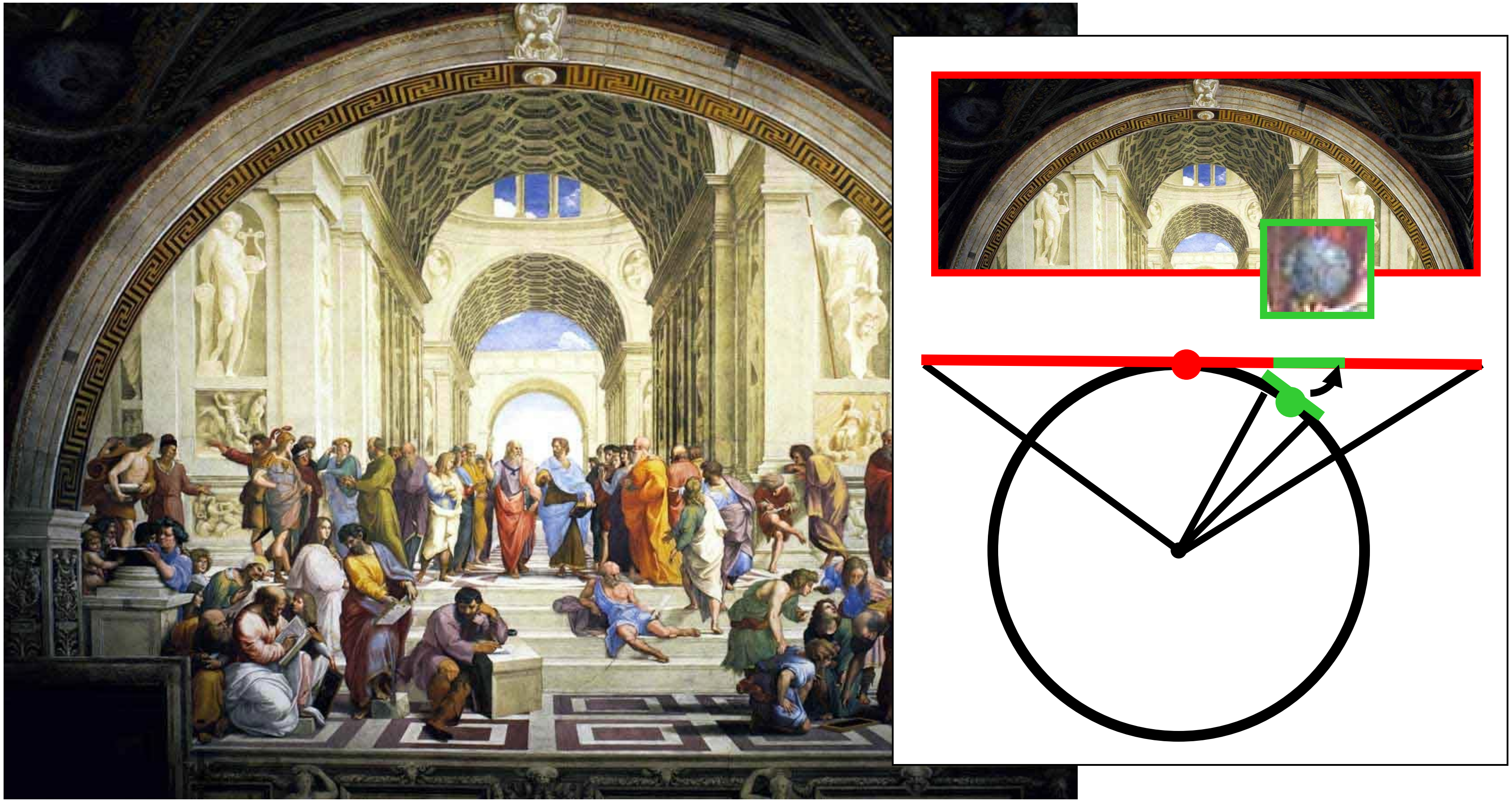
Renaissance Painters' solution



"School of Athens", Raffaello Sanzio ~1510

Give a separate treatment to different parts of the scene!!

Personalized projections



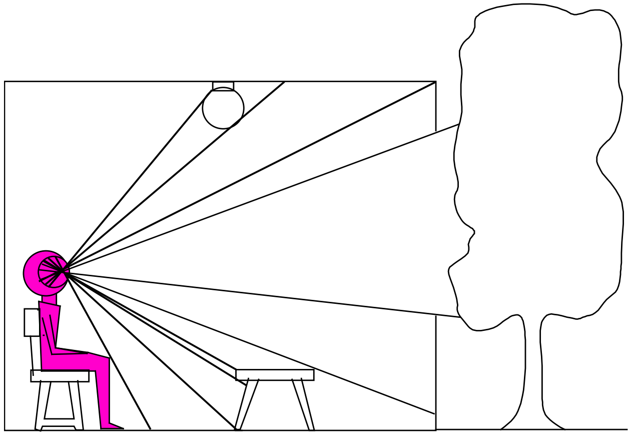
“School of Athens”, Raffaello Sanzio ~1510

Give a separate treatment to different parts of the scene!!

Our visual world as a set of light rays

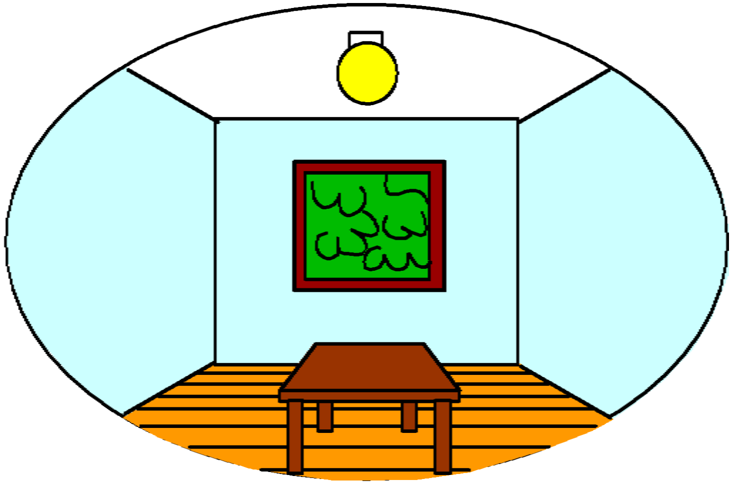
What do we see?

3D world



Point of observation

2D image

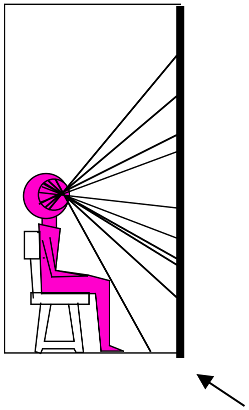


Figures © Stephen E. Palmer, 2002

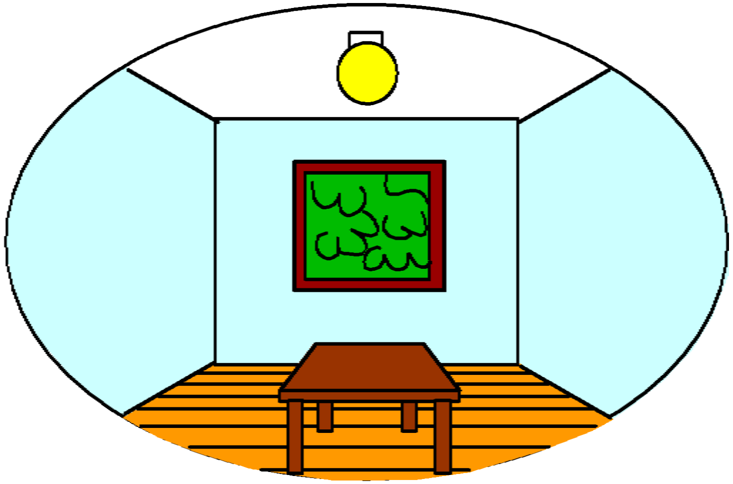
What do we see?

3D world

2D image



Painted backdrop



On Simulating the Visual Experience

Just feed the eyes the right data

- No one will know the difference!

Philosophy:

- Ancient question: "Does the world really exist?"

Science fiction:

- Many, many, many books on the subject, e.g. *slowglass* from "Light of Other Days"
- "Latest" take: *The Matrix*

Physics:

- *Slowglass* might be possible?

Computer Science:

- Virtual Reality

To simulate we need to know:

What does a person see?

The Plenoptic Function

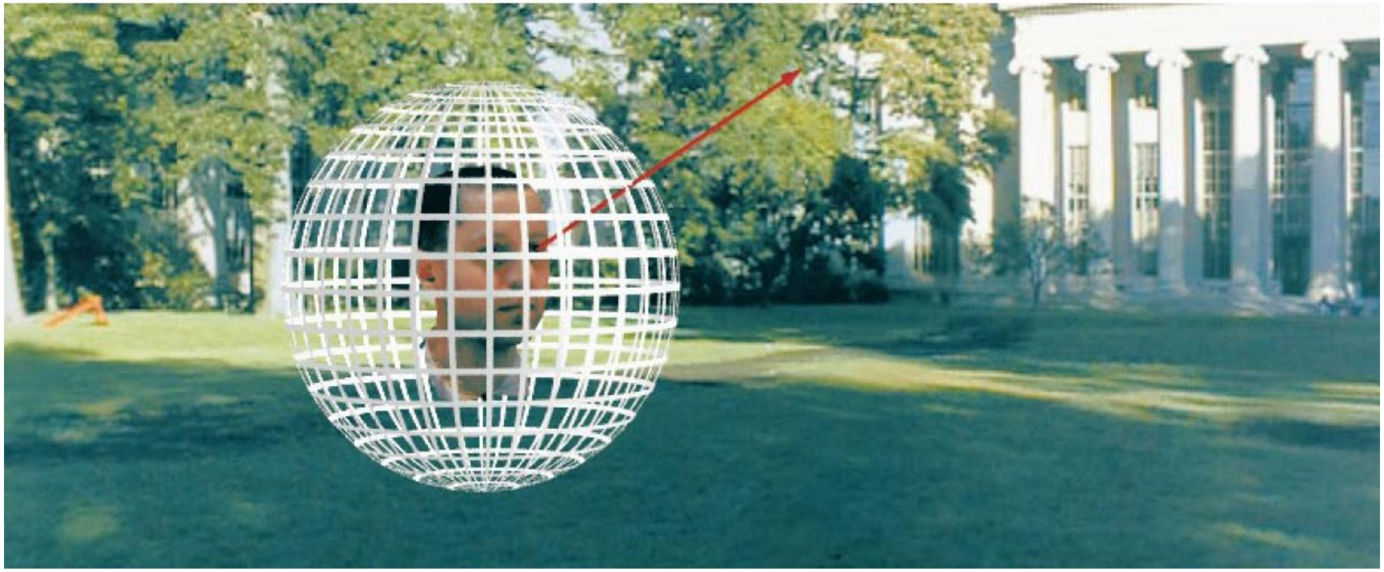


Figure by Leonard McMillan

Q: What is the set of all things that we can ever see?

A: The Plenoptic Function (Adelson & Bergen)

Let’s start with a stationary person and try to parameterize <u>everything</u> that he can see…

Grayscale snapshot

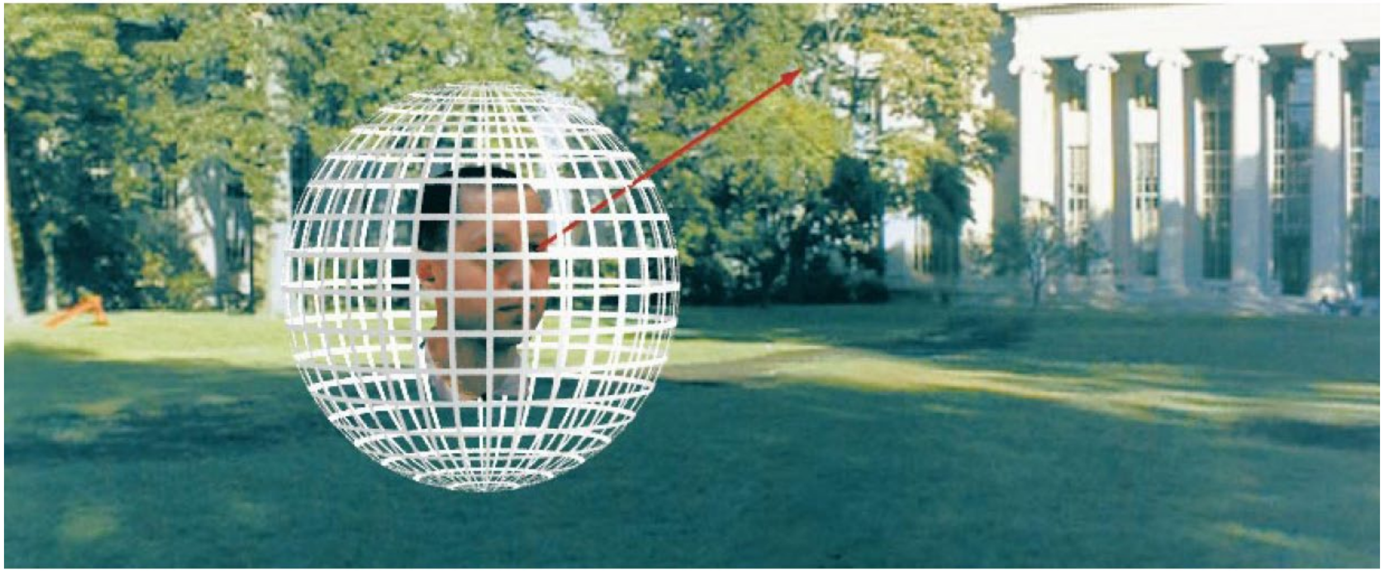
$$P(\theta,\phi)$$

is intensity of light

- Seen from a single view point
- At a single time
- Averaged over the wavelengths of the visible spectrum

(can also do *P(x,y),* but spherical coordinate are nicer)

Color snapshot

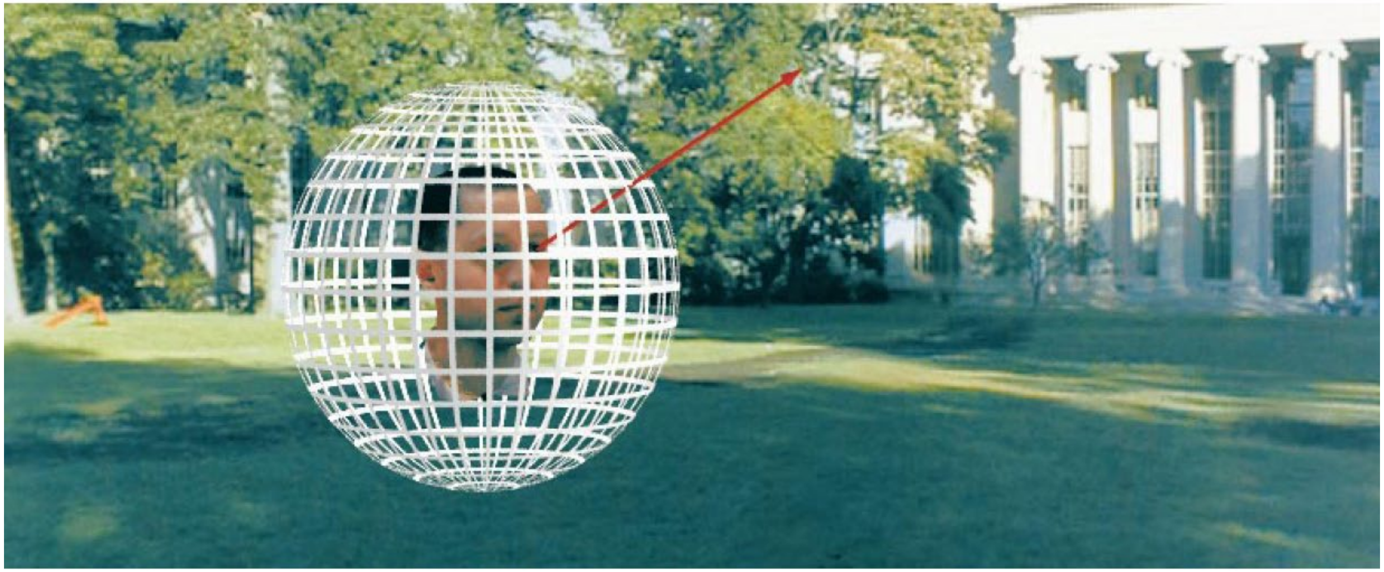


$$P(\theta,\phi,\lambda)$$

is intensity of light

- Seen from a single view point
- At a single time
- As a function of wavelength

A movie

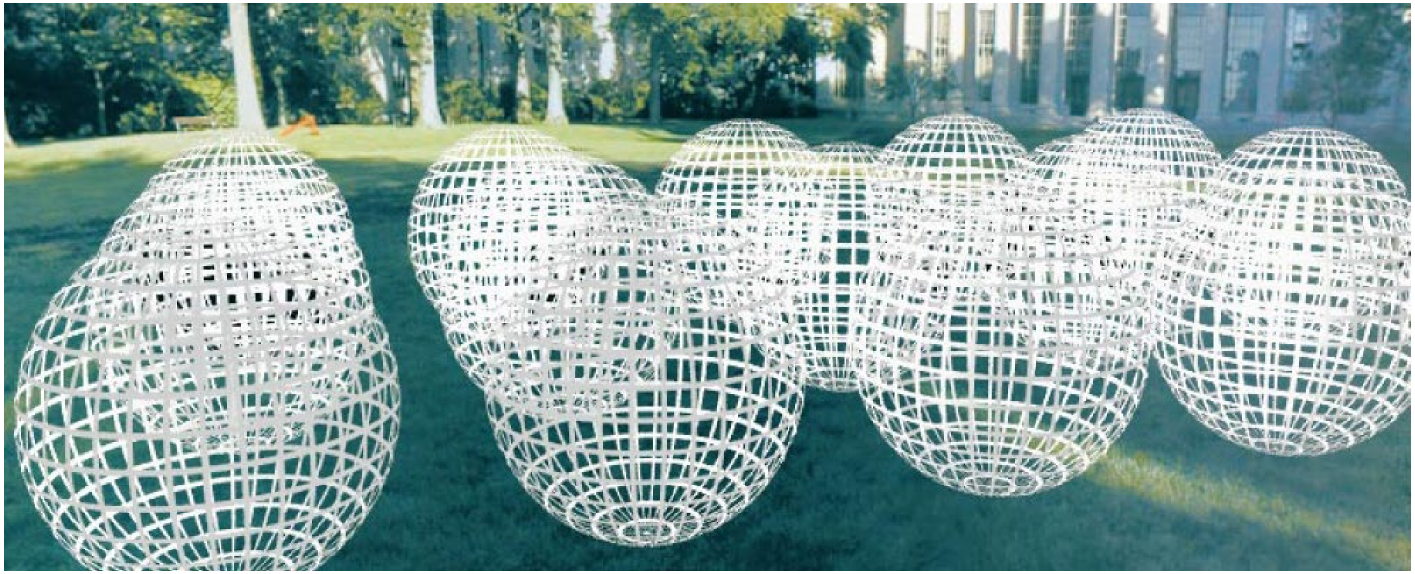


$$P(\theta,\phi,\lambda,t)$$

is intensity of light

- Seen from a single view point
- Over time
- As a function of wavelength

Holographic movie

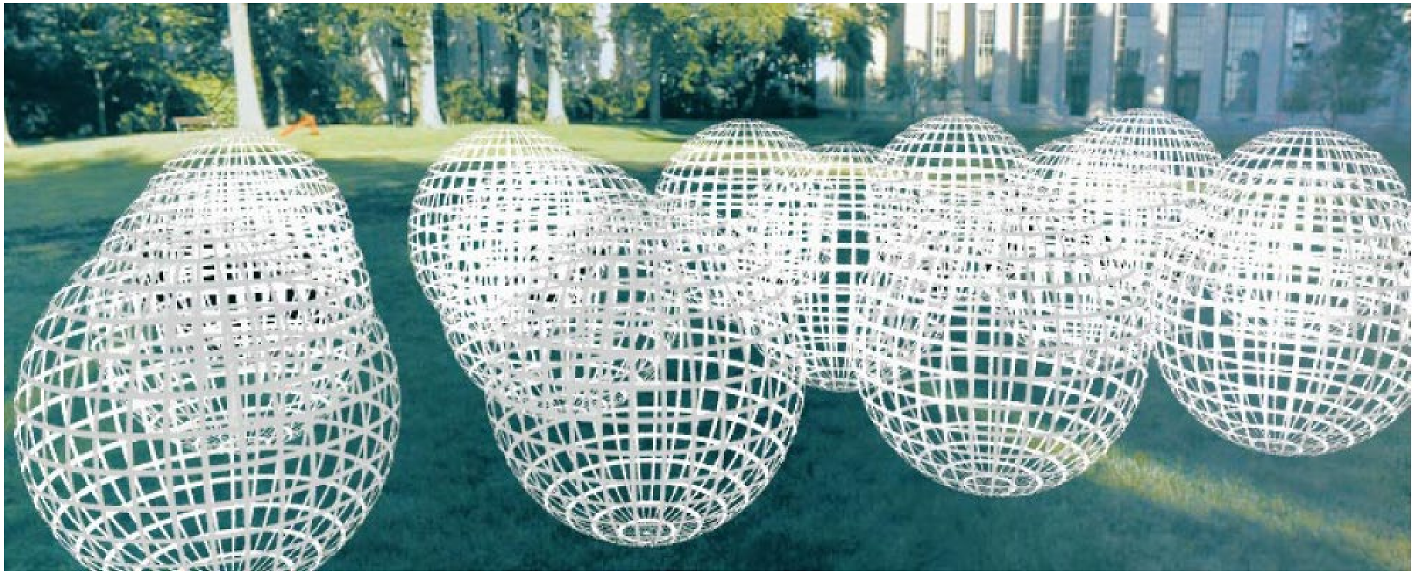


$$P(\theta,\phi,\lambda,t,V_X,V_Y,V_Z)$$

is intensity of light

- Seen from ANY viewpoint
- Over time
- As a function of wavelength

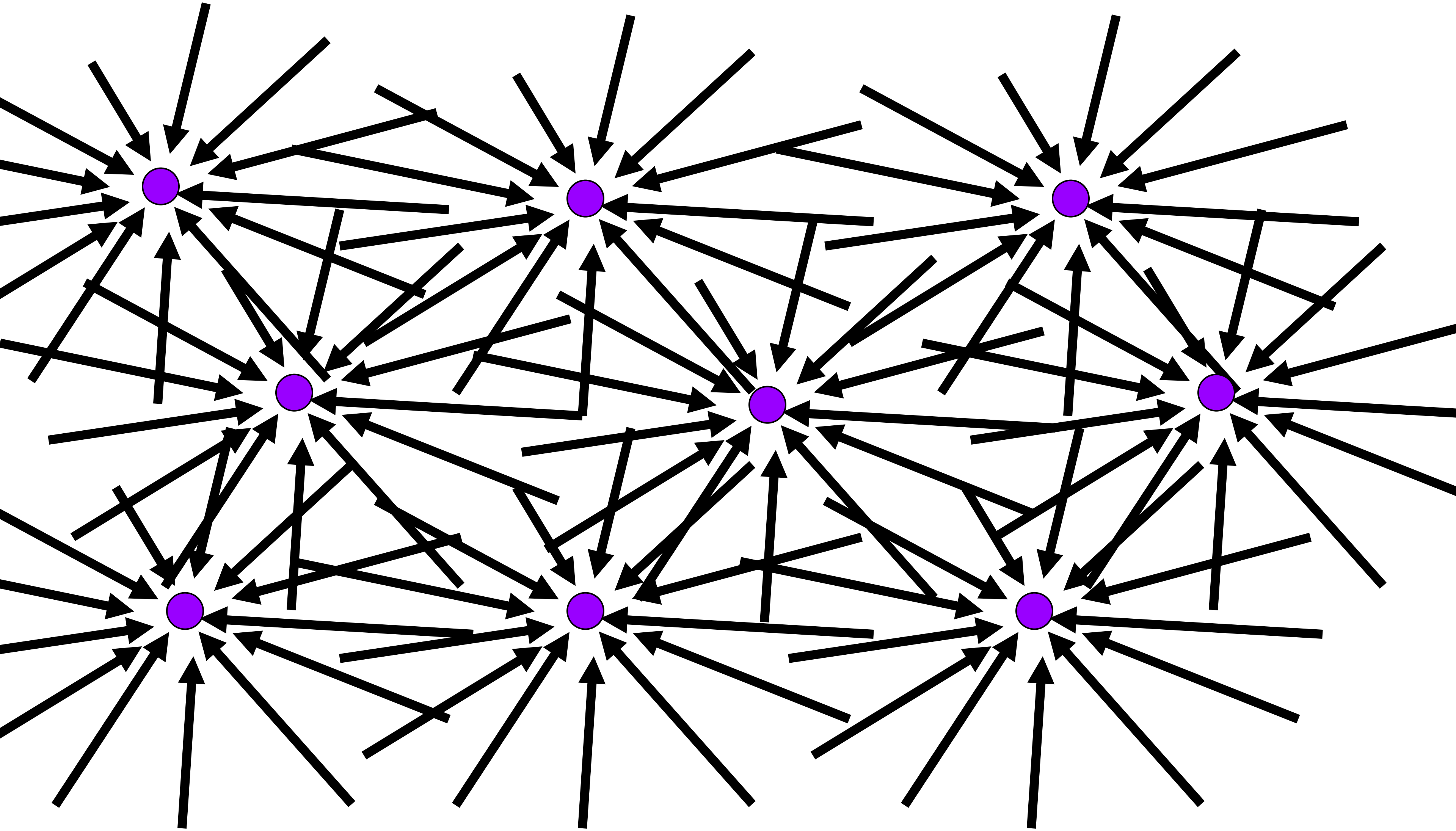
The Plenoptic Function



$$P(\theta,\phi,\lambda,t,V_X,V_Y,V_Z)$$

- Can reconstruct every possible view, at every moment, from every position, at every wavelength
- Contains every photograph, every movie, everything that anyone has ever seen! it completely captures our visual reality!   Not bad for a function…

Sampling Plenoptic Function (top view)

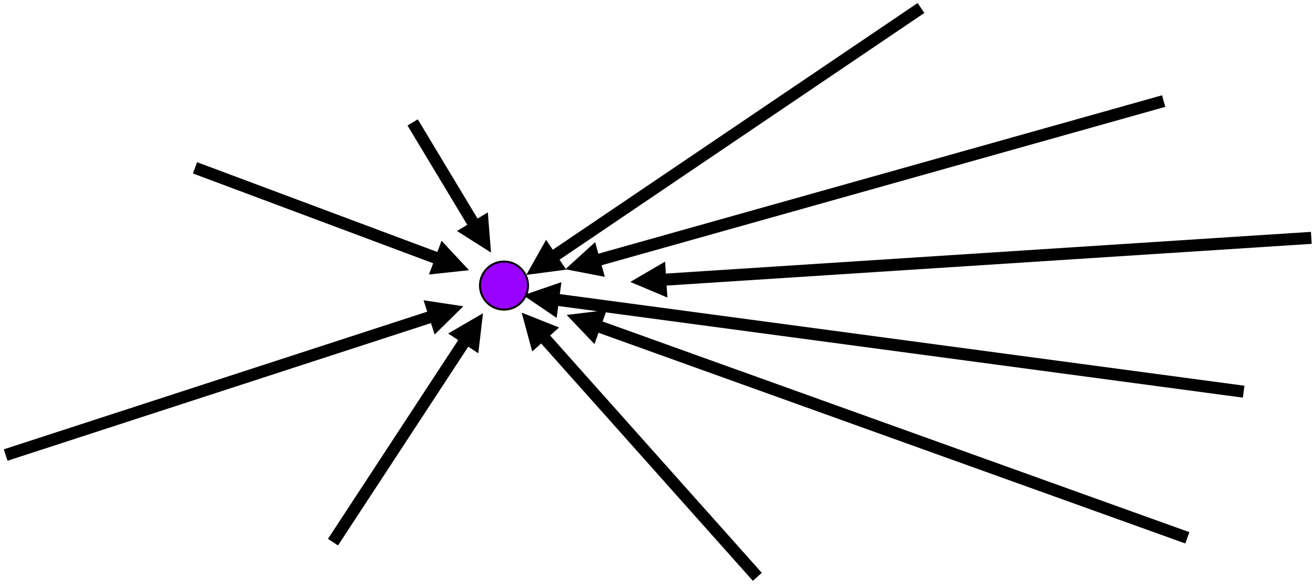


Just lookup -- Quicktime VR

QuickTime VR 1995

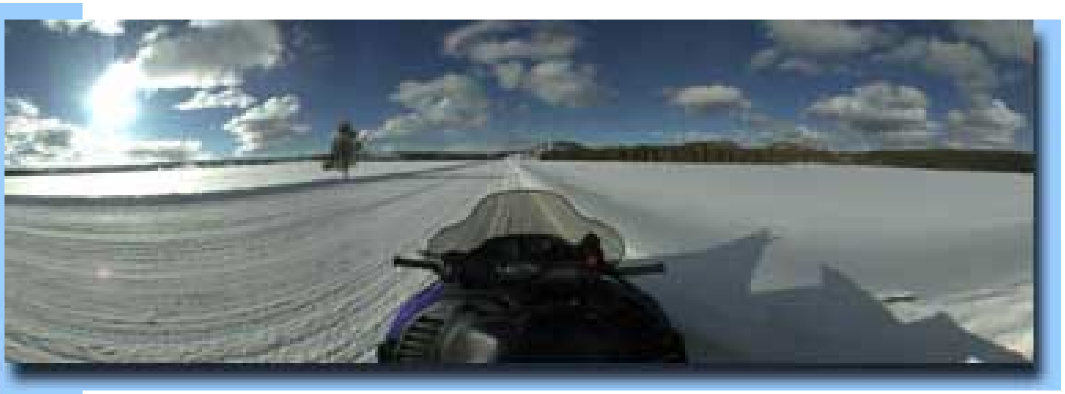
Apple Quicktime VR

What is an image?



Slide by Rick Szeliski and Michael Cohen

Spherical Panorama



All light rays through a point form a ponorama

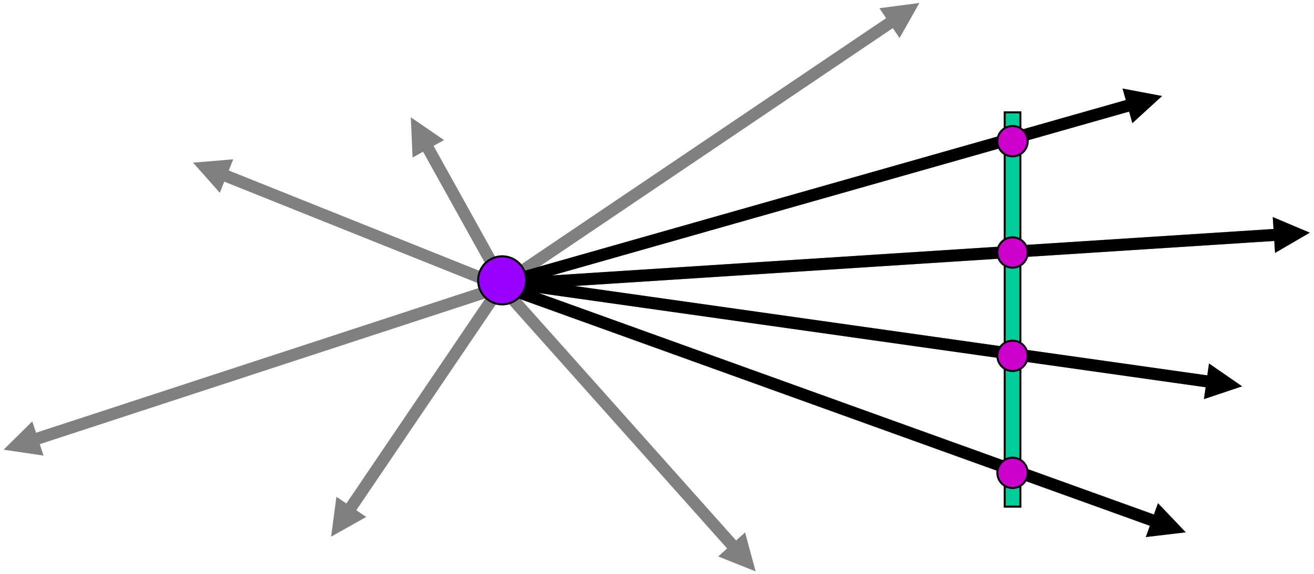
Totally captured in a 2D array -- $P(\theta,\phi)$

Where is the geometry???

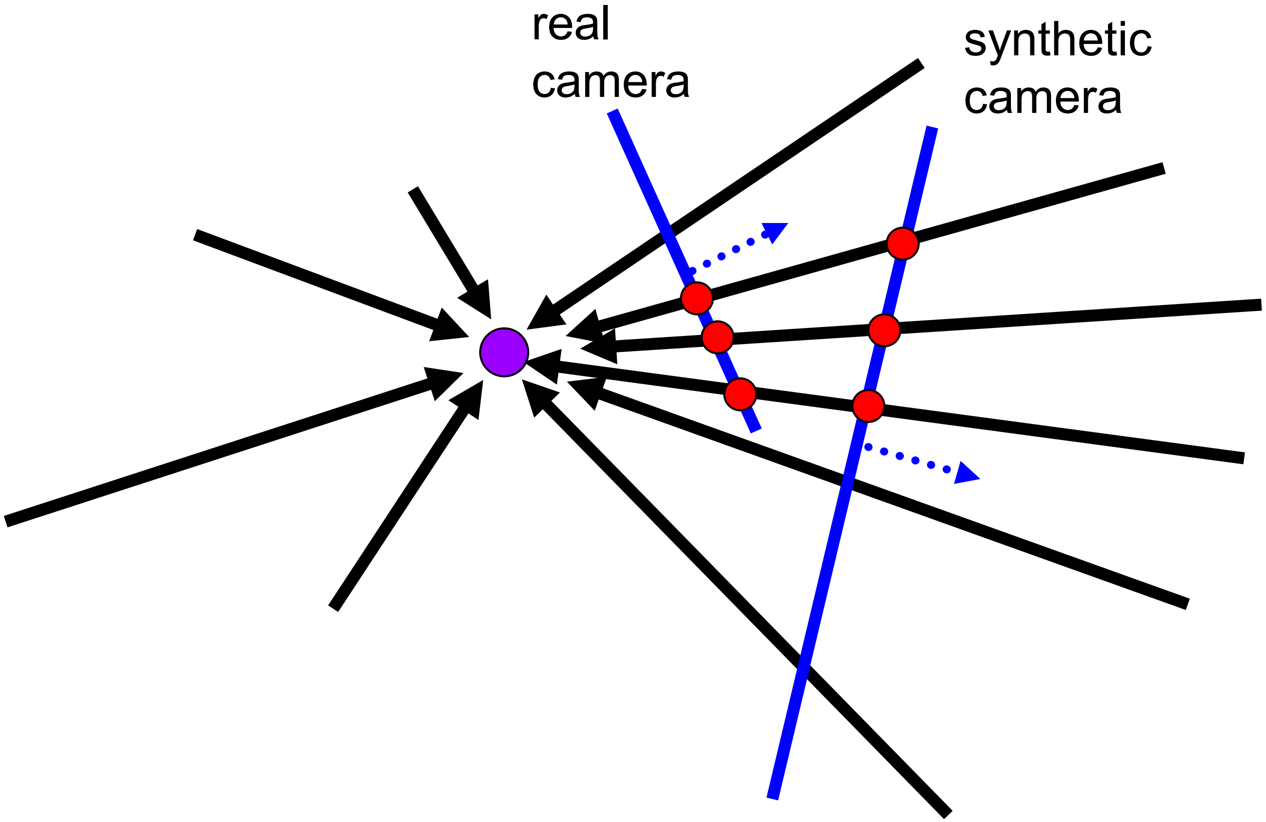
https://www.360cities.net/curated_sets/90-new-year's-eve-celebrations

https://www.360cities.net/image/new-year-in-prague-2015-2016?curated_set=90

What is an Image?



A pencil of rays contains all views

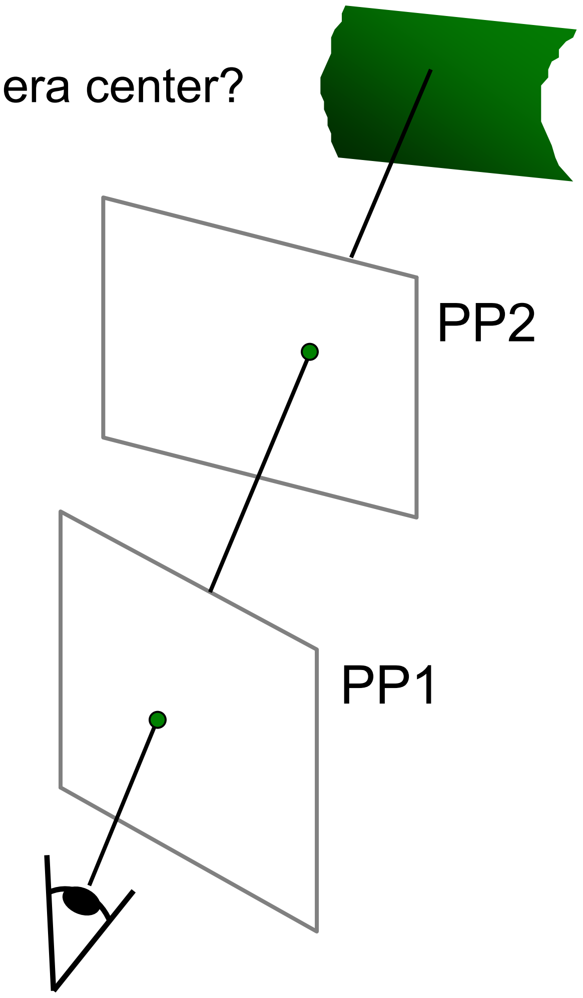


Can generate any synthetic camera view
as long as it has **the same center of projection**!

Image reprojection

Basic question

- How to relate two images from the same camera center?
  - how to map a pixel from PP1 to PP2



Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2

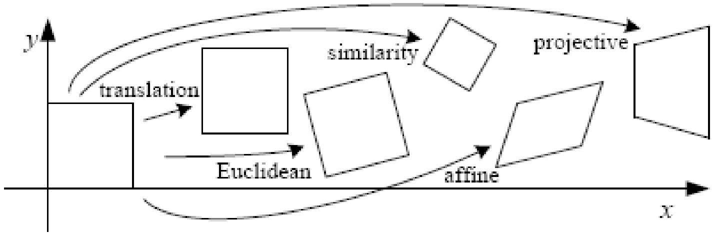
But don’t we need to know the geometry of the two planes in respect to the eye?

Observation:
Rather than thinking of this as a 3D reprojection,
think of it as a 2D **image warp** from one image to another

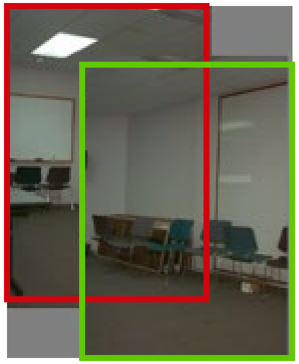
Back to Image Warping

Which t-form is the right one for warping PP1 into PP2?

e.g. translation, Euclidean, affine, projective

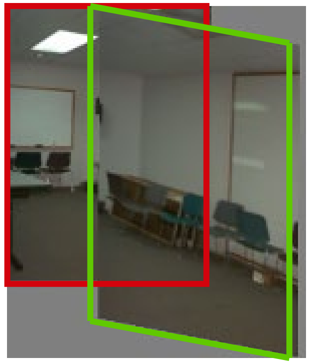


Translation



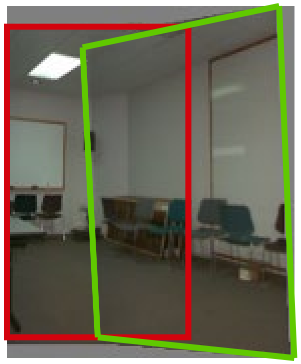
2 unknowns

Affine



6 unknowns

Perspective



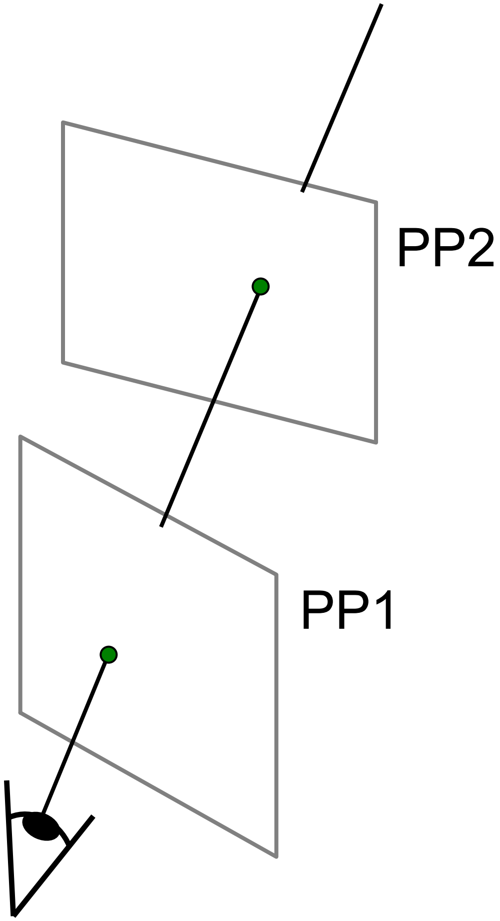
8 unknowns

Homography

A: Projective – mapping between any two PPs with the same center of projection

- rectangle should map to arbitrary quadrilateral
- parallel lines aren't
- but must preserve straight lines
- same as: unproject, rotate, reproject

called Homography



$$\underset{\mathbf{p'}}{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}} = \underset{\mathbf{H}}{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}} \underset{\mathbf{p}}{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}$$

To apply a homography **H**

- Compute **p' = Hp** (regular matrix multiply)
- Convert **p'** from homogeneous to image coordinates

Image warping with homographies

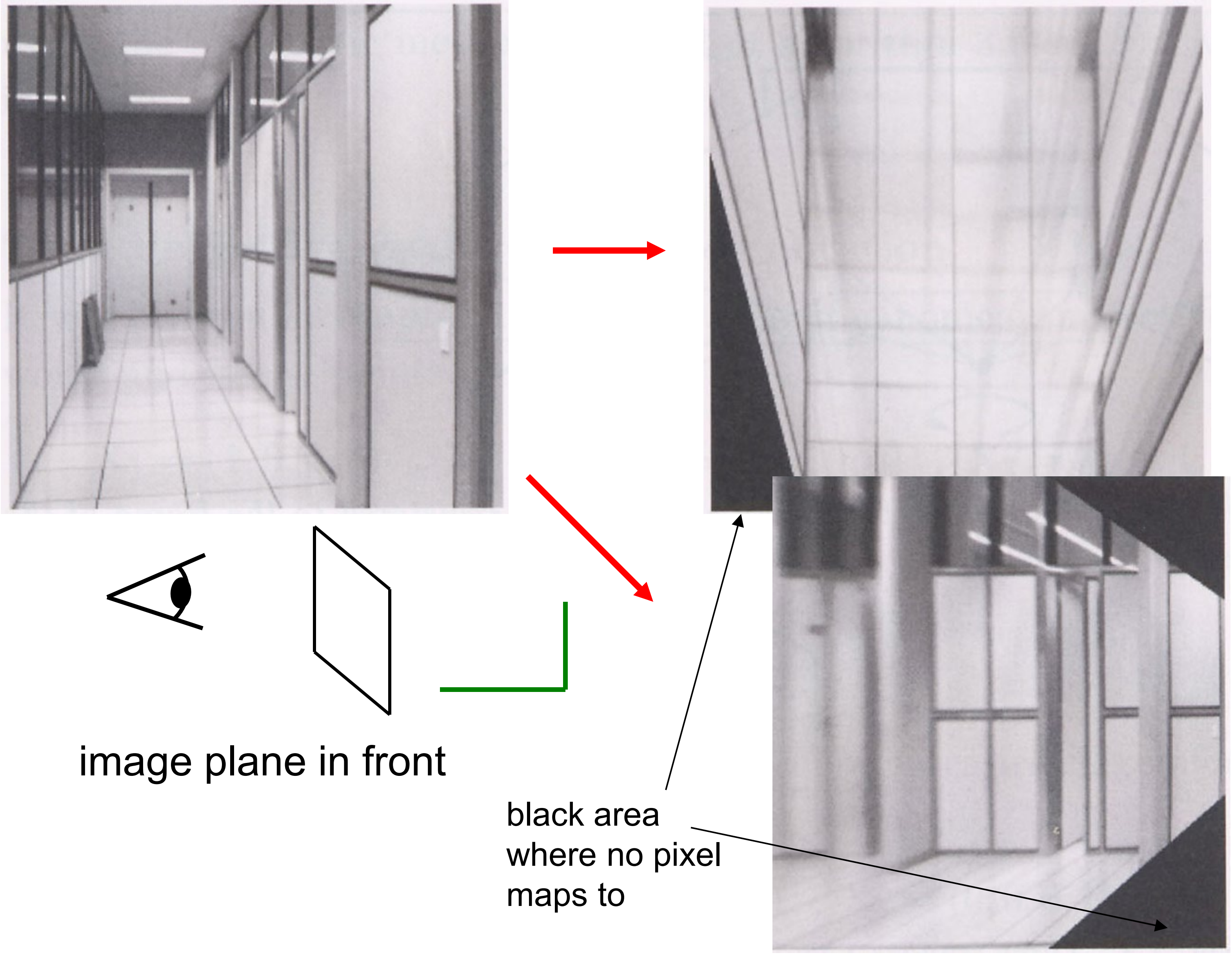
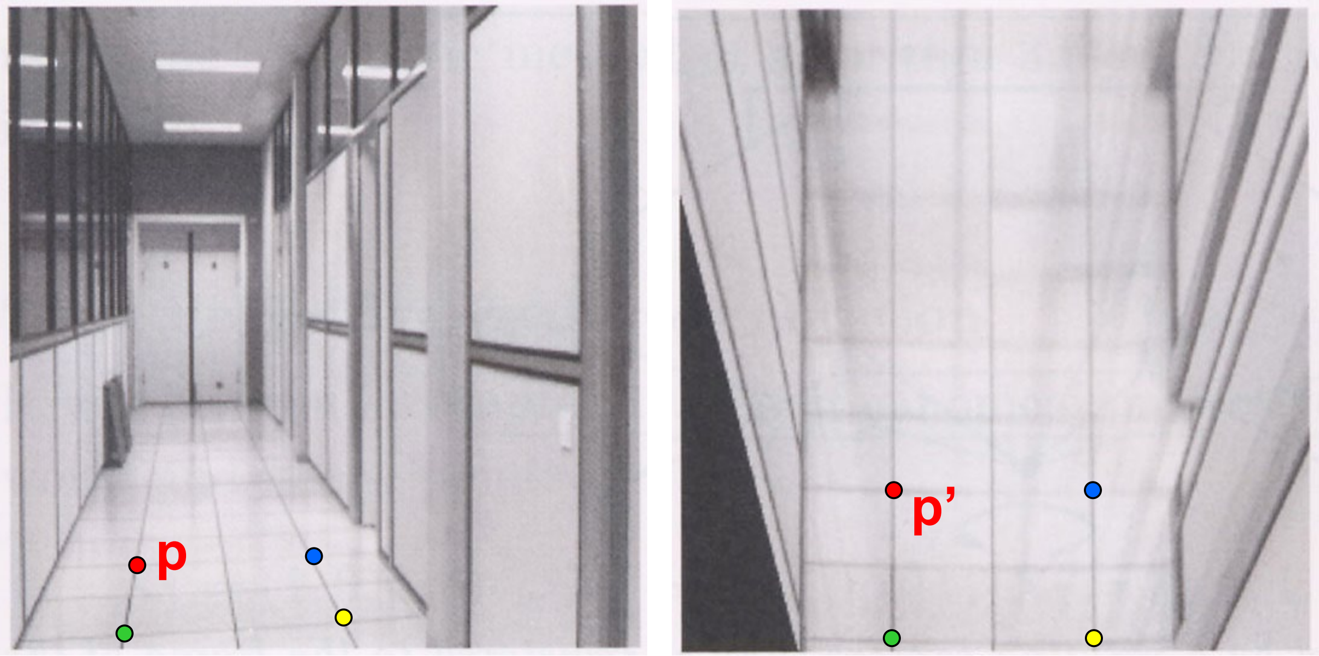


image plane in front

black area where no pixel maps to

Image rectification



To unwarp (rectify) an image

- Find the homography **H** given a set of **p** and **p’** pairs
- How many correspondences are needed?
- Tricky to write H analytically, but we can solve for it!
  - Find such H that “best” transforms points p into p’
  - Use least-squares!

Least Squares Example

Say we have a set of data points (p1,p1'), (p2,p2'), (p3,p3'), etc. (e.g. person's height vs. weight)

We want a nice compact formula (a line) to predict p' from p: $px_1 + x_2 = p'$

We want to find x_1 and x_2

How many (p,p') pairs do we need?

$$p_1x_1 + x_2 = p_1^{'}$$
$$p_2x_1 + x_2 = p_2^{'}$$

$$\begin{bmatrix} p_1 & 1 \\ p_2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p_1^{'} \\ p_2^{'} \end{bmatrix}$$

$$\mathrm{Ax} = b$$

Least Squares Example

Say we have a set of data points (p1,p1'), (p2,p2'), (p3,p3'), etc. (e.g. person's height vs. weight)

We want a nice compact formula (a line) to predict p' from p: $px_1 + x_2 = p'$

We want to find x_1 and x_2

How many (p,p') pairs do we need?

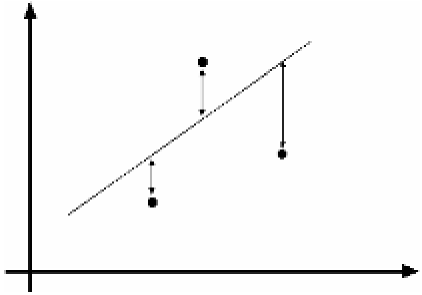
$$p_1 x_1 + x_2 = p_1^{'}$$
$$p_2 x_1 + x_2 = p_2^{'}$$

$$\begin{bmatrix} p_1 & 1 \\ p_2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p_1^{'} \\ p_2^{'} \end{bmatrix} \qquad \mathrm{Ax} = b$$

What if the data is noisy?

$$\begin{bmatrix} p_1 & 1 \\ p_2 & 1 \\ p_3 & 1 \\ \cdots & \cdots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p_1^{'} \\ p_2^{'} \\ p_3^{'} \\ \cdots \end{bmatrix}$$

$$\min \| Ax - b \|^2$$



overconstrained

Least-Squares

- Solve:

$$A\,\mathbf{x} = \mathbf{b}$$

$$(N,d)(d,1) = (N,1)$$

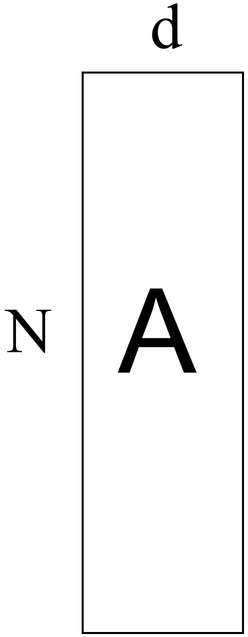
- Normal equations

$$A^TA\,\mathbf{x} = A^T\mathbf{b}$$

$$(d,N)(N,d)(d,1) = (d,N)(N,1)$$

- Solution:

$$\mathbf{x} = (A^TA)^{-1}A^T\mathbf{b}$$



$\mathrm{rank}(A) \le \min(d,N)$
assume $\mathrm{rank}(A)=d$
implies $\mathrm{rank}(A^TA)=d$
A^TA is invertible

Solving for homographies

p' = Hp

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Can set scale factor *i*=1. So, there are 8 unkowns.

Set up a system of linear equations:

Ah = b

where vector of unknowns h = [a,b,c,d,e,f,g,h]^T

Need at least 8 eqs, but the more the better…

Solve for h. If overconstrained, solve using least-squares:

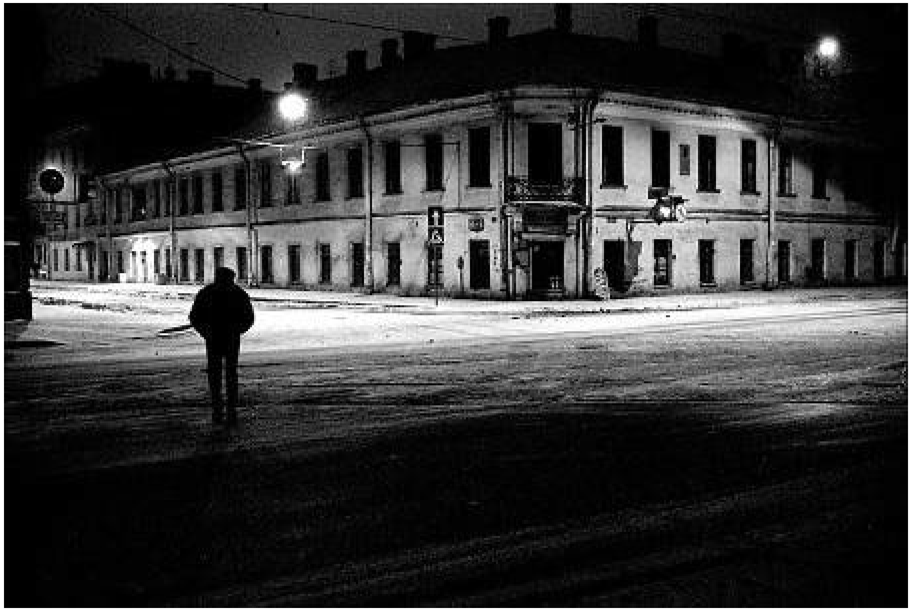
$$\min \left\| Ah - b \right\|^2$$

Can be done in Matlab using "\" command

- see "help lmdivide"

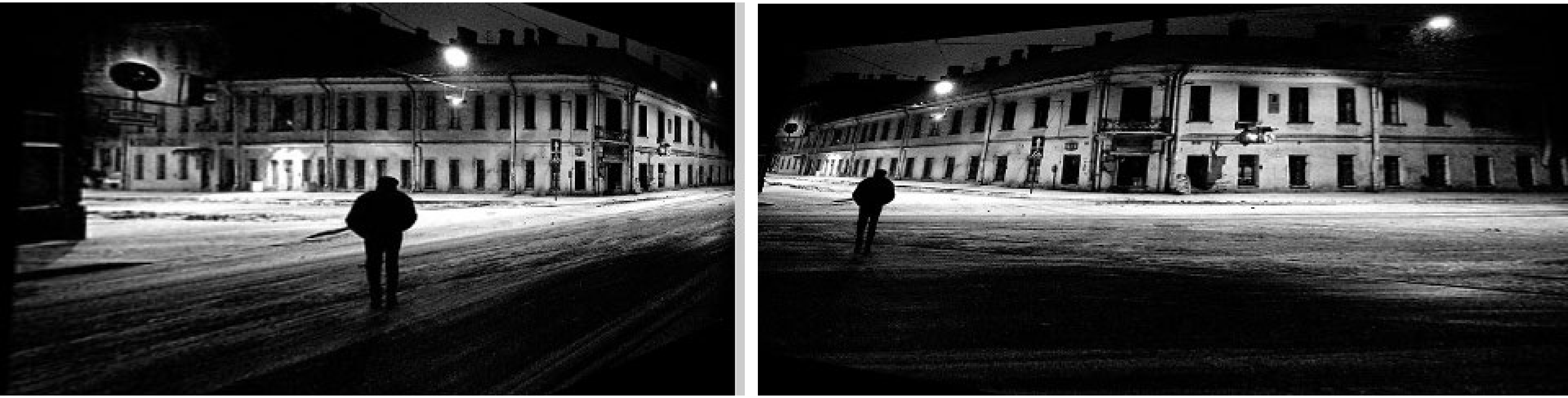
Image rectification

Original image



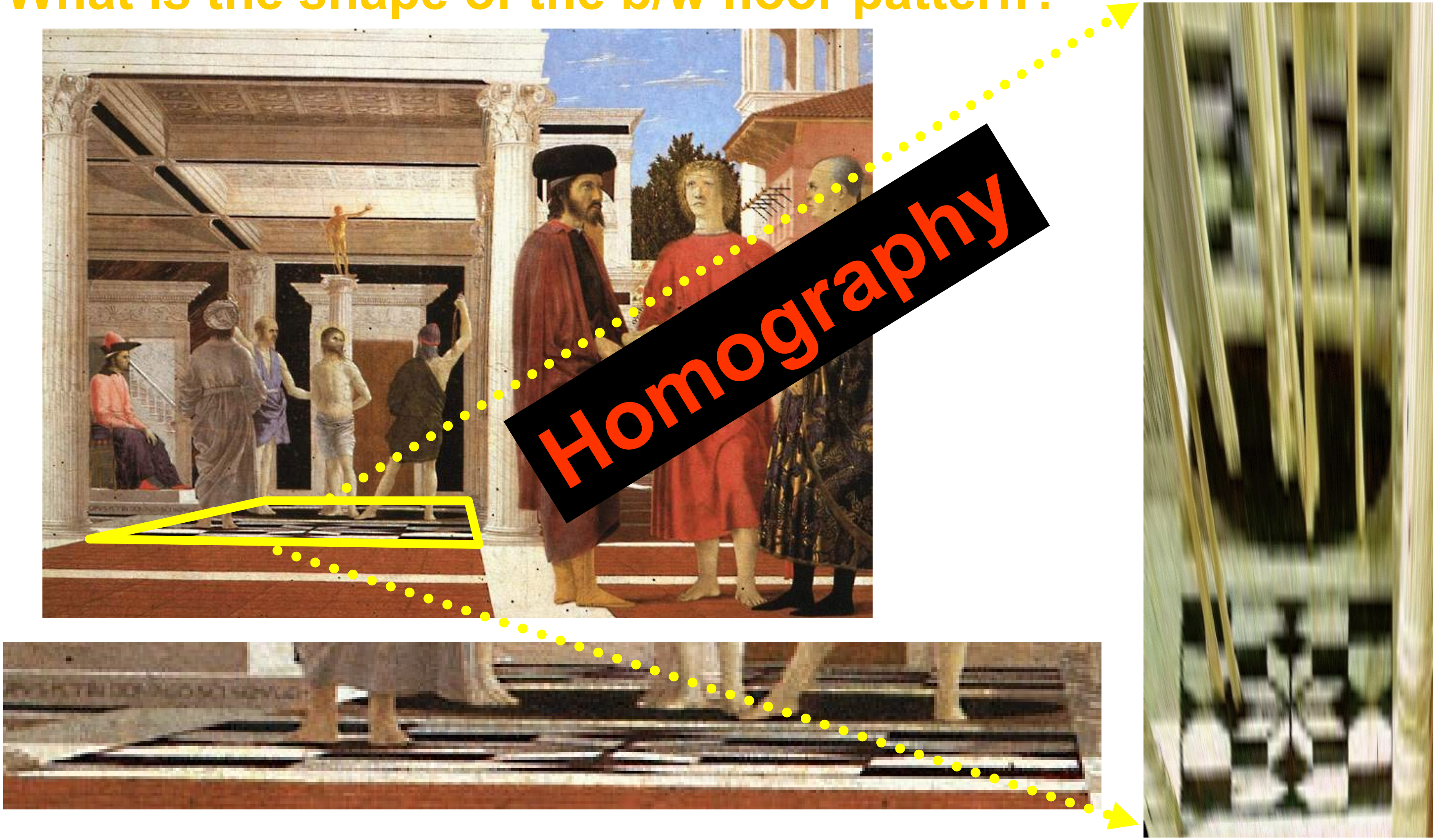
St.Petersburg
photo by A. Tikhonov

Virtual camera rotations



Rectification for Art Analysis

What is the shape of the b/w floor pattern?



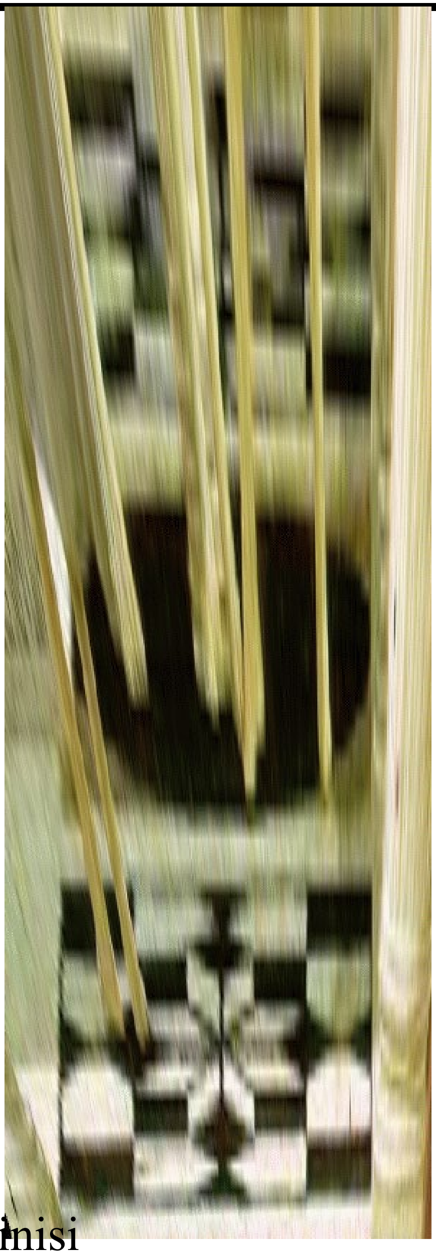
The floor (enlarged)

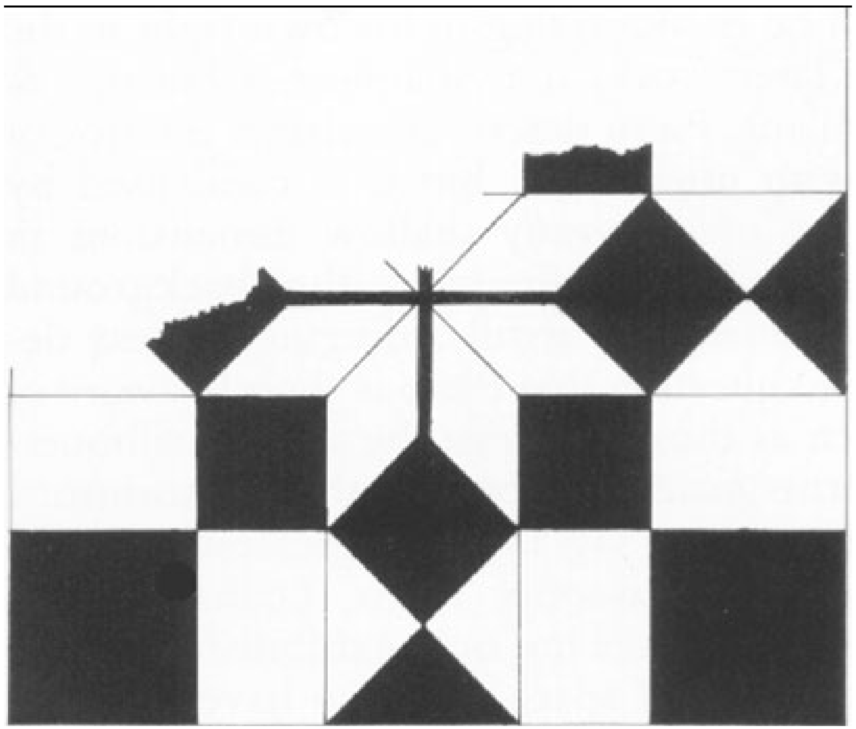
Automatically rectified floor

Slide from Criminisi

Analysing patterns and shapes

Automatic rectification



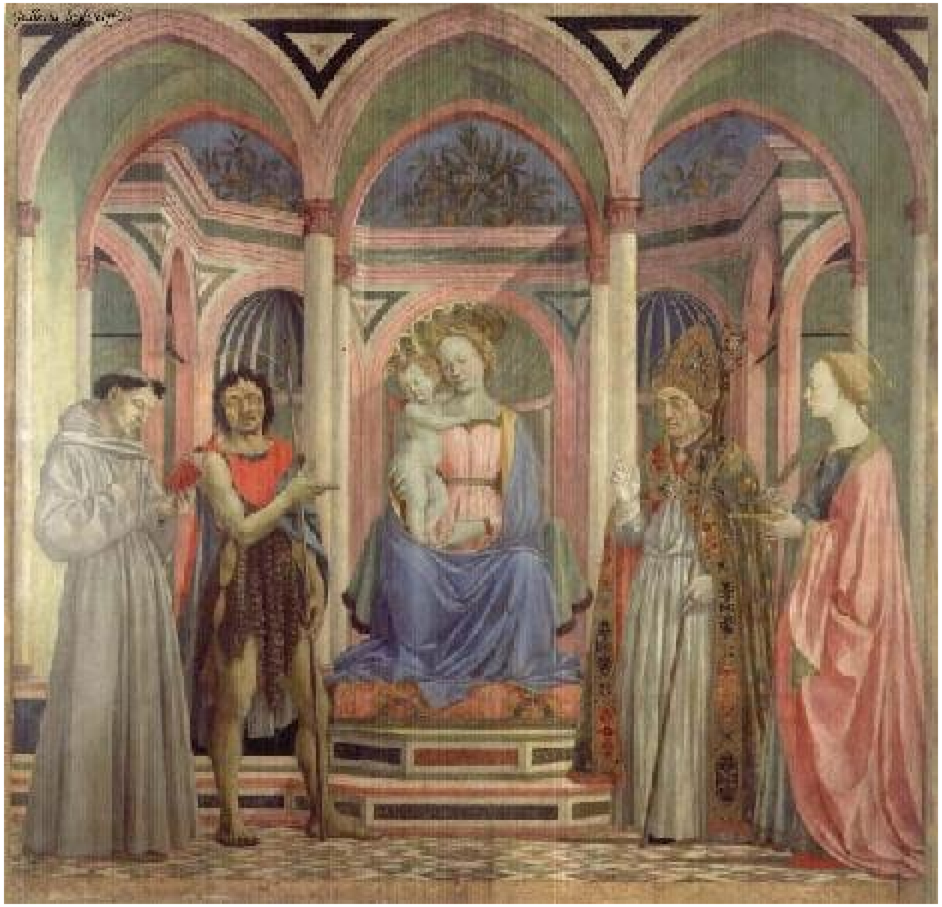


From Martin Kemp *The Science of Art* *(manual reconstruction)*

2 patterns have been discovered !

Slide from Criminisi

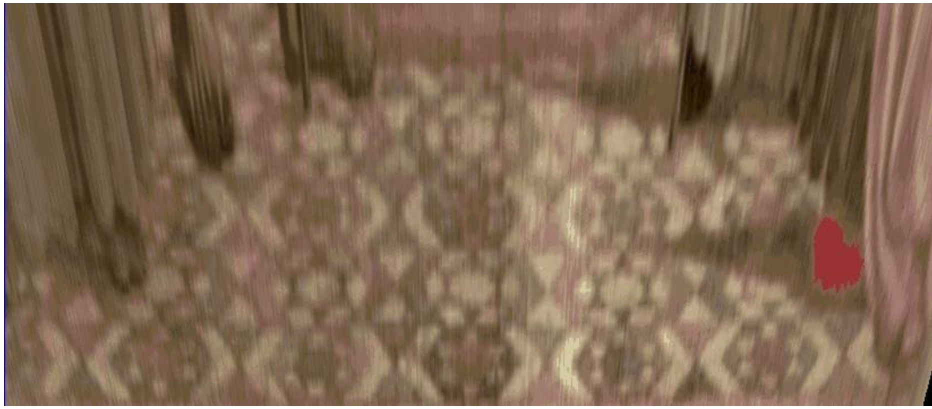
Analysing patterns and shapes



St. Lucy Altarpiece, **D. Veneziano**

Slide from Criminisi

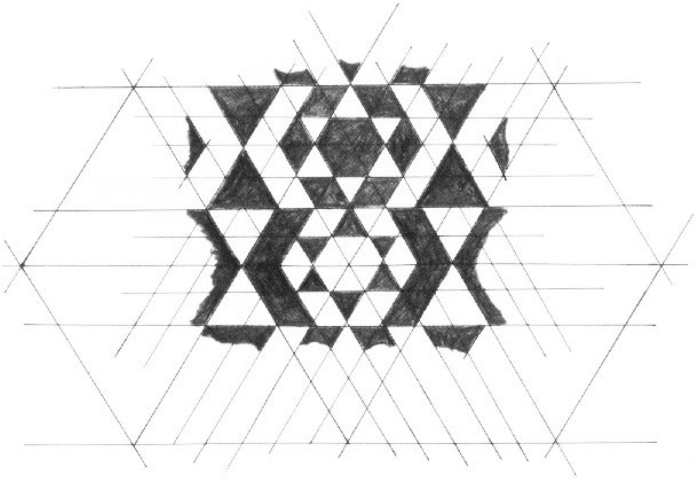
What is the (complicated) shape of the floor pattern?



Automatically rectified floor

Analysing patterns and shapes

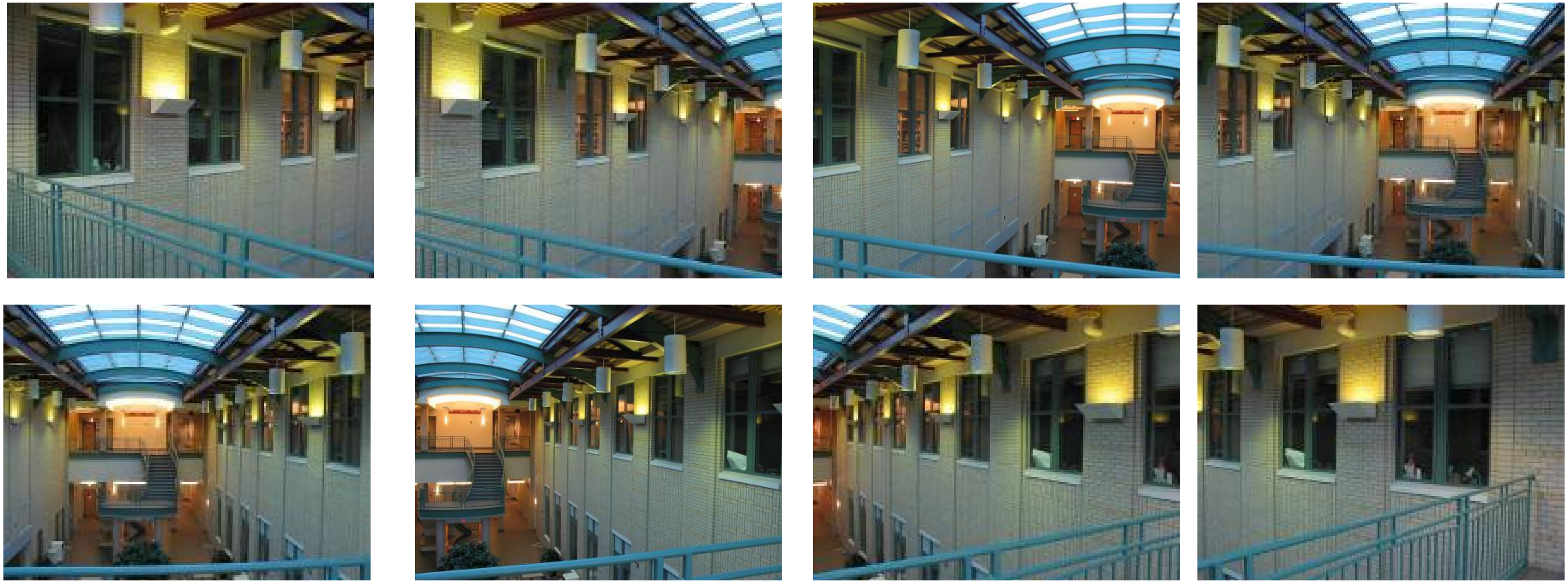
Automatic rectification

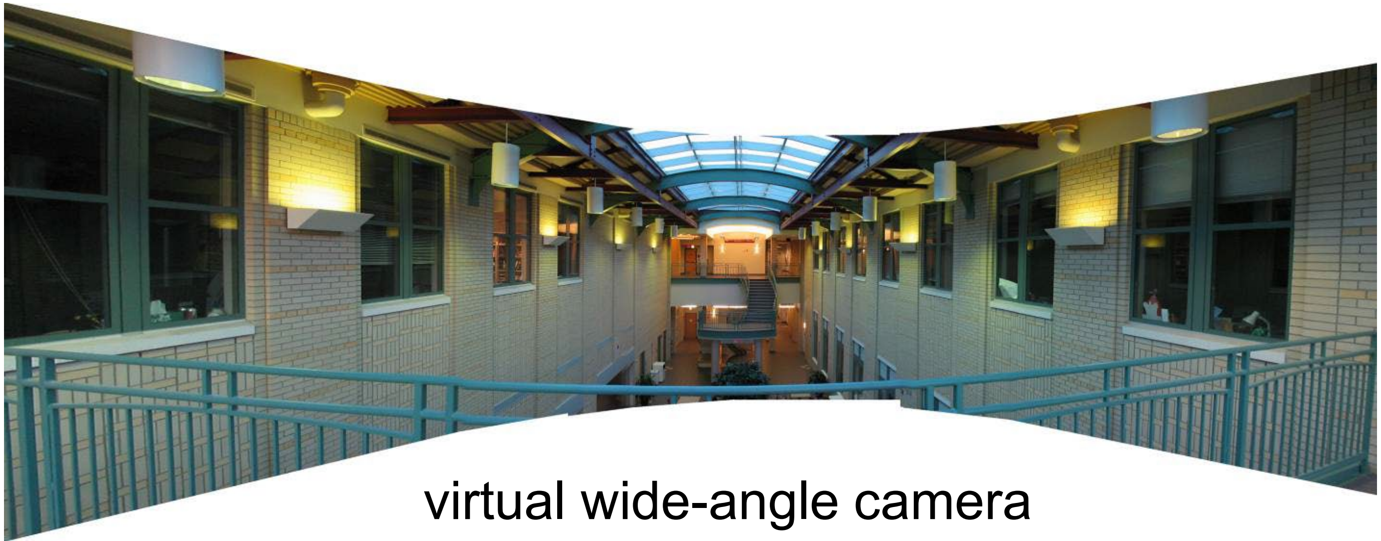


From Martin Kemp, *The Science of Art* *(manual reconstruction)*

Slide from Criminisi

Mosaics: stitching images together





virtual wide-angle camera

Why Mosaic?

Are you getting the whole picture?

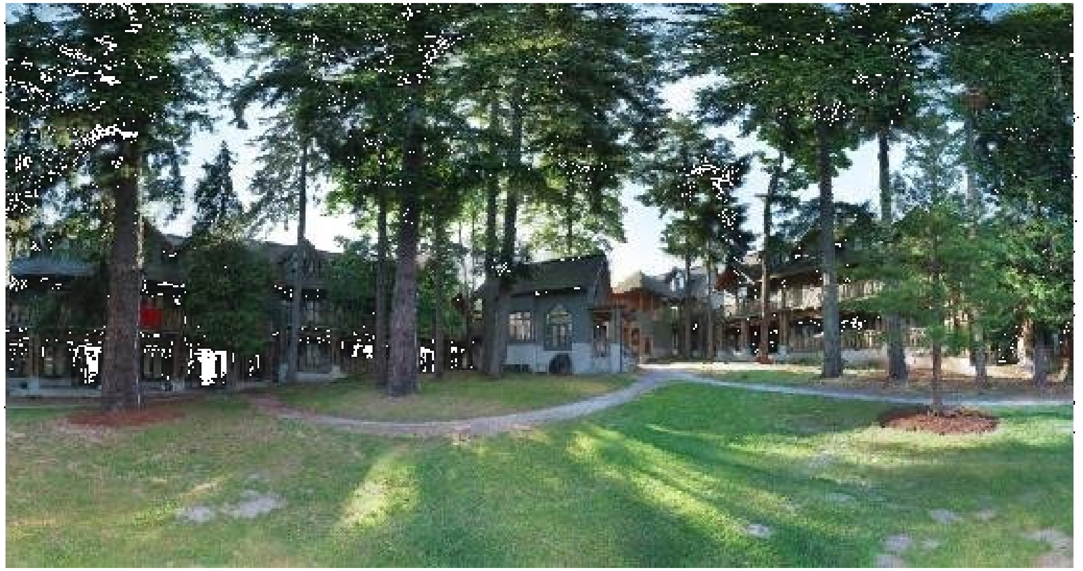
- Compact Camera FOV = 50 x 35°

Slide from Brown & Lowe

Why Mosaic?

Are you getting the whole picture?

- Compact Camera FOV = 50 x 35°
- Human FOV = 200 x 135°



Slide from Brown & Lowe

Why Mosaic?

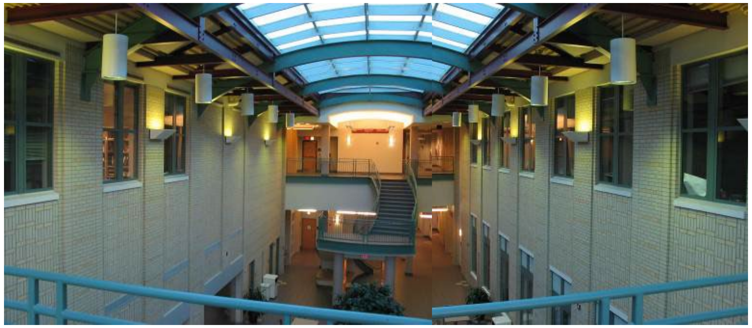
Are you getting the whole picture?

- Compact Camera FOV = 50 x 35°
- Human FOV = 200 x 135°
- Panoramic Mosaic = 360 x 180°

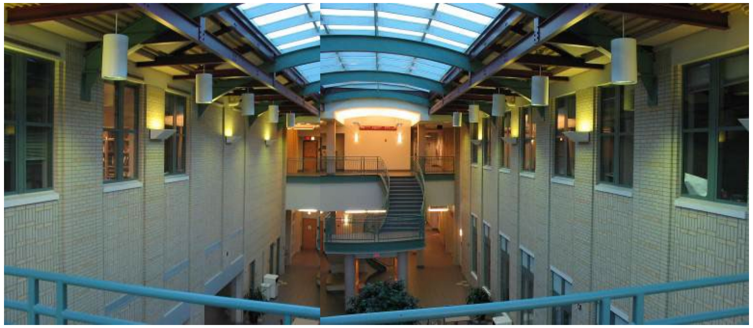
Slide from Brown & Lowe

Naïve Stitching

left on top



right on top



Translations are not enough to align the images

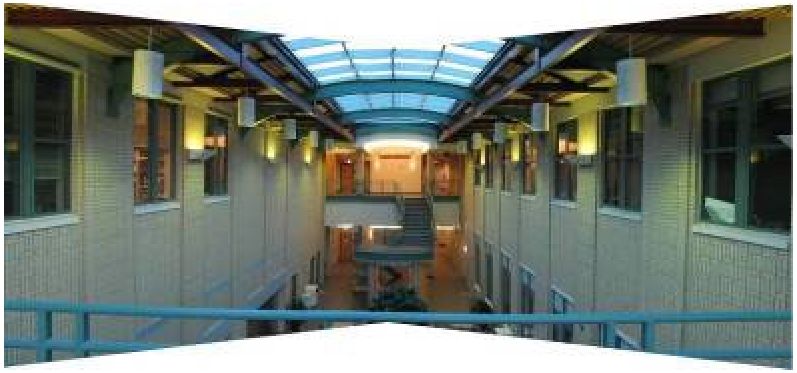
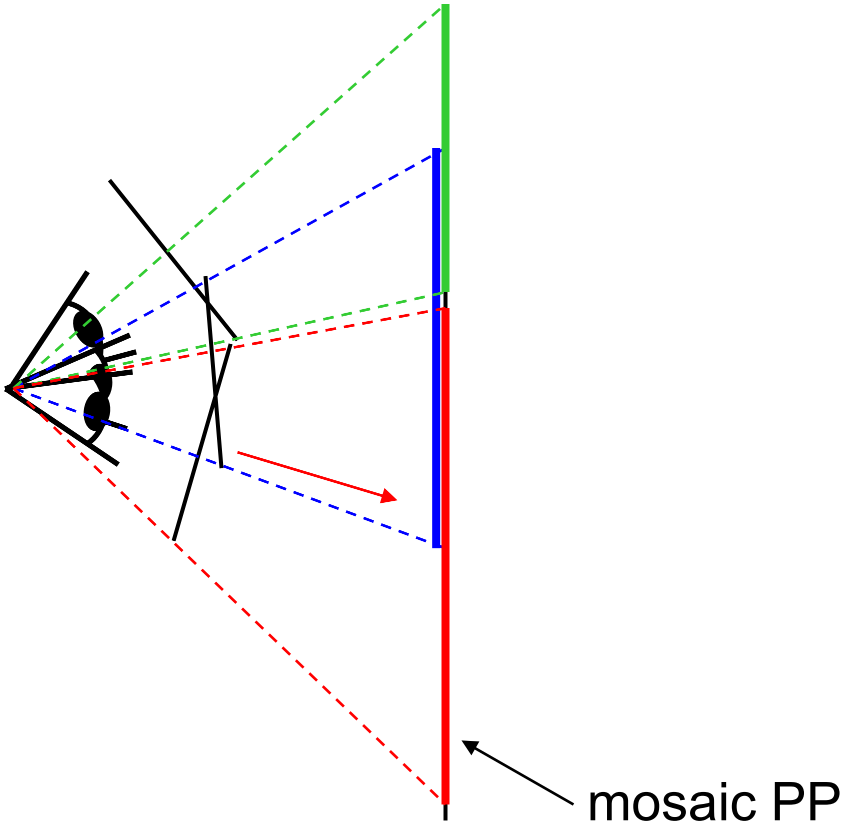


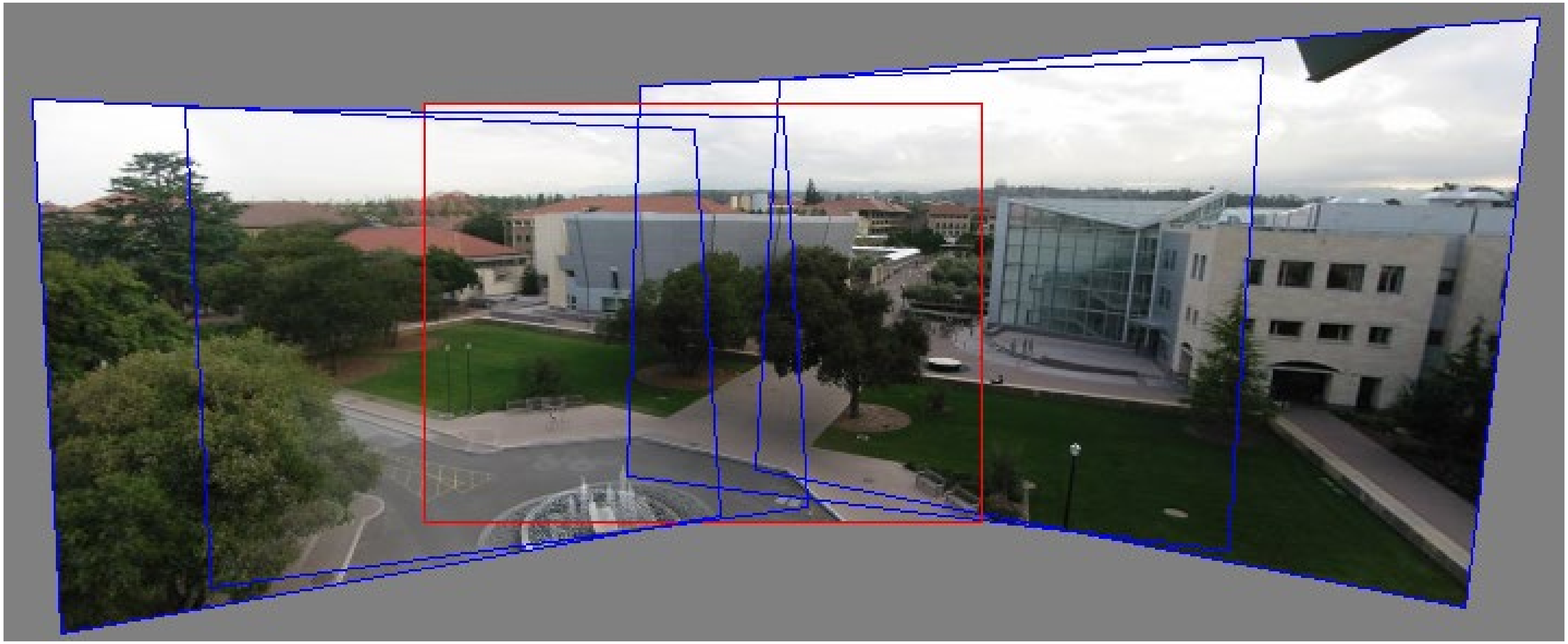
Image reprojection



The mosaic has a natural interpretation in 3D

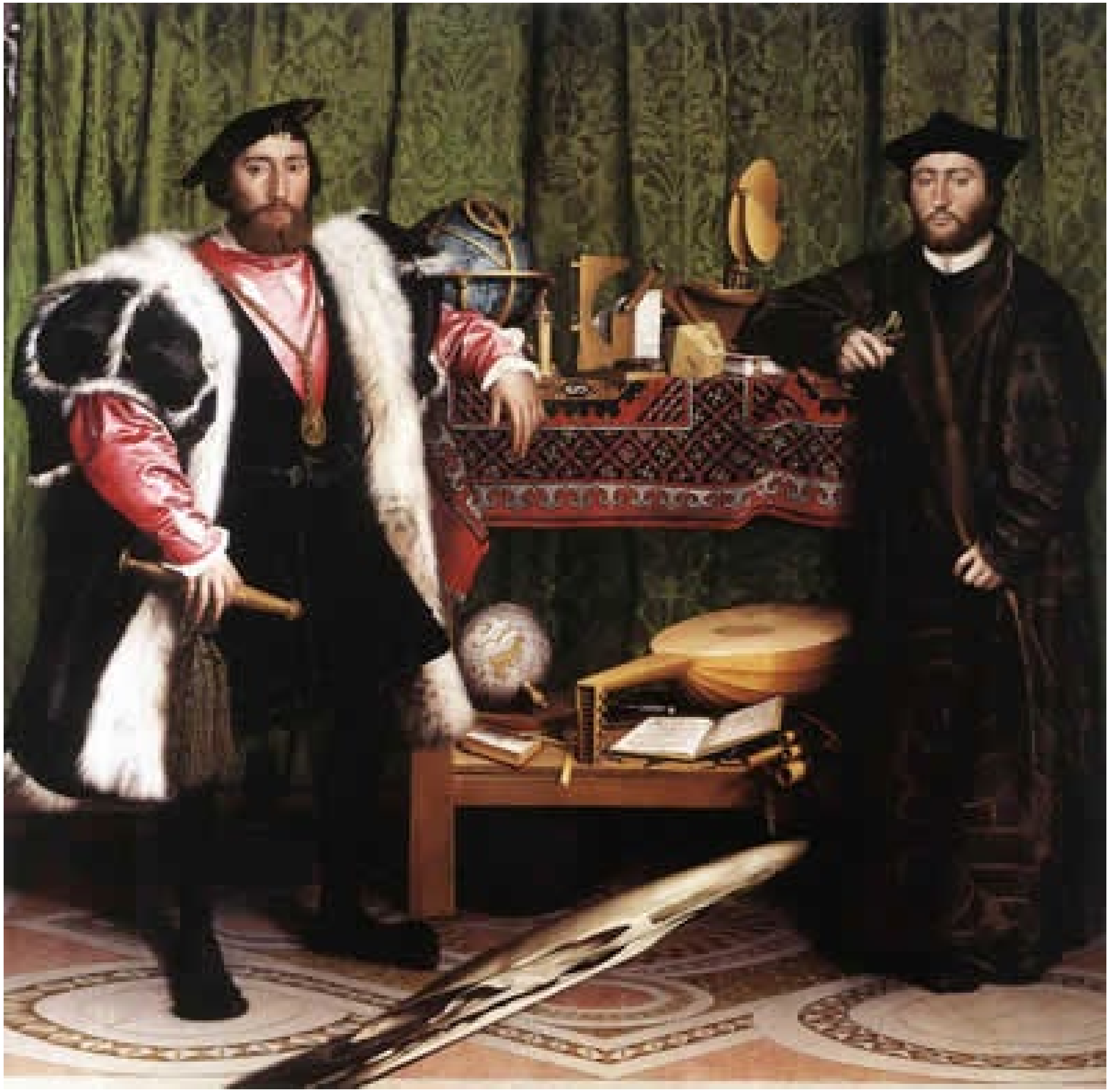
- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*

Panoramas

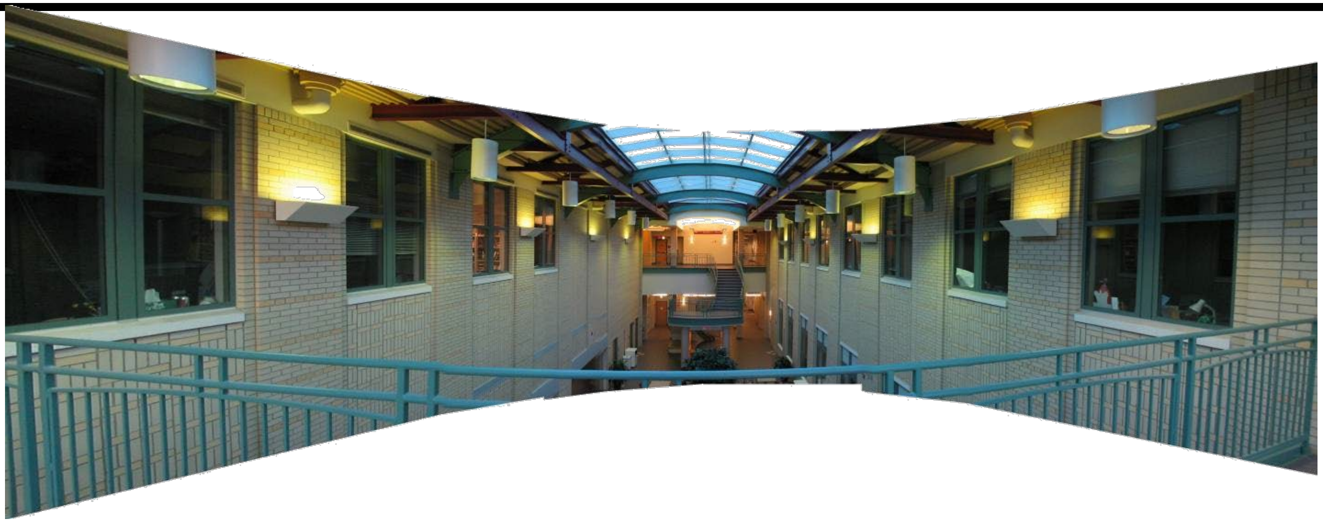


1. Pick one image (red)
2. Warp the other images towards it (usually, one by one)
3. blend

Holbein, *The Ambassadors*



Programming Project #4 (part 1)



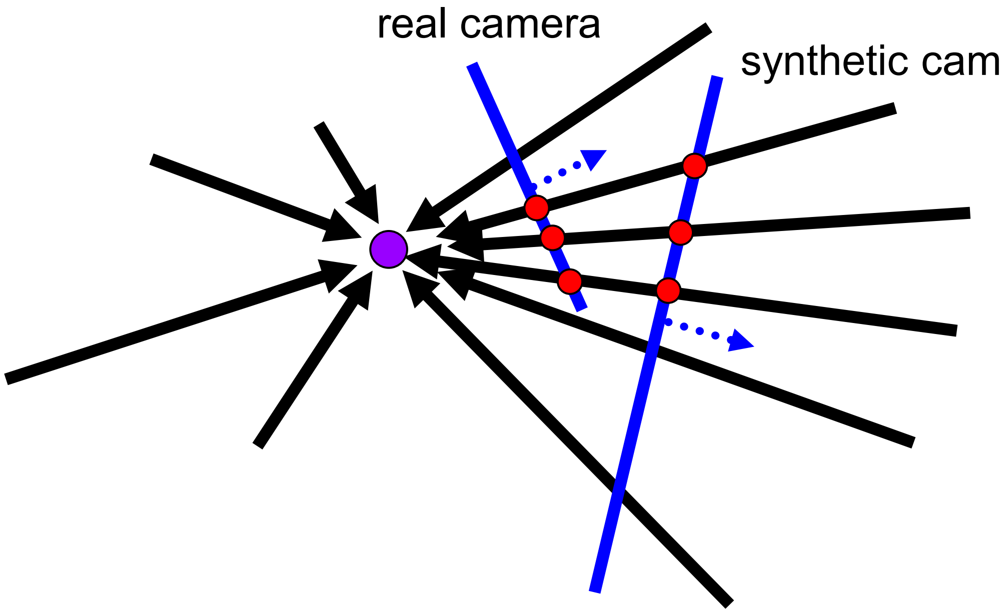
Homographies and Panoramic Mosaics

- Capture photographs (and possibly video)
  - Might want to use tripod
- Compute homographies (define correspondences)
  - will need to figure out how to setup system of eqs.
- (un)warp an image (undo perspective distortion)
- Produce panoramic mosaics (with blending)
- Do some of the Bells and Whistles

Example homography final project

Think about this: When is this not true?

We can generate any synthetic camera view as long as it has **the same center of projection**!



What happens if there are two center of projection?
(you move your head)