

# Flow Matching

Discussion #10

Written by:  
Justin, Konpat

## 1 Warmup: Noise Shapes

At their core, diffusion and flow matching models sample from a base probability *distribution* (usually Gaussian noise) and transport it to a target data distribution.

Note: In this sheet we use the “flow” formulation. Don’t confuse it with the “diffusion” one!

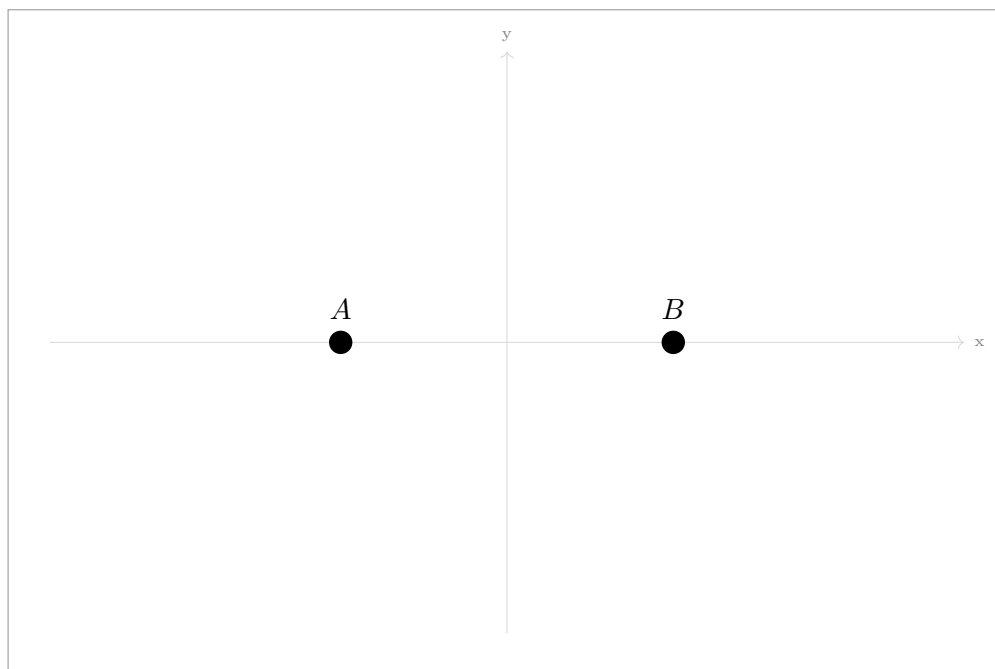
For each task below, state the shape of the input noise  $x_0$  (a.k.a.  $x_{\text{noise}}$ ).

0) Generating a grayscale,  $1920 \times 1080$  image

1) Generating a random point  $(x, y)$  within a square

## 2 Visualizing Flow in 2D

We want to train a flow matching model to generate samples from a data distribution consisting of exactly two points,  $A = (-1, 0)$  and  $B = (1, 0)$  (50% probability each).



**1.0) Define the noise region.** Sketch in the plot above the contour of  $\mathcal{N}(0, 1)$  at 1-std deviation (to roughly denote the high probability region of the noise).

**Conditional Flow** During each iteration of diffusion model training, we select a data point  $d$  (either A or B) at random and combine it with random noise  $r$  according to timestep  $t \in [0, 1]$  like  $x_t = d \cdot t + (1 - t) \cdot r$ .

**1.1** Pick **one** “random”  $x_0$  (full noise) and **plot** them in the chart.

**1.2** Say  $x_1 = A$ , **plot**  $x_{0.9}$ .

**1.3** For all point  $x_t$ ’s, **draw** the velocity vector  $(x_{\text{clean}} - x_{\text{noise}})$  at each point.

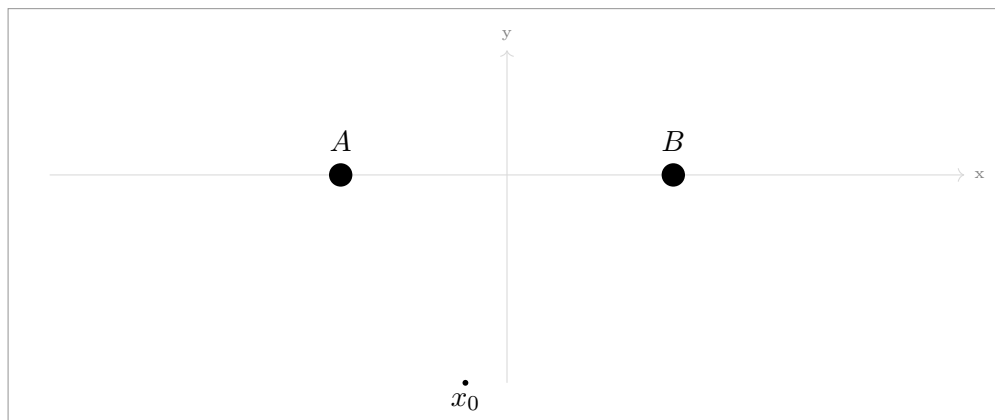
**Marginal Flow.** Our diffusion model models marginal flow, which for a given  $x_t$  is the **weighted average** over all conditional flows from the datapoints we train on.

**1.4** For the point  $x_0$ , **plot** marginal flows (roughly).

**1.5** For the point  $x_{0.9}$ , **plot** marginal flows (roughly).

**1.6** In English, explain the direction of the marginal flow at  $t = 0$  and  $t = 0.9$ .

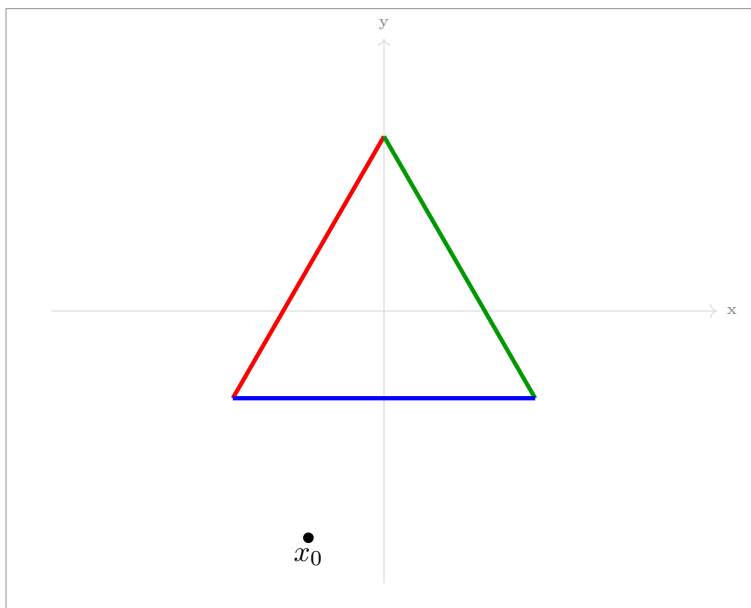
**1.7** Start from  $x_0$ , draw out the flow sampling procedure with 4 sampling steps. (Hint: follow the marginal flow at each step)



**1.8** What if I wanted to generate samples of point  $A$  twice as frequently as point  $B$ ? How could I modify the standard training pipeline to achieve this?

### 3 Conditioning and Guidance

Now suppose we want to draw from a distribution corresponding to sides of a triangle (think of this like our little image manifold). In addition, we want to *control* what type of point our model generates: **red**, **green**, or **blue**. (not the color, but the location)



**3.1 Adding control.** The most common way to control diffusion models is by additional *conditioning* input, like language, to the model.

Explain how one could sample from a specific color without additional model input by changing training.

Classifier-Free Guidance (CFG) modifies the vector field to nudge samples more towards red compared to an unconditional sample:

$$\tilde{u}_\theta(x, t, c) = u_\theta(x, t, \emptyset) + w \cdot (u_\theta(x, t, \text{red}) - u_\theta(x, t, \emptyset))$$

where  $u_\theta$  is our neural network model, which learns marginal flow.

**3.2 Prompted Flow: Draw** in the diagram where  $u_\theta(x, t, \text{red})$  and  $u_\theta(x, t, \text{green})$  point.

**3.3 Unprompted Flow: Draw** in the diagram where  $v(x_0, t, \emptyset)$  points

**3.4 Guidance: Draw** the CFG vector  $\tilde{u}_\theta$  for a large  $w$ . How does it differ from the standard prompted vector?