Image Formation

Discussion #2

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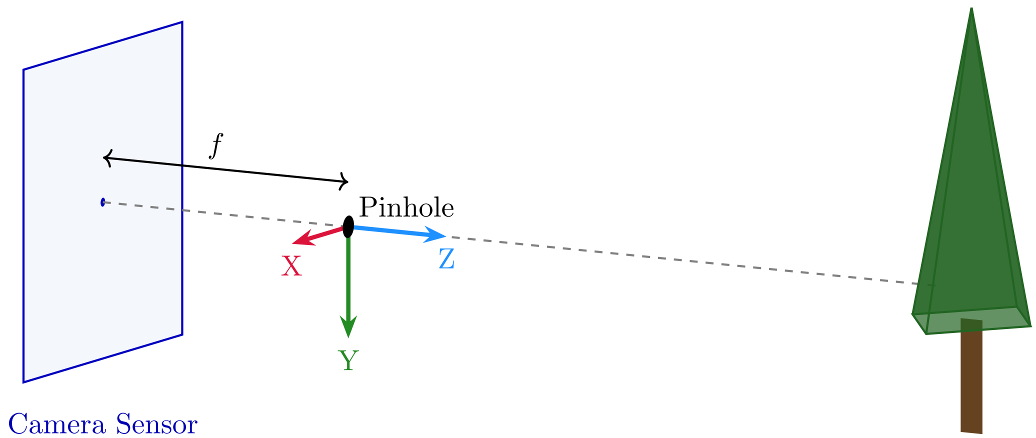
Topics

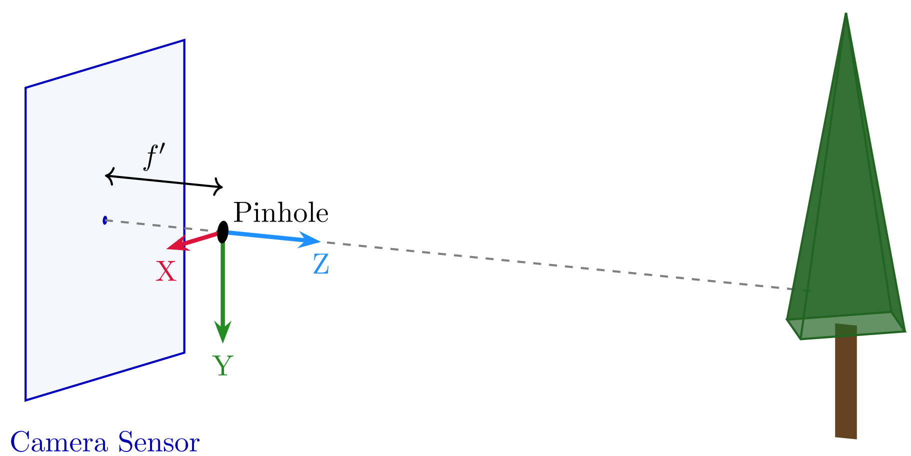
This section covers the pinhole camera model.

1 Warmup

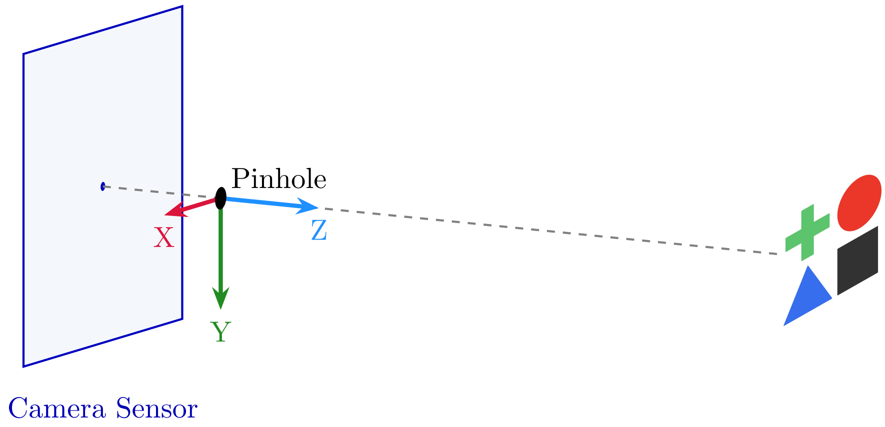
Problem 1.1: Chung Min points a pinhole camera with focal length f directly at a beautiful ponderosa pine tree. How is this object projected onto the image? Draw on the sensor below.



Problem 1.2: Unhappy with the original composition, Chung Min shortens the focal length of the camera. How does the image change? Draw on the sensor below.



Problem 1.3: Let's try on a harder object. Draw the image as it appears on the sensor below.



Problem 1.4: Assume the object has a height of H, and is located distance d away from the camera. Given focal length f, how tall is the projected object on the image plane?

$\frac{fH}{d}$

2 Dolly Zoom

In Project 0, we saw the dolly zoom effect, where multiple camera parameters are adjusted to keep a subject the same size in the image.

Problem 2.1: ***Project 0 recap.*** Between Problem 1.1 and 1.2, Chung Min decided to shorten the camera's focal length. In words, what else needs to be done to achieve a dolly zoom effect?

Shortening focal length means zoom out. To keep subject size the same, distance needs to subject needs to be decreased.

Problem 2.2: ***How to zoom?*** A camera has initial focal length f, and is placed a distance d from a subject. If the camera is moved to distance d', what focal length f' would maintain the subject's size in the image?

This can be solved by drawing similar triangles, or algebraically. One option is to start from the answer to 1.4:

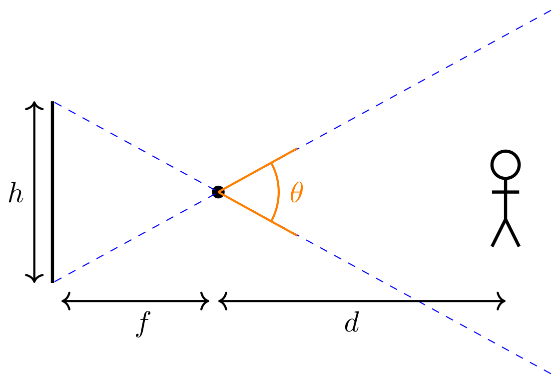
$$\frac{fH}{d} = \frac{f'H}{d'}$$

Then, to solve for f': $f' = f\frac{d'}{d}$

Problem 2.3: ***Zoom vs crop.*** Can you achieve the dolly zoom effect without changing the physical focal length of the camera system? Assume $d' > d$.

Yes, you can just crop the image.

Problem 2.4: ***Angles.*** Real-world computer vision systems often have to grapple with multiple conventions for camera geometry. One that we saw in lecture was field-of-view, which can be expressed in radians as θ:



Given initial field-of-view θ, initial subject distance d, updated subject distance d', how can we compute updated FOV θ' to achieve the dolly zoom effect?

Hint: you can start from the answer to 2.2, but your final answer should not depend on f or h.

Tangent relationship (you can draw a midline):

$$\tan(\theta/2) = \frac{h}{2}/f$$

Rearrange to get f and θ:

$$f = \frac{h}{2\tan(\theta/2)}$$
$$\theta = 2\arctan\left(\frac{h}{2f}\right)$$

These same relationships also hold for θ' and f'.

$$\begin{aligned}\theta' &= 2\arctan\left(\frac{h}{2f'}\right)\\ &= 2\arctan\left(\frac{h}{2}\left(f\frac{d'}{d}\right)^{-1}\right)\\ &= 2\arctan\left(\frac{h}{2}\left(\frac{h}{2\tan(\theta/2)}\frac{d'}{d}\right)^{-1}\right)\\ &= 2\arctan\left(\tan\left(\frac{\theta}{2}\right)\frac{d}{d'}\right)\end{aligned}$$