Filtering and Frequencies

Discussion #3

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Topics

This section covers filters, convolution, and frequency decomposition.

Logistics

Remember Project 2 is due Friday, 9/26 at 11:59pm!

1 Convolution

Problem 1.1: ***Basic convolution.*** Consider the following 5×5 image and 3×3 filter:

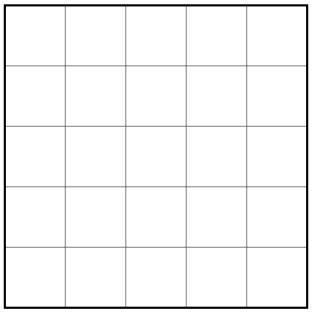
Image:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Filter:

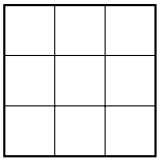
$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Compute the convolution of the image with the filter by using 0-padding. What is the resulting 5x5 output image?

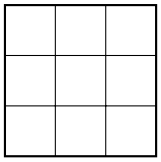


Problem 1.2: ***Feel the filter.*** Give any 3x3 example of each of the following types of image convolution filters:

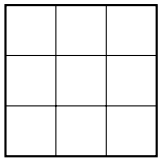
(a) Image darkening



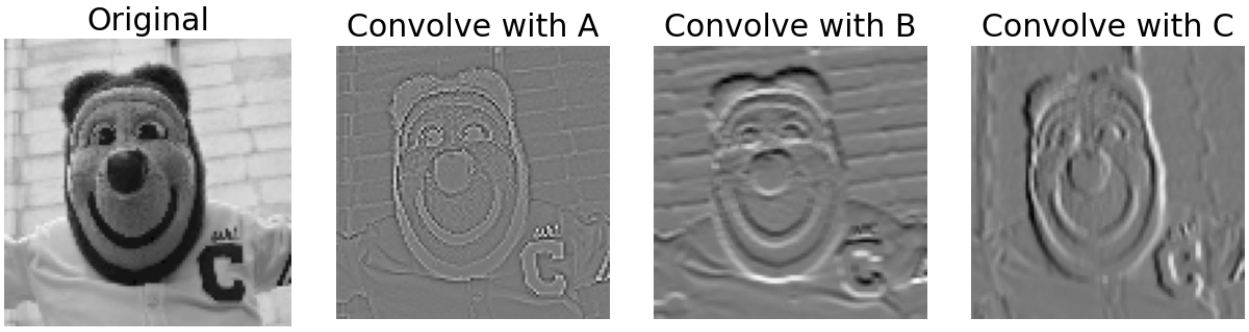
(b) 1-pixel left shift



(c) Smoothing

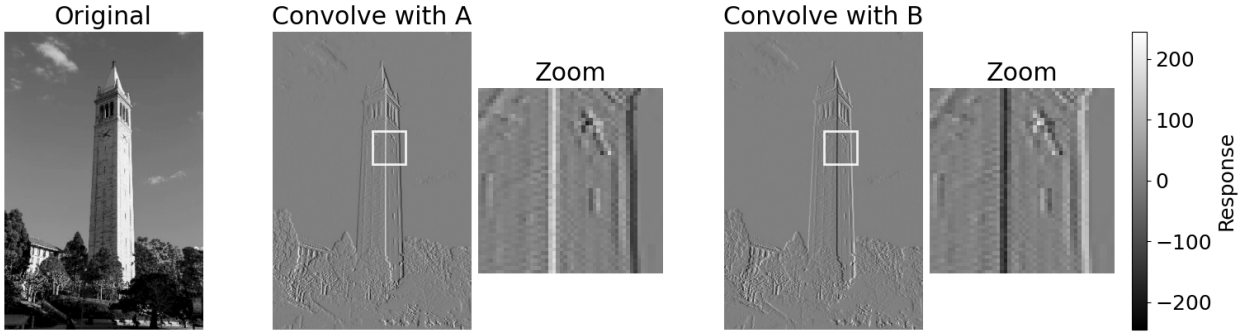


Problem 1.3: ***Up down, left right?*** Match the filters to their names.

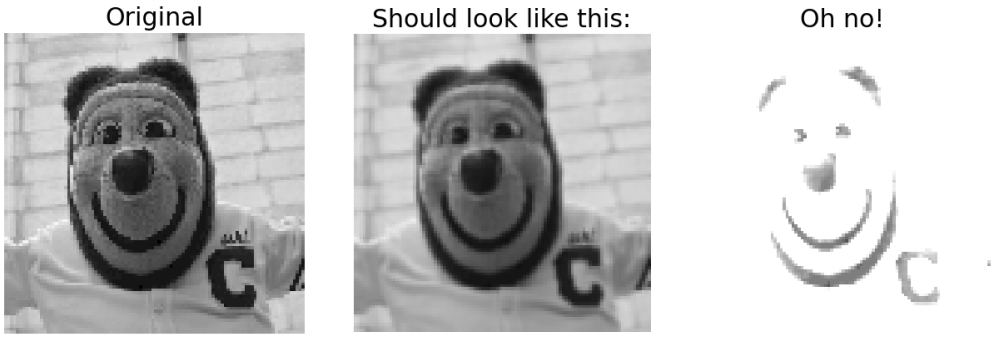


$$\underline{\qquad\qquad}: \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \qquad \underline{\qquad\qquad}: \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \qquad \underline{\qquad\qquad}: \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Problem 1.4: ***Finding direction (on campus).*** Write the 1x3 convolution filters for A and B. *Hint: one has values* $[1, 0, -1]$*, and the other has* $[-1, 0, 1]$*. Pay close attention to the colors!.*



Problem 1.5: ***Did you forget a step?*** A student made a mistake constructing an image blur, and got the resulting image. What did they forget to do?



Problem 1.6: ***Invariance***

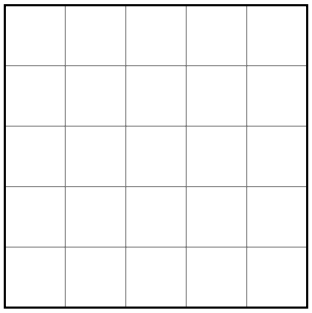
1. Is the Sobel filter invariant to constant image brightness shifts? (i.e., $+/-$ pixel values)

2. Is the Sobel filter invariant to image contrast? (recall histogram equalization from Lecture)

2 Frequency

Problem 2.1: Intuition
(a) Which has on average higher frequency? `I` or `blur(I)`?

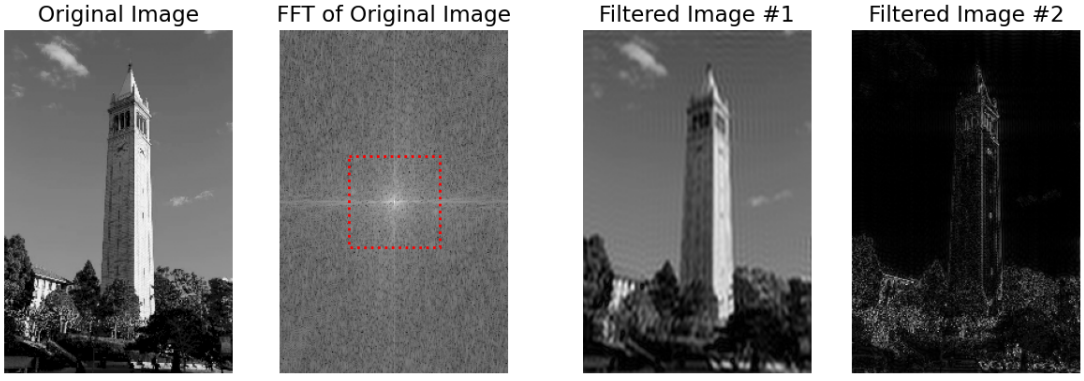
(b) Suppose I want to fill in this 5x5 image in a way that produces high frequencies in the FFT. How could I do that? (There are lots of correct answers)



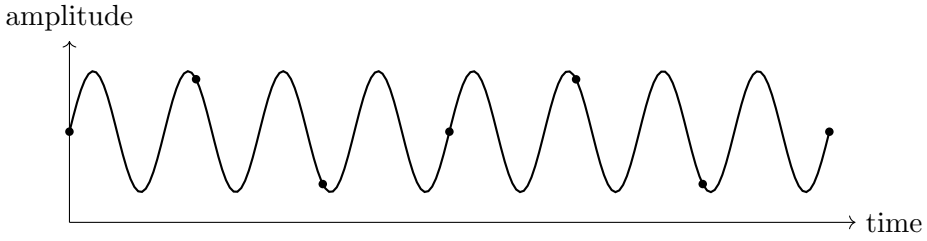
In words, what sort of image features produce high frequencies?

(c) Suppose I downsample an image by a factor of 2. What is the highest frequency that this new image can represent in terms of the previous highest frequency h_f?

(d) Below is an image and its FFT transform. Find the corresponding images if we reconstructed it using inverse FFT after (a) masking out the inside of the red box and (b) masking the outside of the red box.



Problem 2.2: ***Aliasing*** Consider this 1D signal sampled at regular intervals (black dots show sample points):



What lower-frequency signal would you reconstruct if you connected these sample points? Sketch it below:

Why might this be a problem when constructing image pyramids?