CS 180 Discussion #4

2D Image Transformations

Pixels vs. Coordinates
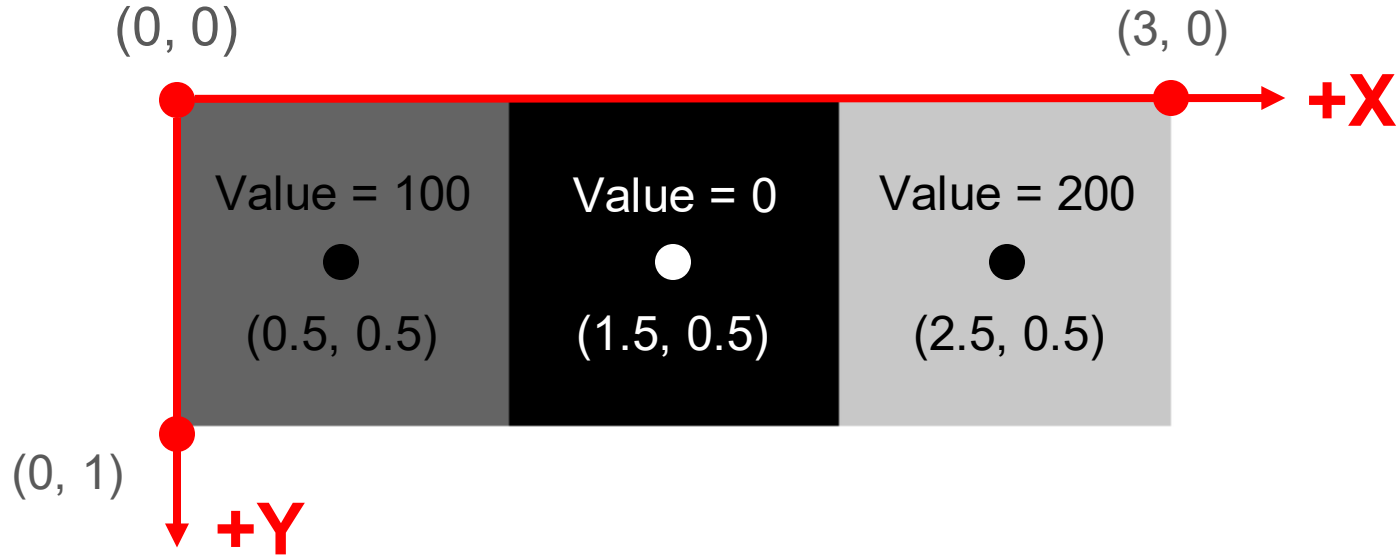
Image = Discrete

XY Coord. = Continuous
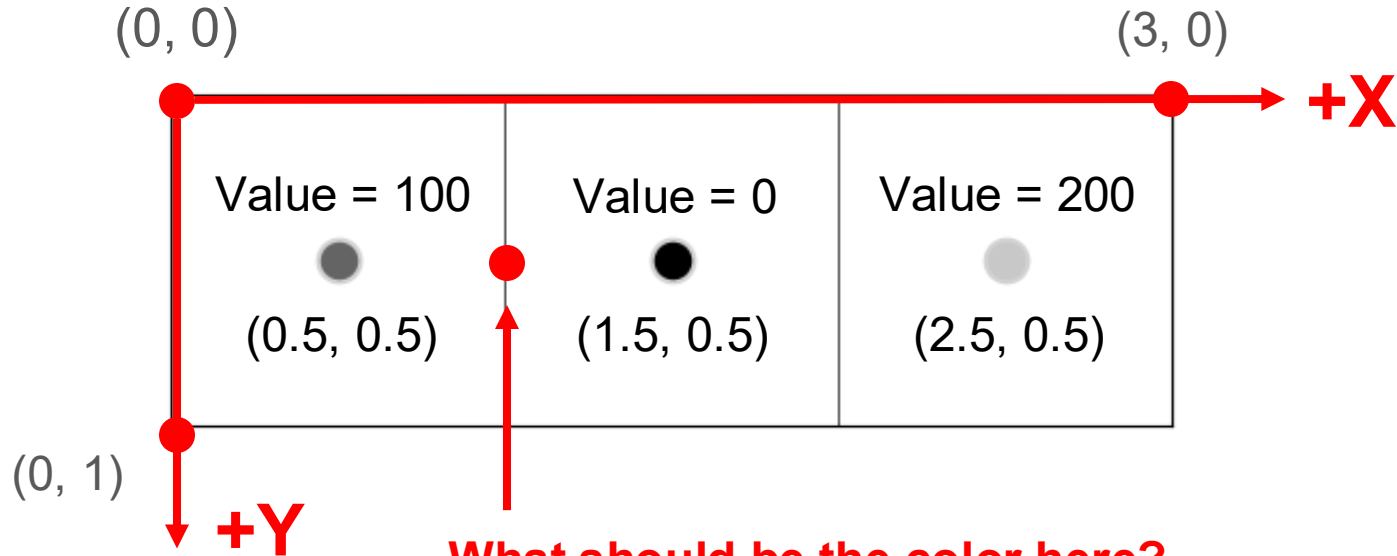
(0, 0)

+X

(3, 0)

(0.5, 0.5)

Pixel is at the middle of the grid

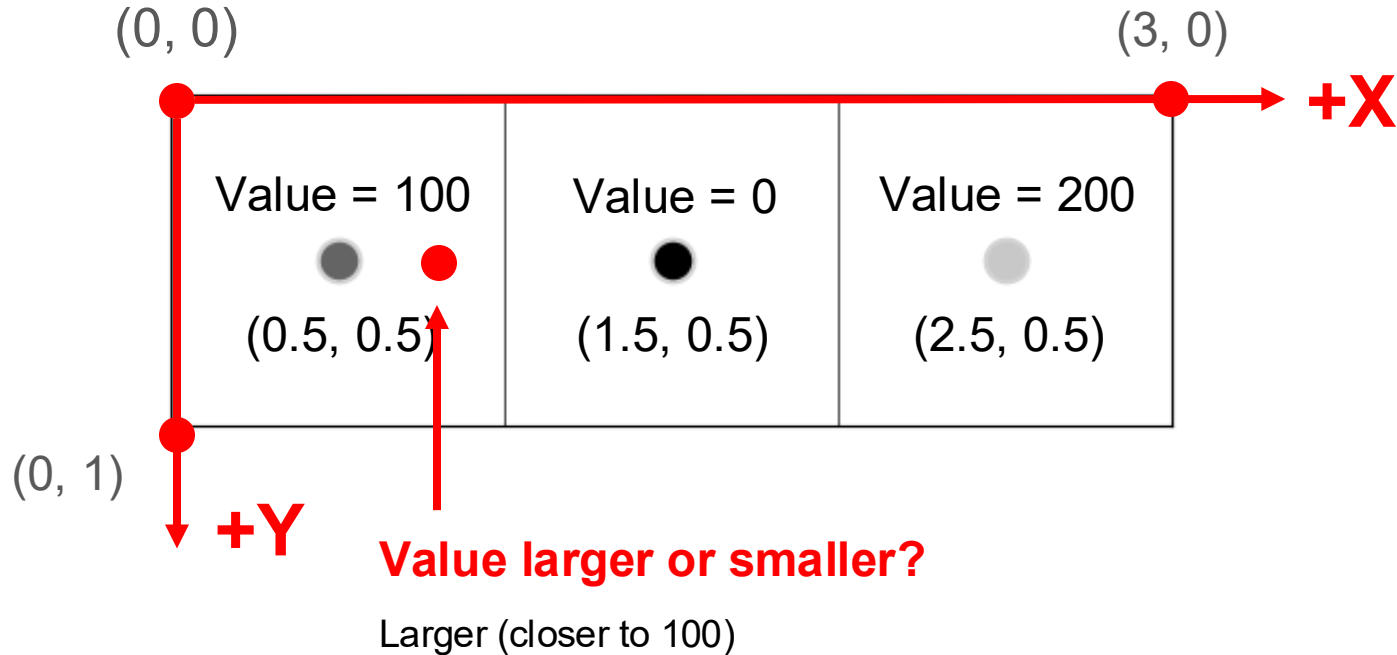(x = col + 0.5, y = row + 0.5)

+Y

(0, 4)
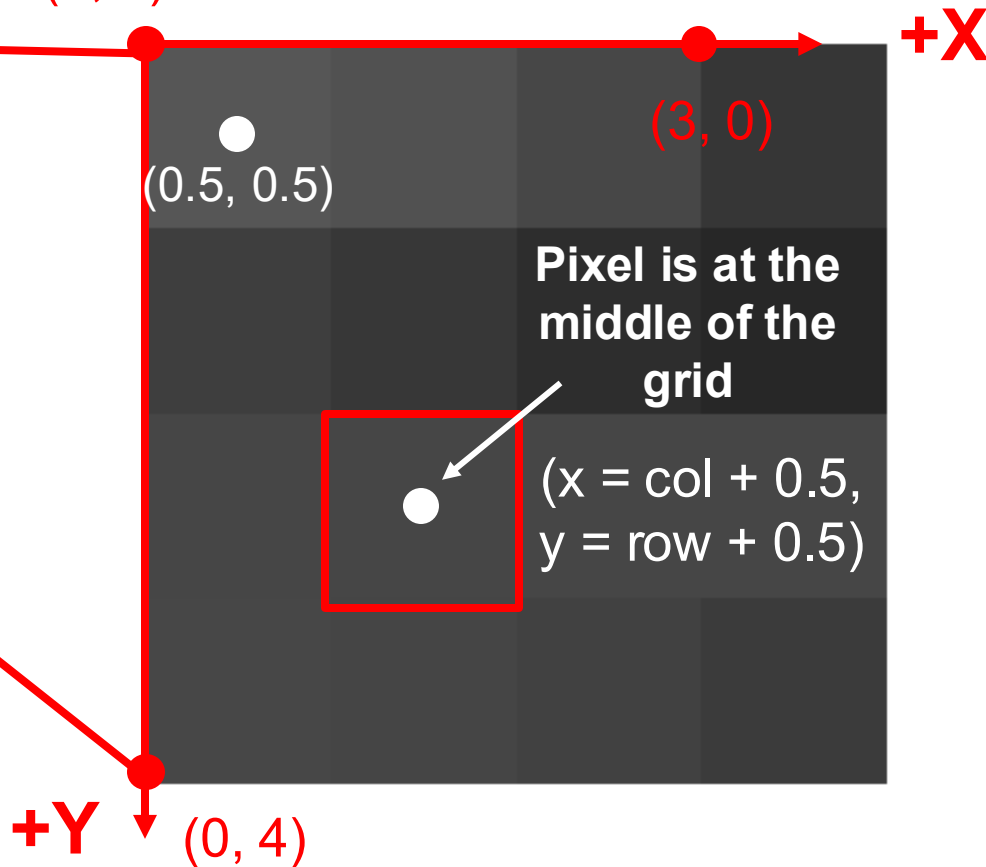
*Image array is (row, col)!

What is the color between pixels?

What is the color between pixels?

Value = 50 (mid way between 0 and 100)

What is the color between pixels?

(0, 0)

(3, 0)

+X

Value = 100

(0.5, 0.5)

Value = 0

(1.5, 0.5)

Value = 200

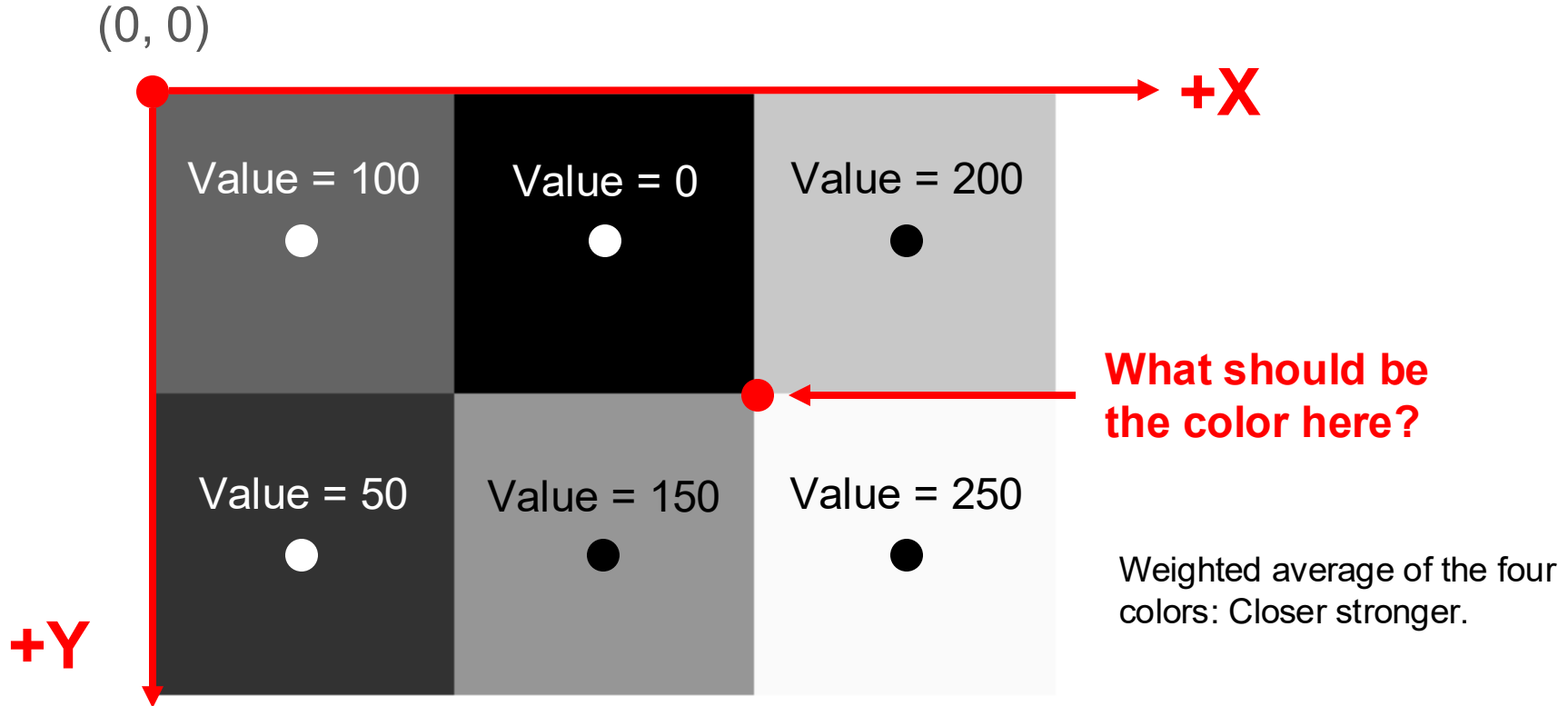(2.5, 0.5)

(0, 1)

+Y

Value larger or smaller?

Larger (closer to 100)

Linear interpolation is like we have this...
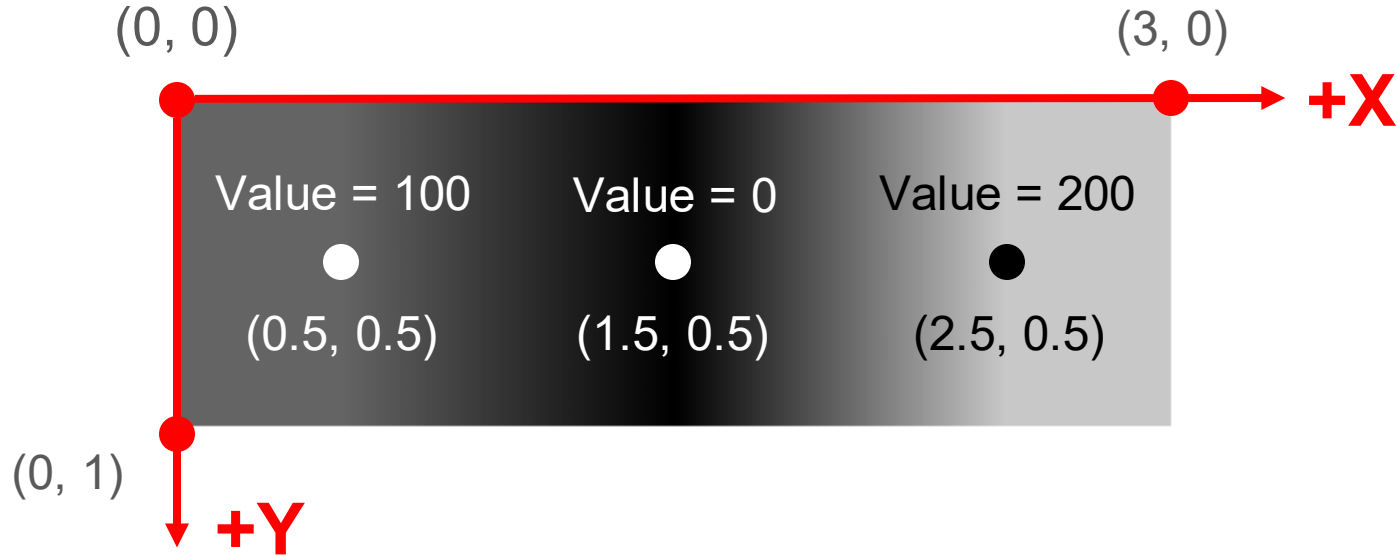
Wow, we have infinite resolution!

What is the color between pixels? 2D

(0, 0)

+X

Value = 100

Value = 0

Value = 200

What should be the color here?

Value = 50

Value = 150

Value = 250

+Y

Weighted average of the four colors: Closer stronger.

Bilinear interpolation looks like this...

(0, 0)

+X

Value = 100   Value = 0   Value = 200

Value = 50   Value = 150   Value = 250

+Y
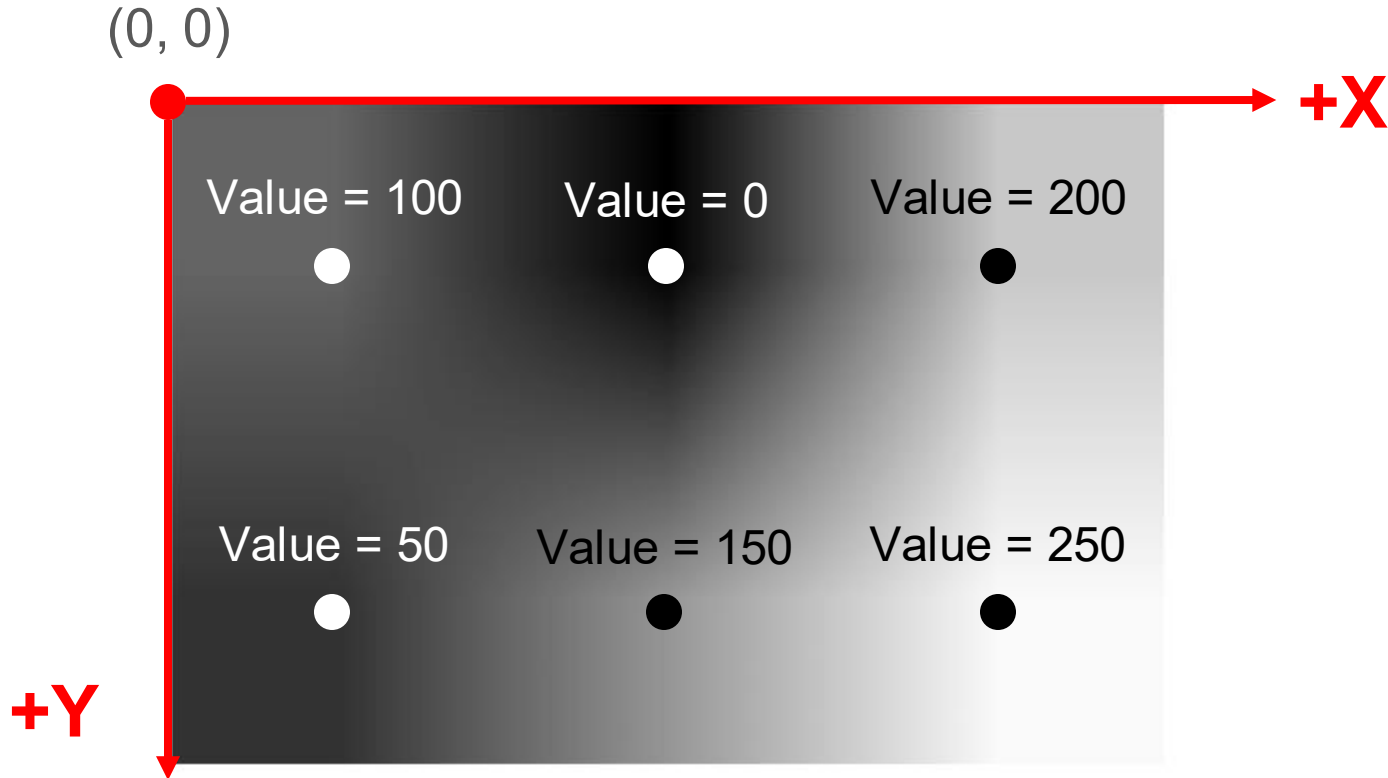
*There are other kinds of interpolations

Filters vs. Transformations
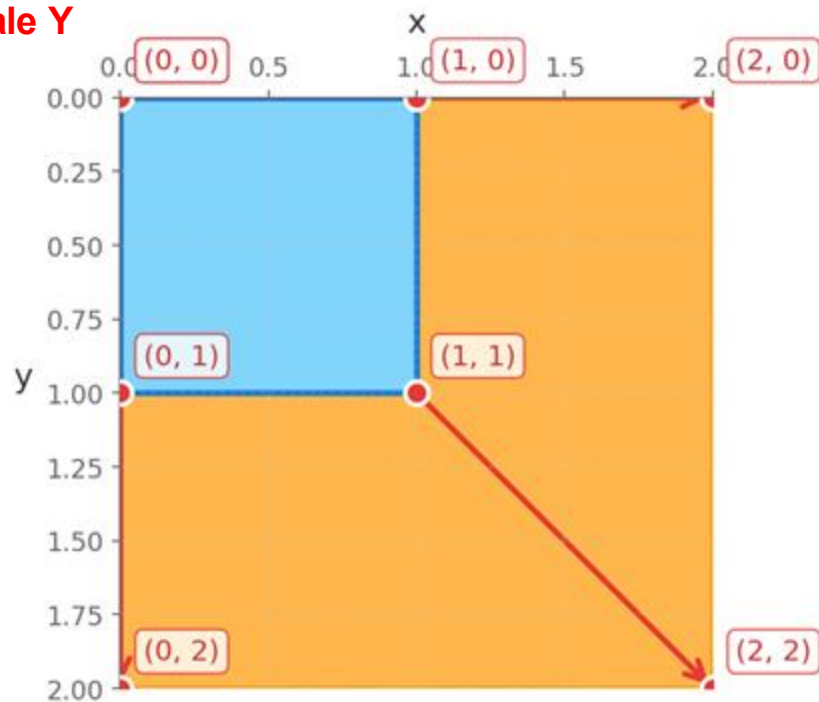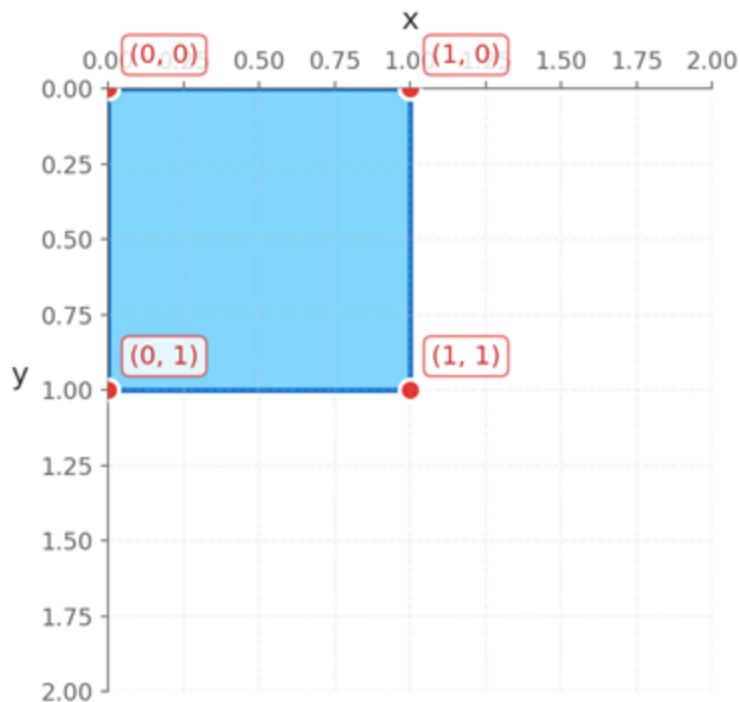
Filter:
Change values..

Darken
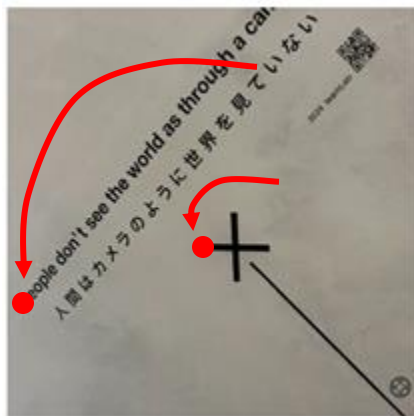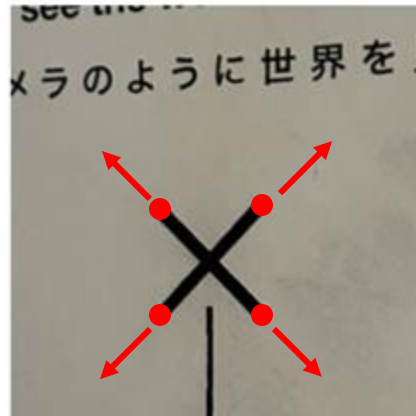
Transform:
Change coordinates!

Rotation

Zoom

Scaling

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Scale X

Scale Y

Translation (needs special treatment)

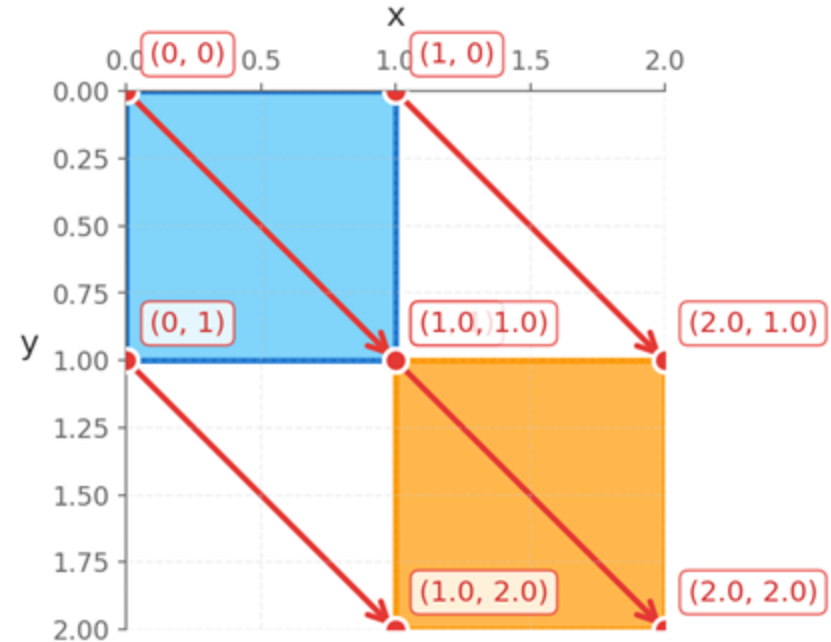$$\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{\text{homogeneous coords}} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Translate X

$$\begin{pmatrix} x' \\ y' \\ \boxed{1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \boxed{t_x} \\ 0 & 1 & \boxed{t_y} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Delete

Translate Y

*Translation is NOT a linear operation.

Other transformations...

Shearing

Rotation 30 deg + Translation
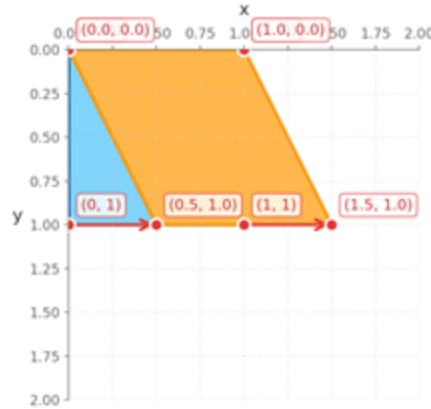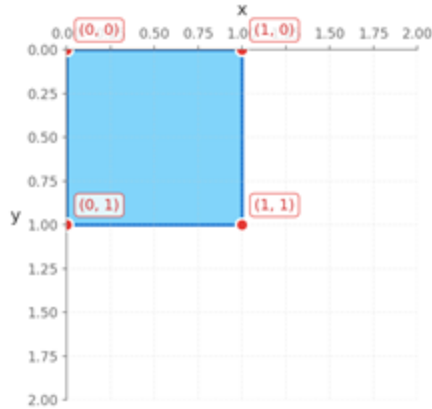
Shear X

Shear Y

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0.5 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Rotate not at (0, 0)

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos 30° & -\sin 30° & t_x \\ \sin 30° & \cos 30° & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}.$$

Walkthrough image (2x, 2x) upscaling...

Input

Output

3 x 3

6 x 6

Step 1: What's the transformation?

Input

Output

Scaling up 2x
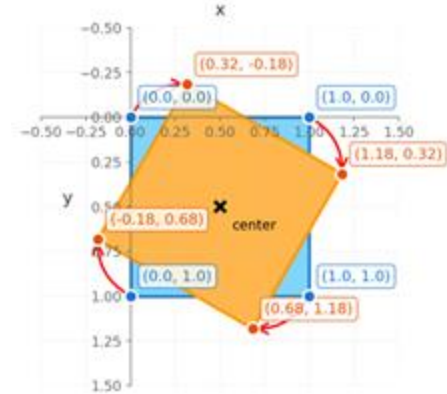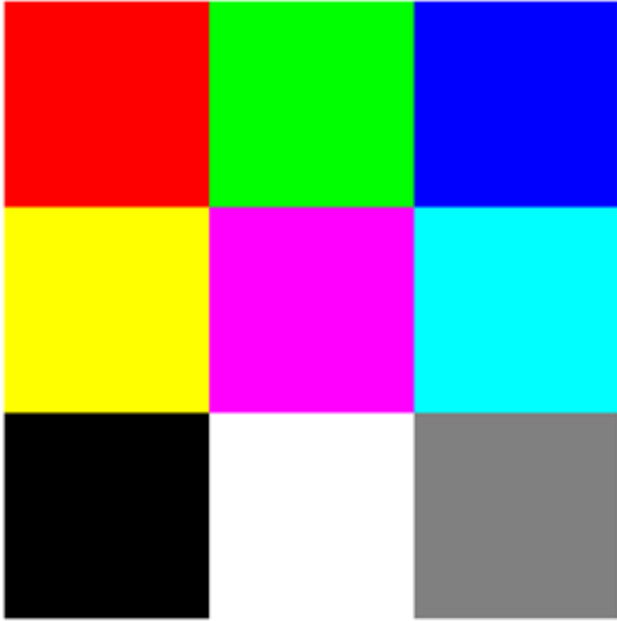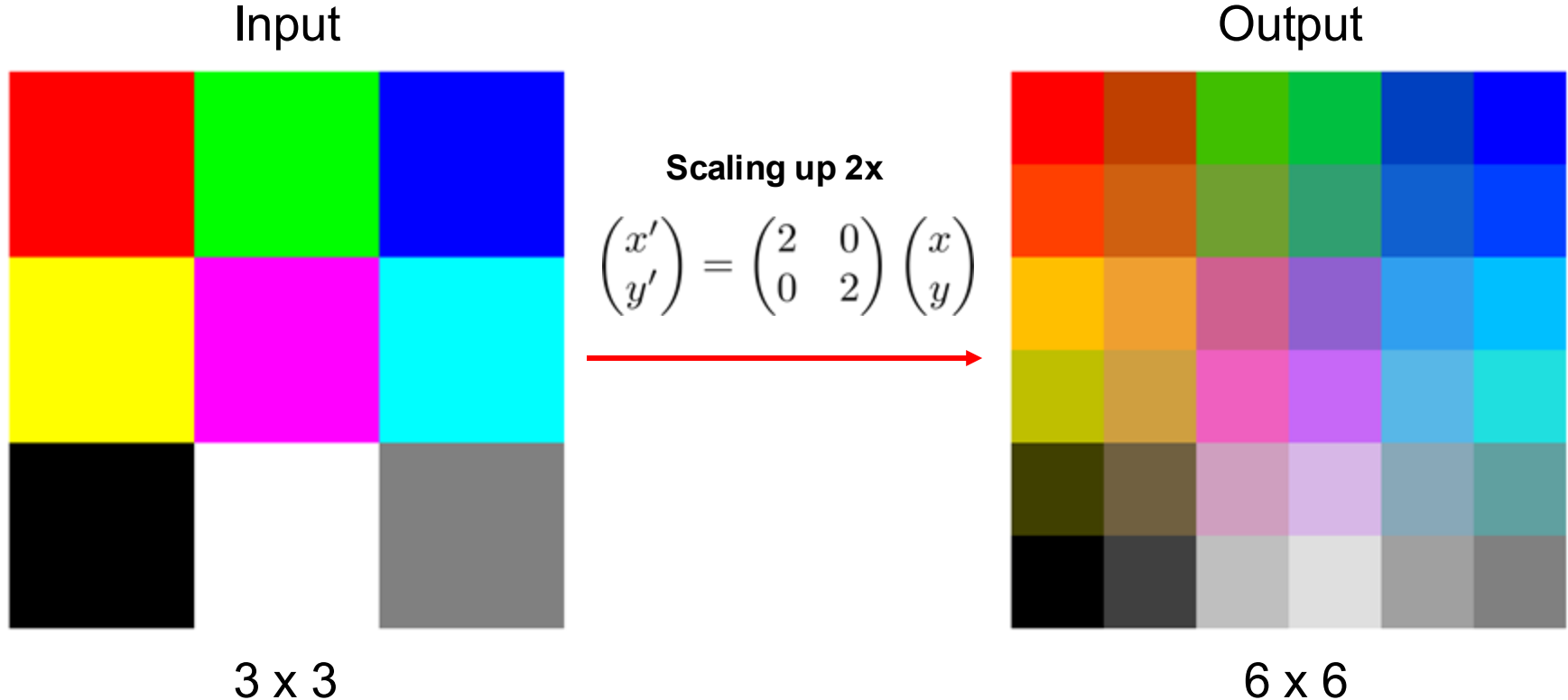
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

3 x 3

6 x 6

Step 2: Think of the image as a grid of dots...

Input

Grid of dots

(0,0)

(0.5, 0.5)   (1.5, 0.5)

(0.5, 1.5)

3 x 3

3 x 3
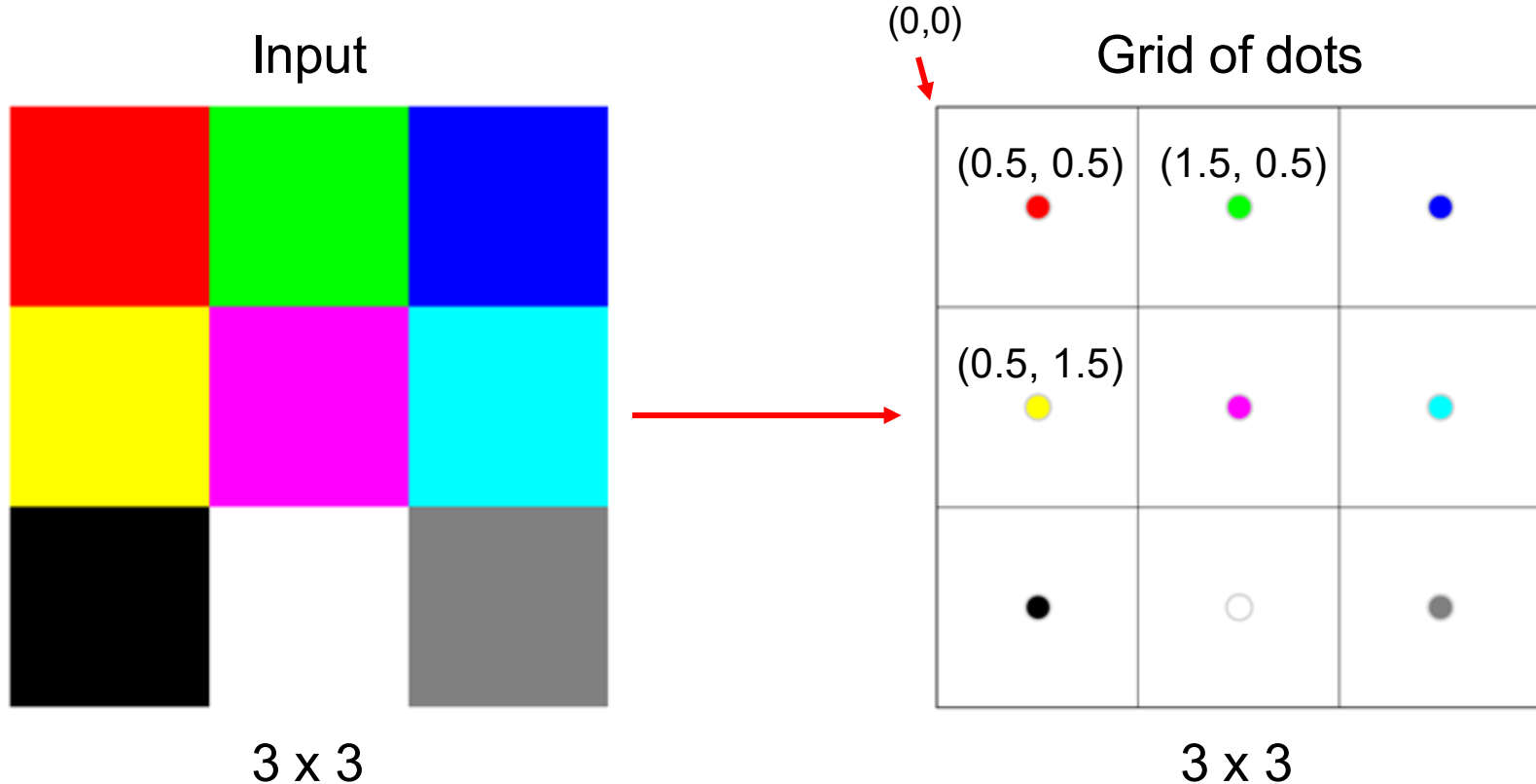
Step 3: Prepare a larger blank "canvas" for output

(0,0)

Grid of dots

(0.5, 0.5) (1.5, 0.5)

(0.5, 1.5)

3 x 3

Canvas

(0, 0) (1.5, 0.5)

(0.5, 0.5)

(0.5, 1.5)

6 x 6
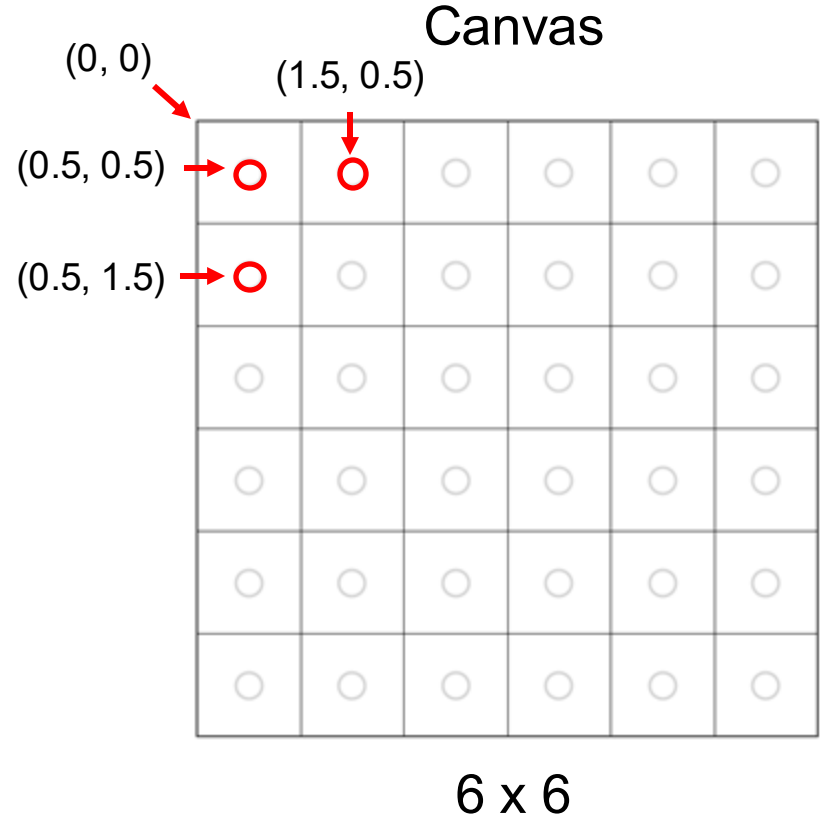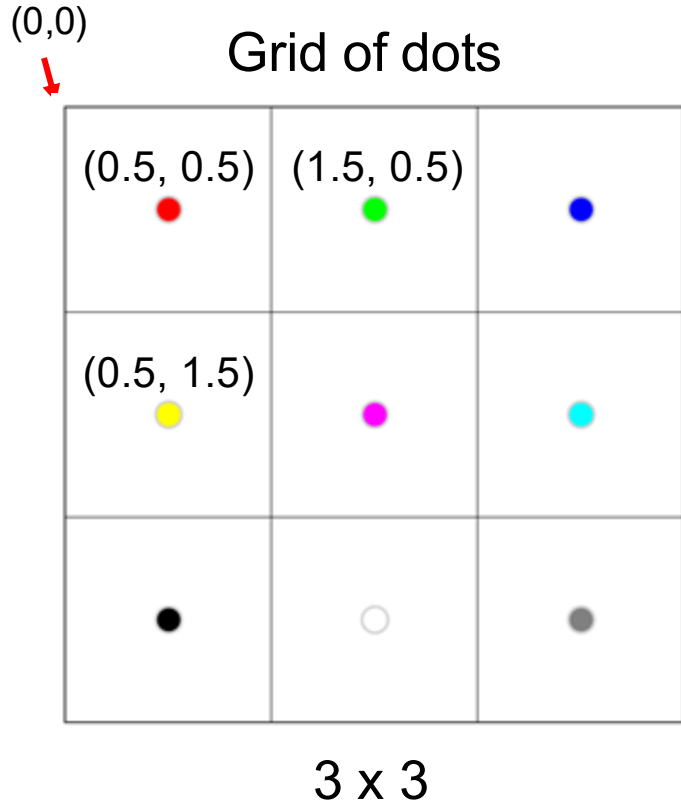
Step 4: Find the inverse transformation
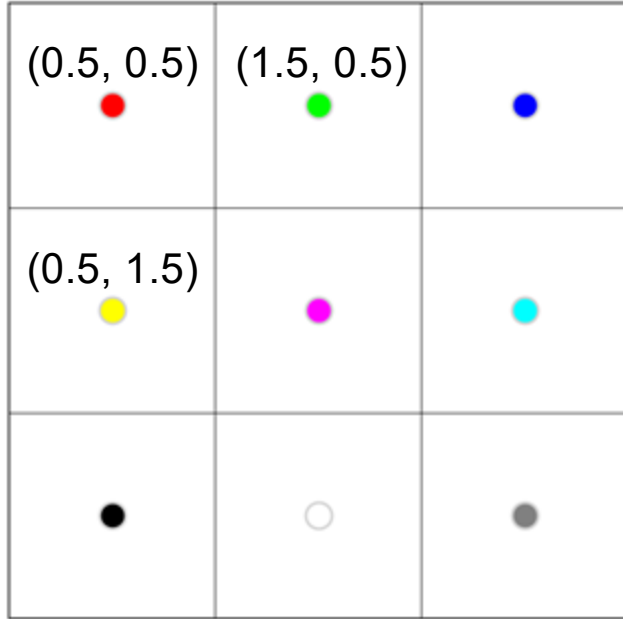
Grid of dots

(0,0)

(0.5, 0.5) (1.5, 0.5)

(0.5, 1.5)

3 x 3

Canvas

(0, 0) (1.5, 0.5)

(0.5, 0.5)

(0.5, 1.5)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

6 x 6

Step 5: Transforms to source

Grid of dots

(0.25, 0.25) (0.75, 0.25)

(0, 0) (1.5, 0.5)

Canvas

(0.5, 0.5)

(0.5, 1.5)

(0.25, 0.75)
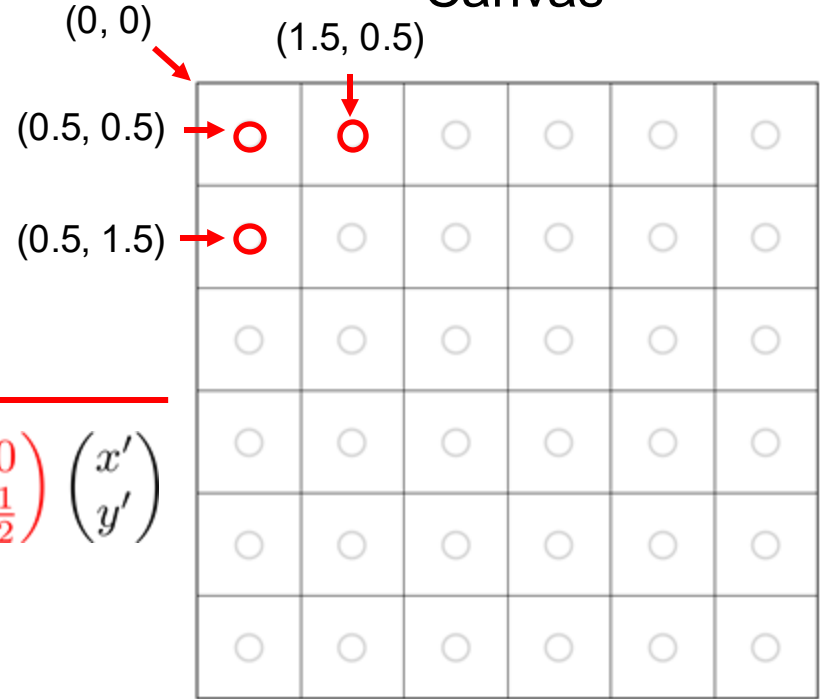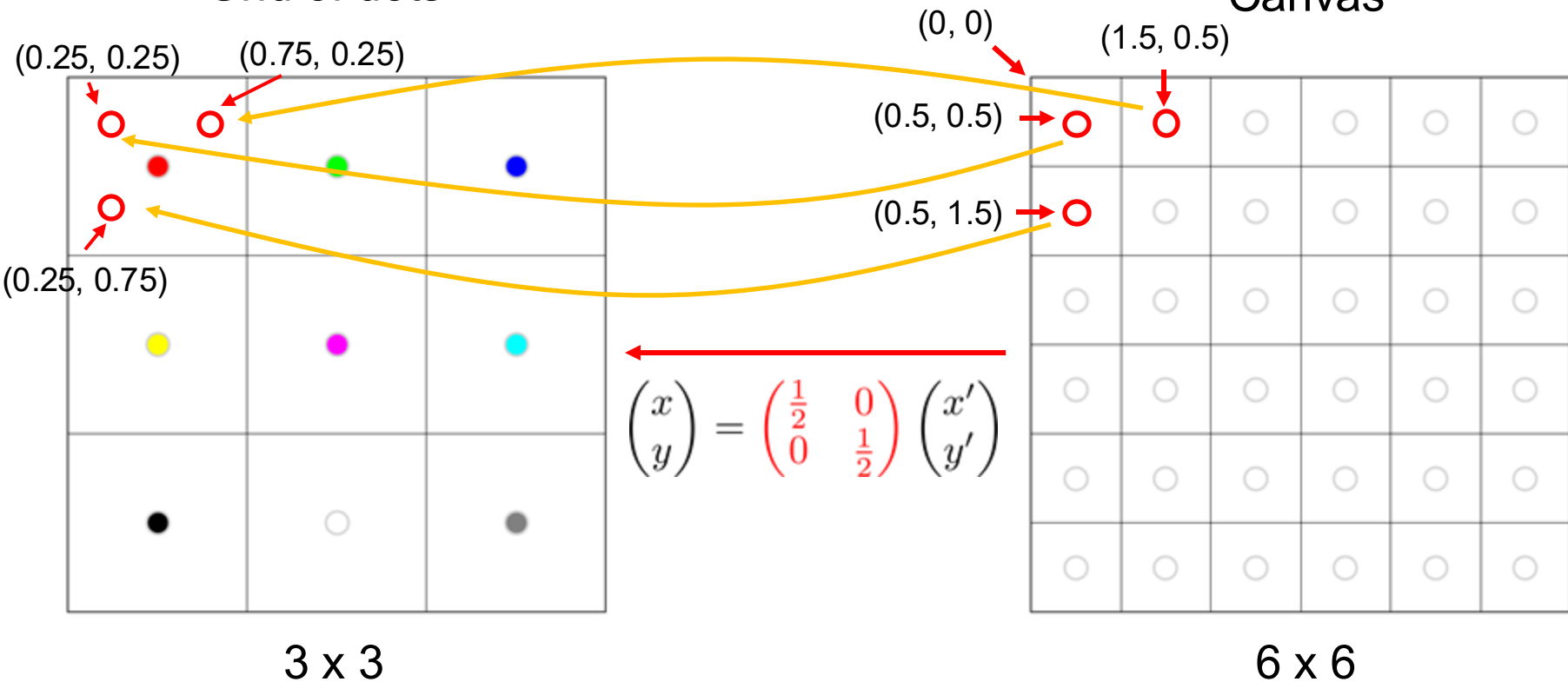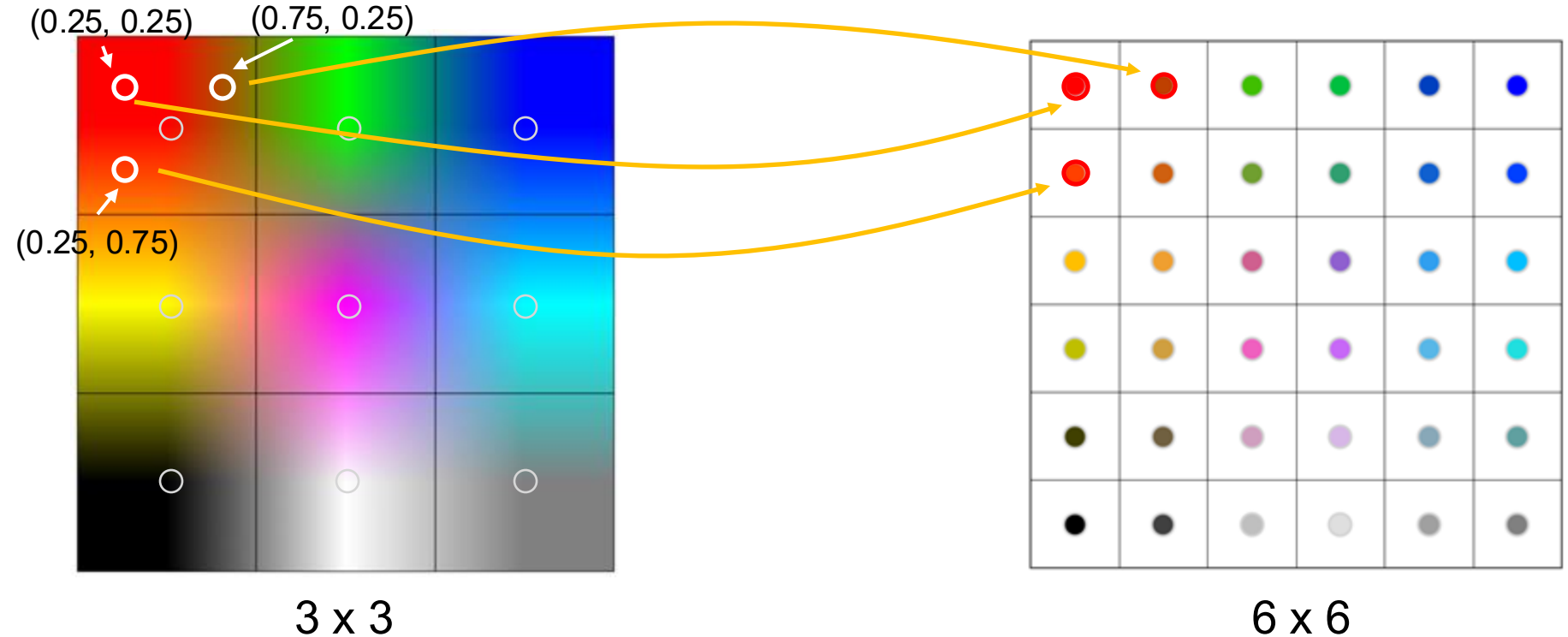
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

3 x 3

6 x 6

Step 6: Get the "interpolated" colors from the source

Grid of dots

Canvas

(0.25, 0.25) (0.75, 0.25)

(0.25, 0.75)

3 x 3

6 x 6

Done!

Output

=

Canvas

6 x 6

Batch the transformation

Pack the pixel coordinates in [2 x N]

Batch of points

$$\begin{pmatrix} x'_1 & x'_2 & \cdots & x'_n \\ y'_1 & y'_2 & \cdots & y'_n \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \end{pmatrix}$$

Let's work on the sheets!