3D Coordinates and Epipolar Geometry

Discussion #8

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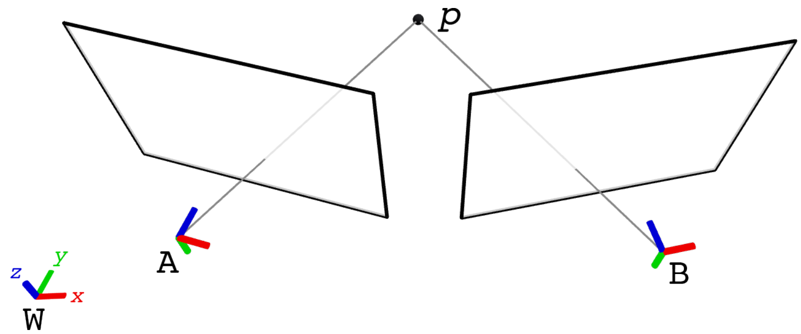
This worksheet reviews 3D transformations and the basics of epipolar geometry.

1 Rigid Transforms

Let's consider a point p and three coordinate frames: the world W, camera A, and camera B:



Notation. We'll denote the coordinates of p in the world frame $p_{\mathrm{W}} \in \mathbb{R}^3$. The extrinsics[1] of camera A and B can be written as T_{WA} and T_{WB} respectively, where

$$\begin{bmatrix} p_{\mathrm{W}} \\ 1 \end{bmatrix} = T_{\mathrm{WA}} \begin{bmatrix} p_{\mathrm{A}} \\ 1 \end{bmatrix}.$$

(a) The same point p can be written as either p_{W}, p_{A}, and p_{B}. In words, what is the difference between these three vectors?

(b) Express p_{W} in terms of p_{A} and T_{WA}.

[1]Following the "world-from-camera" / "camera-to-world" convention.

(c) Express p_{B} in terms of p_{W} and T_{WB}.

(d) How would you compute a transform that converts coordinates relative to camera B from camera A, in terms of T_{WA} and T_{WB}?

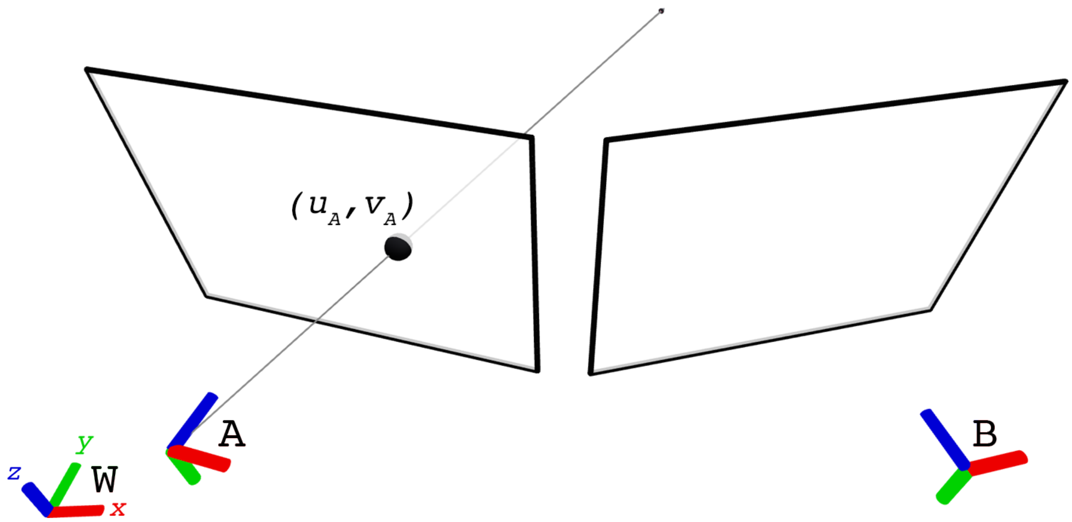
(e) Consider the case where camera A has intrinsics $K \in \mathbb{R}^{3\times 3}$. Given p_{W}, K, and T_{WA}, what are the image-space coordinates (u, v) of p viewed from camera A? You can leave your solution in homogeneous coordinates.

(f) Let $T \in \mathrm{SE}(3)$ be composed of rotation $R \in SO(3)$ and $t \in \mathbb{R}^3$. We know that rotation matrices are orthonormal; $R^{-1} = R^\top$. Use this fact to derive an expression for the inverse T^{-1} without using matrix inversion.

2 Rays and Epipolar Geometry

3D computer vision algorithms typically begin with 2D observations, like coordinates extracted from corner and feature detectors.

We once again have cameras A and B, located in the world frame W. Consider the case where we observe a feature in the image from camera A, located at 2D coordinates (u_A, v_A):



The pixel at (u_A, v_A) corresponds to the projection of an unknown 3D point in the scene, p.

(a) Annotate the figure above with 3 possible locations for p.

(b) Annotate the image plane of camera B with the projection of your points from part (a). What pattern do these projected 2D points follow?

(c) Given (u_A, v_A) and intrinsics $K \in \mathbb{R}^{3\times3}$, write an expression for all possible values of $p_A \in \mathbb{R}^3$.

(d) Given $(u_{\text{A}}, v_{\text{A}})$, $K \in \mathbb{R}^{3\times3}$, T_{WA}, and T_{WB}, what are all possible coordinates for $p_{\text{B}} \in \mathbb{R}^3$?

(e) Given the same inputs as part (d), how would you find all possible values of the point projected onto the image of camera B, $(u_{\text{B}}, v_{\text{B}})$? Describe an approach in words.