

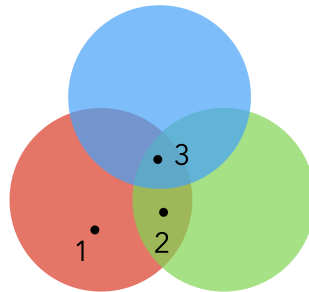
# NeRF Fundamentals

Discussion #9

Written by:  
Justin, Chung Min

## 1 Alpha Compositing

Suppose we're trying to overlay 3 colored R G B circles with translucent opacity on a white background. The formula for overlaying (compositing) color  $c_a$  with opacity (alpha)  $\alpha_a$  over an opaque  $c_b$  is:  $c_{new} = c_a \cdot \alpha_a + c_b \cdot (1 - \alpha_a)$ .



**1.1: Enhance!** Expand the alpha compositing formula to express the output pixel values at points 1, 2, and 3. Don't forget the background! Also don't worry about simplifying, just expand.

Point 1:  $\alpha_r \cdot C_r + (1 - \alpha_r) \cdot C_w$

Point 2:  $\alpha_g \cdot C_g + (1 - \alpha_g) \cdot (\alpha_r \cdot C_r + (1 - \alpha_r) \cdot C_w)$

Point 3:  $\alpha_b \cdot C_b + (1 - \alpha_b) \cdot (\alpha_g \cdot C_g + (1 - \alpha_g) \cdot (\alpha_r \cdot C_r + (1 - \alpha_r) \cdot C_w))$

**1.2: Simplifying** Write your answer for point 3 with 4 terms for r, g, b, and white.

$$C_3 = \alpha_b \cdot C_b + \underline{\hspace{2cm}} \alpha_g \cdot C_g + \underline{\hspace{2cm}} \alpha_r \cdot C_r + \underline{\hspace{2cm}} C_w$$

$$C_3 = \alpha_b \cdot C_b + (1 - \alpha_b) \cdot \alpha_g \cdot C_g + (1 - \alpha_b)(1 - \alpha_g) \cdot \alpha_r \cdot C_r + (1 - \alpha_b)(1 - \alpha_g)(1 - \alpha_r) \cdot C_w$$

**1.3: Transmittance!** How does the form of this compositing formula look similar to transmittance from a NeRF?  $T(n) = \prod_{k=1}^{n-1} (1 - \alpha_k)$ .

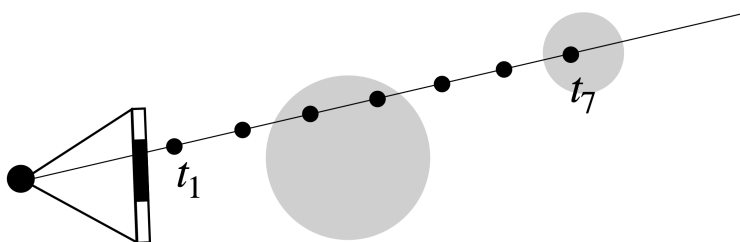
Transmittance accounts for the underlined terms! It describes how much light “leaks through” from that layer given the opacities of the layers above it.

## Flatland NeRF

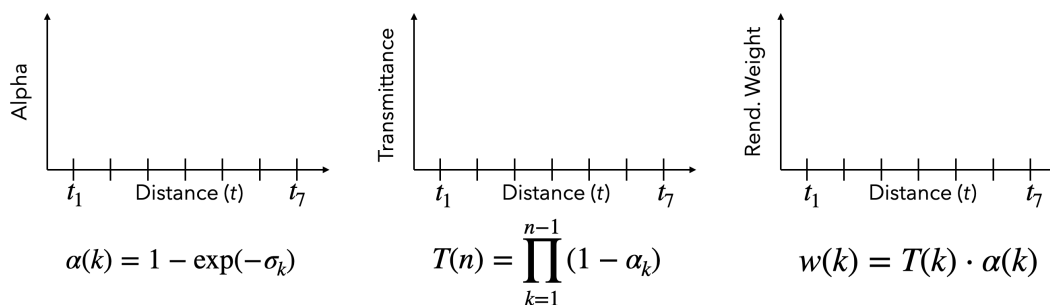
In this sheet, we consider a simplified NeRF in a flat plane. In this setup, **1)** Each camera has a 1D sensor array which measures color. **2)** All poses and rays exist only in 2 dimensions. **3)** The background is known to be white. **4)** Only output color  $c$  and density  $\sigma$  per-point (we ignore view direction).

## 2 Volume Rendering

Density in NeRF represents where geometry lies in a scene. During NeRF rendering there are three quantities we derive from density to render color from a ray: **alpha** (opacity), **transmittance**, and **rendering weights**. For simplification, the equations below *omit ray sample spacing*  $\delta_k$  as it is constant.



**2.1: Rendering a cloud** In the plots below, plot the values for the scenario above assuming the circles have low density values  $\sigma = \ln 2$  ( $\alpha \approx 0.5$ ).



For alpha, points 3,4,7 have 0.5 alpha.

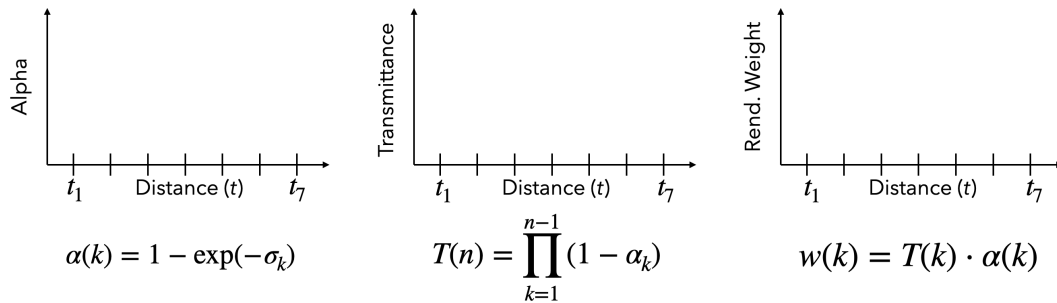
For transmittance, cumulatively multiply the alpha values from left to right.

For rendering weight, multiply the transmittance by the alpha values element-wise.

**2.2: Rendering a solid** In the plots below, plot the values for the scenario above assuming the circles have very high density values  $\sigma = 1000$ .

For alpha, points 3,4,7 have near 1.0 alpha.

For transmittance, cumulatively multiply the alpha values from left to right. (So here it will



decay from 1 to 0 after the 3rd point)

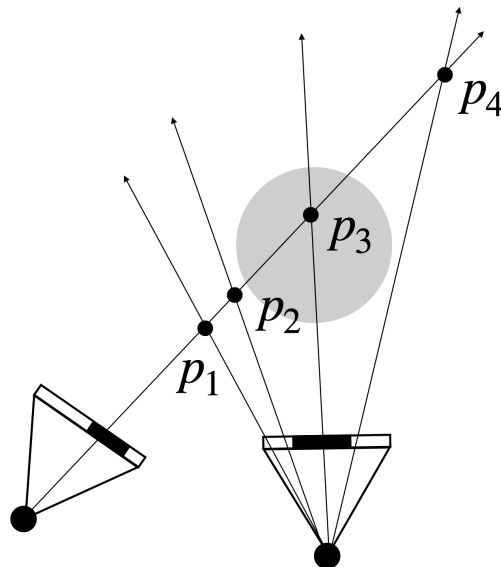
For rendering weight, multiply the transmittance by the alpha values element-wise.

**2.3: Analysis** Explain which quantity accounts for occlusion in 3D, and how it accomplishes this.

Transmittance accounts for occlusion by blocking out light from behind highly dense points.

### 3 Multiview in Flatland

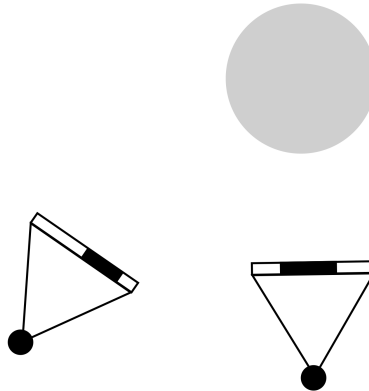
The diagram below shows 2 cameras, each observing a circle in 2D, which projects to a black stripe in their images. On the right are illustrated 4 points in 2D along with their projections into the two cameras.



**3.1: 4-point NeRF.** Based on these 4 points, come up with an assignment of density and color to each point  $p_i$  which satisfies the input images.

$p_1, p_2, p_4$  have 0 density, and  $p_3$  has very high density with color black.

**3.2: Where's the density?** Extending the logic from 3.1, indicate in the diagram below *all possible* region over which the density in the scene could lie. (Hint: It's larger than the circle.)



Draw 2 rays from each camera to the sides of the circle, the polygon where they intersect includes all density in the scene.

**3.3: Adding a new view** Based on the existing 2 views, where should I take a new image to give me the most information about where density lives in the scene? Draw the new view and sketch the new region of density.

Add a camera roughly orthogonal to these views, eg 90 degrees towards the left or to the right.

