2D Transformations and Warping

Discussion #4

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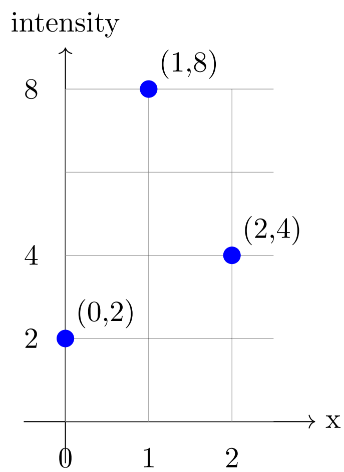
Topics

This section covers 2D transformations, interpolation, and image warping.

1 Interpolation

Problem 1.1: ***1D Linear Interpolation.***
Consider a 1D signal with 3 sample points at locations 0, 1, and 2 with values 2, 8, and 4 respectively:



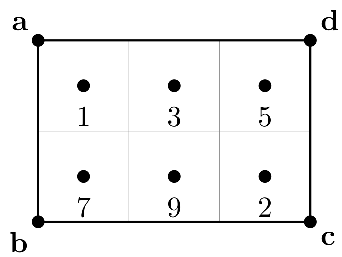
The signal is sampled at the following x locations using linear interpolation. What should the interpolated values be?

Location 0.0: __________

Location 0.5: __________

Location 1.75: __________

Problem 1.2: ***2D Bilinear Interpolation.*** Consider a 2×3 grayscale image (2 rows, 3 columns) with the following pixel values (The numbers (1, 3, 5, 7, 9, 2) represent the pixel values at their respective centers):



Part (a): What are the (x, y) coordinates for points a, b, c, and d?

Part (b): The image is sampled at the following locations. What should the bilinearly interpolated values be?

Location (0.5, 0.5): ________

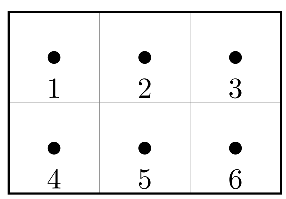
Location (0.5, 0.0): ________

Location (1.5, 1.0): ________

2 Warping

Problem 2.1: ***Reverse Mapping with Inverse Transform***

We have a 2×3 source image with the following pixel values:



We want to apply transformation $T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (scale by 2).

Part (a): Find the inverse transformation T^{-1}.

$$T^{-1} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

Part (b): For output pixel (3,1), use T^{-1} to find which source pixel to sample from with nearest neighbors interpolation.

$$T^{-1} \begin{bmatrix} 3.5 \\ 1.5 \end{bmatrix} = \underline{\hspace{3cm}}$$

Source coordinates: ________

Pixel value: ________

Part (c): For output pixel (2,2), use T^{-1} to find which source pixel(s) to sample from with bilinear interpolation.

$$T^{-1}\begin{bmatrix}2.5\\2.5\end{bmatrix} = \underline{\hspace{4cm}}$$

Source coordinates: ________

Part (d): For (c), show how to use bilinear interpolation to get the pixel value.

Which 4 source pixels are involved? ______________

What are their values? ______________

Interpolated value: ________

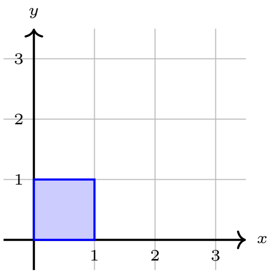
Part (e): Why do we need the inverse transformation for reverse mapping?

3 Transformation Types

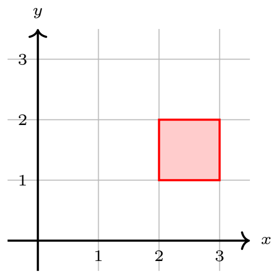
Problem 3.1: ***Match that transformation***

Consider the original square with vertices at (0,0), (1,0), (1,1), and (0,1). Below are three transformations applied to this square:

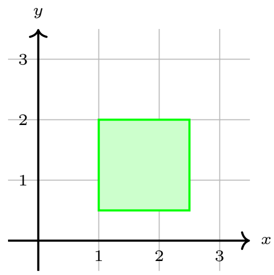
Original



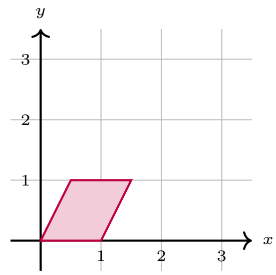
Transform A



Transform B



Transform C



Match each transformation above with its corresponding transformation matrix:

Matrix I: $\begin{bmatrix}1.5 & 0 & 1\\0 & 1.5 & 0.5\\0 & 0 & 1\end{bmatrix}$ matches transformation ____

Matrix II: $\begin{bmatrix}1 & 0.5 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ matches transformation ____

Matrix III: $\begin{bmatrix}1 & 0 & 2\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$ matches transformation ____

Problem 3.2: ***Homogeneous coordinates***

Suppose we want to translate an image by (2, 3).

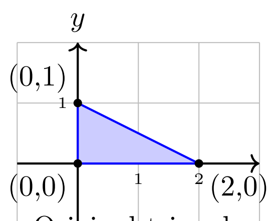
Part (a): Why can't we represent translation $(x, y) \to (x + 2, y + 3)$ as a 2×2 matrix?

Part (b): Write the 3×3 homogeneous matrix for translating by (2, 3).

$$T = \begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix}$$

Part (c): Consider the triangular object with vertices at (0,0), (2,0), and (0,1). Compare what happens when we apply transformations TR versus RT, where T is translation by (3,0) and R is 90° counterclockwise rotation.

Given: $T = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



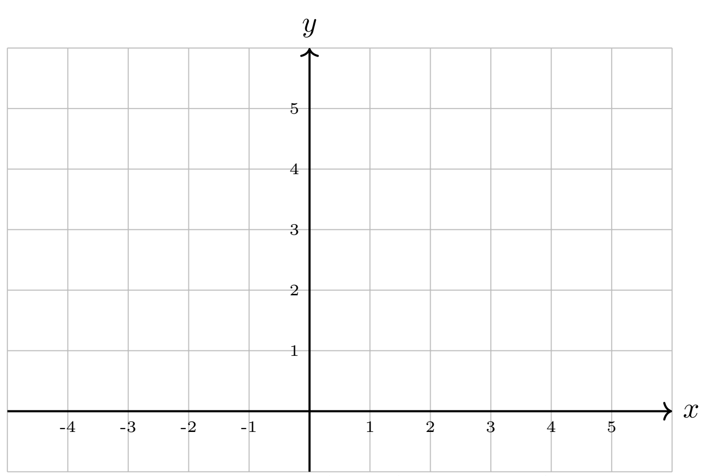
Original triangle

Calculate the composite transformations:

$TR = \begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix}$ New vertices: ____________________

$RT = \begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix}$ New vertices: ____________________

Draw both transformed triangles on the graph below:



Draw TR triangle and RT triangle